Errors

Systematic Error:

Error due to incorrectly calibrated scale.

E.g. 1 meter is mistaken as 1 cm in the ruler.

Random Error:

Error due to estimation.

Can be reduced by repeating the experiment and take the mean.

E.g. Different readings in experiments.

Combining Errors:

If possible error of measurement A is e_A (e.g. 5 ± 0.1 cm), B is e_B, then:

- Error of $A + B$ is $e_A + e_B$.
- Error of $A - B$ is $e_A + e_B$.

If possible percentage error of A is p_A (e.g. 5 mm ± 1%), B is p_B, then:

- Percentage error of A × B is $p_A + p_B$.
- Percentage error of A ÷ B is $p_A + p_B$.

Vectors

Vectors (向量) = Directed Line.

Length / Magnitude of vector is denoted by $|\vec{u}|$.

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \ldots + \overrightarrow{YZ} = \overrightarrow{AZ}$ (Polygon law of addition)

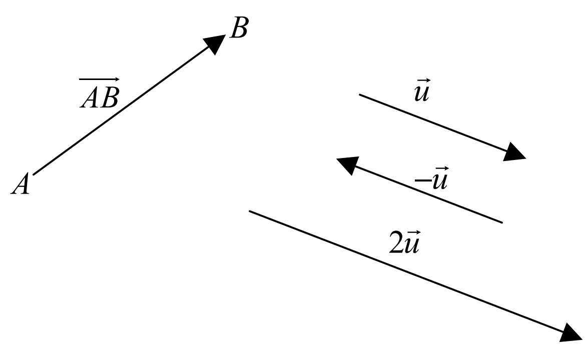
$-\vec{u}$ = Vector $\vec{u}$ in opposite direction. $-\overrightarrow{AB} = \overrightarrow{BA}$

$k\vec{u}$ = Lengthen $\vec{u}$ by a factor of k.

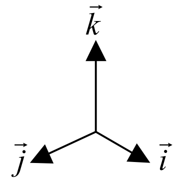
$\vec{i}$ = vector of a unit of the x-axis

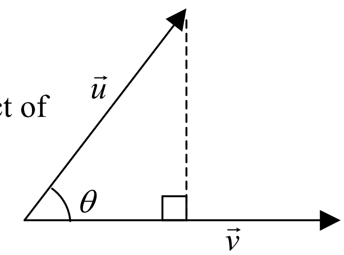
$\vec{j}$ = vector of a unit of the y-axis

$\vec{k}$ = vector of a unit of the z-axis



Any 3D-vector can be resolved as components of $\vec{i}, \vec{j}, \vec{k}$.





The dot product (點積) $\vec{u} \cdot \vec{v}$ is defined to be the product of the magnitude of $\vec{v}$ and the length of the projection of $\vec{u}$ on $\vec{v}$

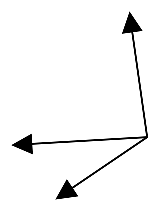
$$\vec{u} \cdot \vec{v} = |u| \times |v| \cos\theta$$

If $\vec{u} = a\vec{i} + b\vec{j} + c\vec{k}, \vec{v} = x\vec{i} + y\vec{j} + z\vec{k}$, then $\vec{u} \cdot \vec{v} = ax + by + cz$.

$$\vec{u} \cdot \vec{u} = |u|^2 .$$

If $\vec{u} \perp \vec{v}$, then $\vec{u} \cdot \vec{v} = 0$

The cross product (叉積) $\vec{u} \times \vec{v}$ is defined be to the vector which is perpendicular to both $\vec{u}$ and $\vec{v}$.



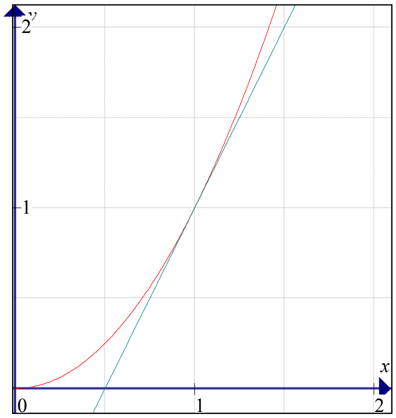
$|\vec{u} \times \vec{v}|$ = Area of //gram formed by $\vec{u}$ and $\vec{v}$.

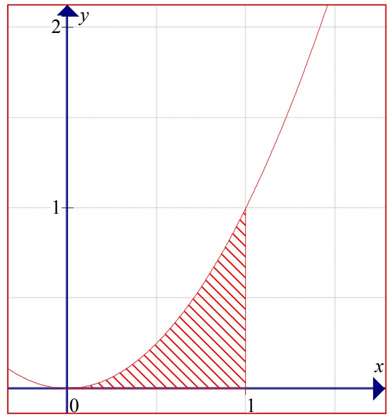
$$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$$

If $\vec{u} = a\vec{i} + b\vec{j} + c\vec{k}, \vec{v} = x\vec{i} + y\vec{j} + z\vec{k}$, then $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ x & y & z \end{vmatrix}$.

Calculus

Given a function $f(x)$.





$f'(x_0)$ or $\frac{df}{dx}(x_0)$ or $\dot{f}(x_0)$ is the slope of the tangent of f at x_0, or

$$f'(x_0) = \frac{f(x_0+\varepsilon) - f(x_0)}{\varepsilon}$$ for some "very small" $\varepsilon > 0$.

$\int_a^b f(x)\,dx$ is the area constructed by f(x) are $x = a$ to b.

Figured: Red line = $f(x) = x^2$.

Green tangent = $f'(1) = 2$.

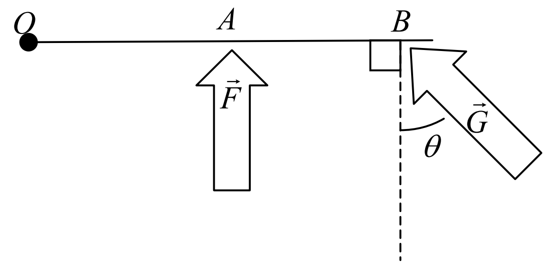
Red area = $\int_0^1 f(x)\,dx = \frac{1}{3}$.

Linear Motions

Moment (力矩)

Moment of a force F about point O

$= \vec{F} \cdot \overrightarrow{OA}$



Moment of G about O

$= \vec{G} \cdot \overrightarrow{OB}$

$= G \times OB \times \cos\theta$

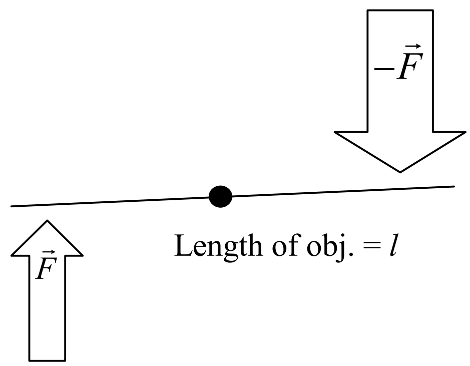
Unit: N m (Newton-meter).

Type: Scalar

- Positive if force applied rotate the object anticlockwise.
- Negative if clockwise.

Couple (偶力)

大小相等、向相反方向作用的一對力.



Torque (轉矩) of couple = $F \times l$.

Equilibrium is achieved if

- No net force (Movement = 0)
- No net moment/torque (Rotation = 0)

Center of Mass (質心)

$= \frac{1}{M} \sum m\vec{x}$

M: Total mass of object

m: Mass of each particle

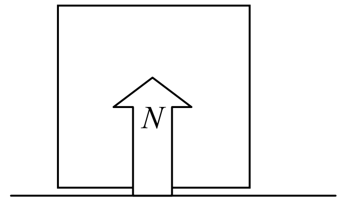
$\vec{x}$: Position of each particle. (With respect to a defined “origin”)

Friction

Limiting friction = 令物體不能動的 Friction

Kinetic friction = 物體移動時的 Friction.

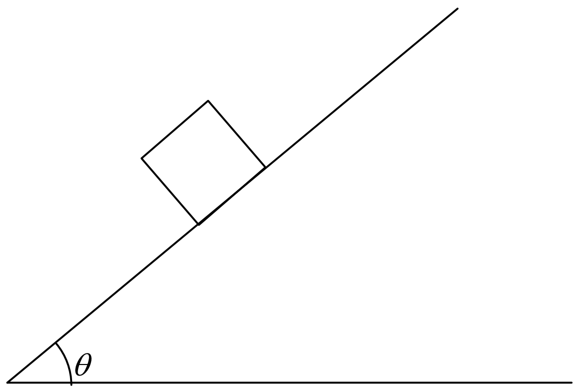
Normal reaction = Reaction force done by the "ground".



If limiting friction = F_L, kinetic friction = F_K, normal reaction = N,

$$\mu_L = \frac{F_L}{N}; \mu_K = \frac{F_K}{N}$$

Where μ_L is the coefficient of limiting friction, μ_K is the coefficient of friction.



If the object is just about to fall, $\mu_L = \tan\theta$. This θ is called the angle of friction.

Movement

For a position function of an object with respect to time $s(t)$, its velocity $v(t)$ and acceleration $a(t)$ are related as:

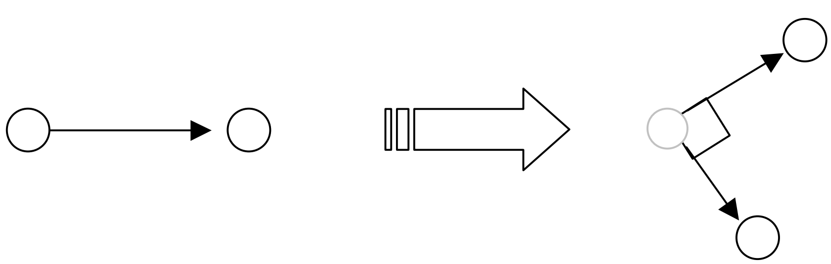
$$s = \dot{v} \qquad s = \ddot{a}$$
$$v = \int s \qquad v = \dot{a}$$
$$a = \iint s \qquad a = \int v$$

If an object V moves at a velocity of $\vec{v}$, which is observed by an observer moving at a velocity of $\vec{u}$, then the actual velocity of V is $\vec{u} + \vec{v}$

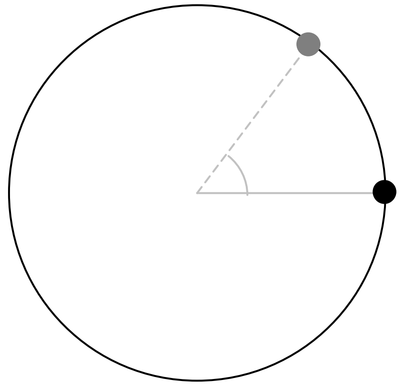
[Apply only if u, v << speed of light.]

Momentum

若有二同質量 (mass) 物體相撞, 其一原不動, 則撞後二物軌跡成直角.



Circular Motions



Angular velocity (角速度) $\vec{\omega}$: Angle (in radian) rotated by a particle in 1 sec.
Unit: rad s^{-1}.

Speed of the particle = Radius × ω.

Angular acceleration (角加速度) $\vec{\alpha}$ = Acceleration in angular velocity.

Linear acceleration of particle = $\frac{v^2}{r} = \omega^2 r$ (r = radius)

Centripetal Force (向心力)

$$F = \frac{mv^2}{r} = m\omega^2 r$$

Force applied to the particle to "pull" it to the center of the circle.

Rotating Body

Moment of inertia $= I = \sum mr^2$ (i.e., For particle with mass m having a distance of r from the center of rotation, I is the sum of mr^2 for all particles)

K.E. of it $= \frac{1}{2} I\omega^2$.

Equations of Uniform Angular Acceleration

For initial ang vel = ω_0, final ang vel = ω, ang accel = α, time taken = t, rotation = θ:

$$\omega = \omega_0 + \alpha t$$

$$\frac{\theta}{t} = \frac{\omega + \omega_0}{2}$$

$$\omega = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

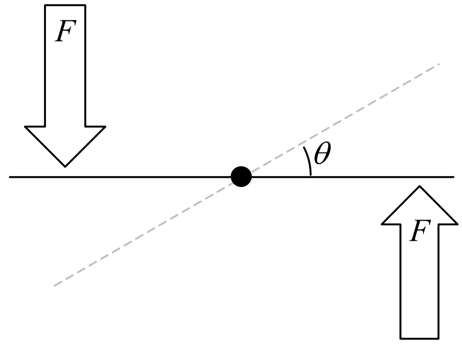
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Work done by couple

Work done

$= 2Fr\theta$

$= T\theta$



r = Radius of obj.

T = Torque of couple

In general, $T\theta = \frac{1}{2}I\omega^2$

Angular Momentum (角動量)

$= I\omega$.

Newton's second law → $T = I\alpha$.

Oscillation

Simple Harmonic Motion (SHM)

If motion is SHM if the accel. of a body is directly prop. to its dist. From a fixed pt. and is always directed towards that point.

$$\vec{a} \propto -\vec{x}$$

$$\vec{a} = -\omega^2 \vec{x}$$

E.g.: Spring, "鞦韆", Pendulum

Let r = max. disp.

Period: $T = \dfrac{2\pi}{\omega}$

Velocity: $\vec{v} = \pm\omega\sqrt{r^2 - x^2} = -\omega r \sin \omega t$

Displacement: $x = r \cos \omega t$.

ω

$$\omega = \sqrt{\frac{\text{Force/unit disp}}{\text{Mass of oscillating system}}} = 2\pi\sqrt{\frac{\text{Mass of oscillating system}}{\text{Force/unit disp}}}$$

Hooke's Law

For a spring, to stretch it from for *x* m, the force needed:

$$F = -kx$$

Where k = the spring constant

Simple Pendulum

For a pendulum with length l,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Holds only if the pendulum doesn't swing for > 10°.

Fluids

Density

$$\rho = \frac{m}{V}$$

(Density = Mass / Volume)

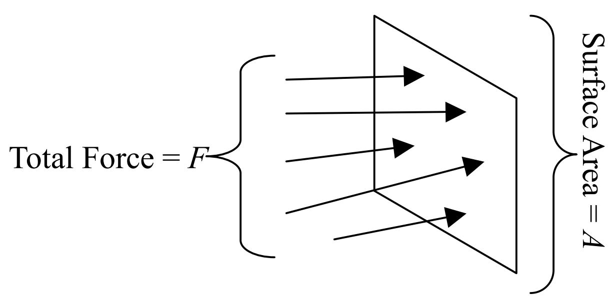
(Unit = kg m^{-3})

Pressure

$$p = \frac{F}{A}$$

(Pressure = Force / Area)

(Unit = Pa)

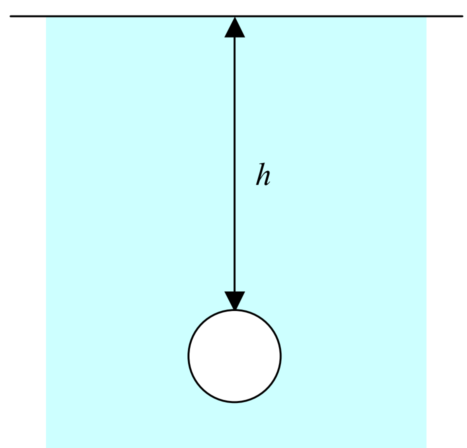


$$p = \frac{dF}{dA}$$

Pressure on the object:

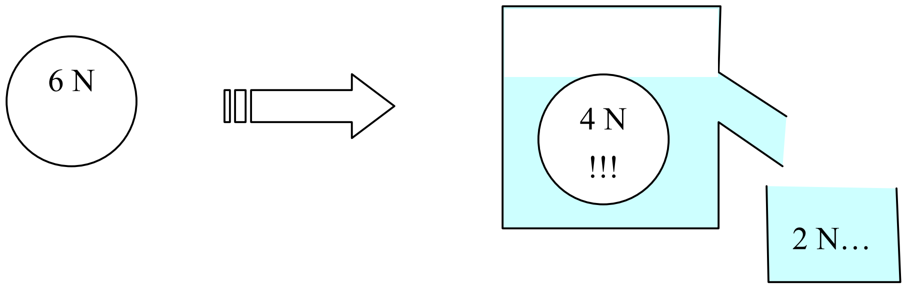
$p = hg\rho$

(g: gravitational accel.)



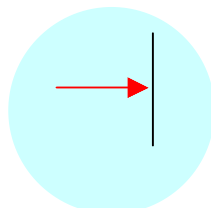
Archimedes' Principle

When a body is completely or partly immersed in a fluid it experiences an up-thrust (上衝), or apparent loss in weight, which is equal to the weight of fluid displaced



A floating body displaces its own weight of fluid.
If the body fails to do so, it sinks.

Surface Tension (表面張力)

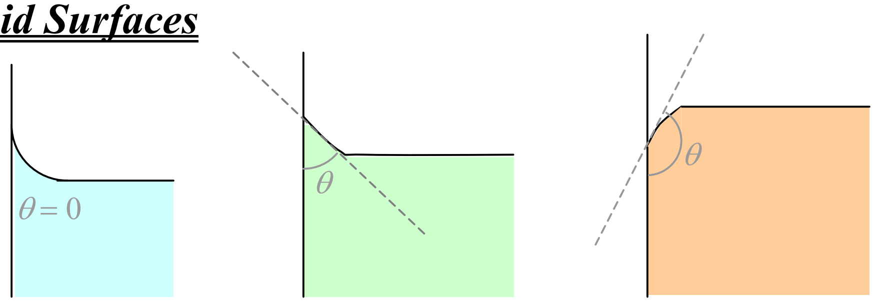


If the force acting on the string with length l m is F N, then the **surface tension** of the fluid:

$$\gamma = \frac{F}{l}$$

Unit: N m^{-1}.

Liquid Surfaces



θ: Angle of contact. For water and many organic liquids, $\theta = 0°$ on clean surface.
Liquid with $\theta < 90°$ are said to "wet" the surface, while > 90° not.

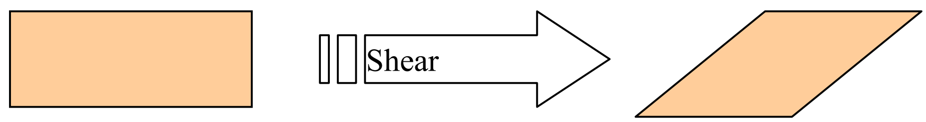
Rise of liquid in a capillary

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$

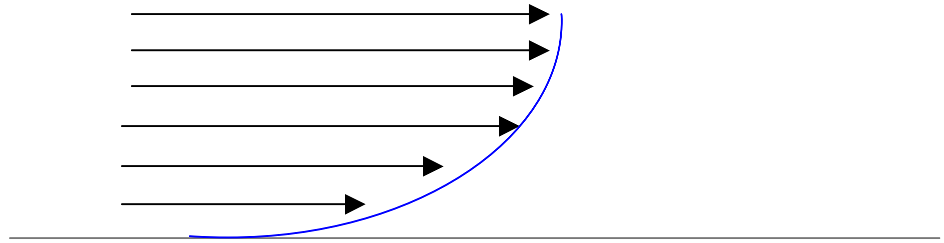
r: Radius of the capillary [Note: if $\theta > 90°$, the liquid actually falls]

Viscosity (黏性)

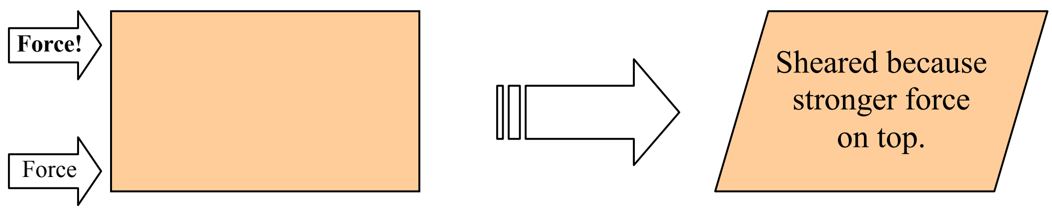
All fluids (except *very* low dens. gases) stick to a solid surface. When they flow, the vel. must gradually dec. to 0 as the wall of the pipe/containg vessel is approached.
A fluid is therefore sheared (displaced laterally) when it flows past a solid surface and the opposition set up by the fluid is called its **viscosity**.



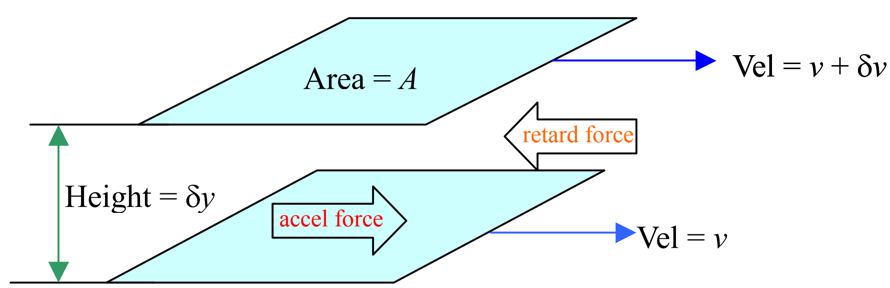
Viscosity is a kind of internal friction exhibited to some degree by all fluids.



When the particles of fluid passing successively through a fluid follow the same path, the flow is said to be **steady**. "Streamlines" can be drawn to show the direction of motion of the particles. For steady flow, the bottom layer in contact with the bottom must be at rest. The length of streamline represents the magnitude of the velocities.



Coefficient of Viscosity



$$\eta = \frac{\frac{F}{A}}{\frac{\delta v}{\delta y}}$$

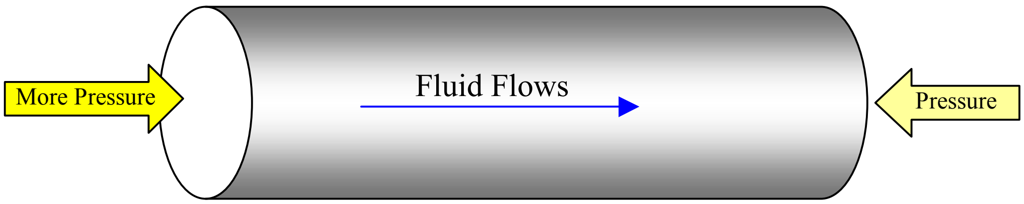
Unit: Pa s.

For fluid that is independent of $\frac{\delta v}{\delta y}$; i.e., η: const, this fluid is called **Newtonian fluid**.

If $\frac{\delta v}{\delta y}$ inc ➔ η dec: The fluid is **thixotrophic**. Example = Paints, Glues, …

Usually tempature inc ➔ η dec rapidly.

Poiseuille's Formula



If a fluid is steady moving in a pipe, then:

$$V = \frac{\pi p r^4}{8\eta l}$$

r: Radius of pipe.

p: Pressure different between two ends of the pipe.

l: Length of the pipe

η: Viscosity coefficient of fluid

V: Volume of fluid passing through the pipe per second.

Steady vs. Turbulent Flow (平靜與狂亂流動)

Reynold's number (Re) is useful in the study of the stability of fluid flow.

$$\text{Re} = \frac{v l \rho}{\eta}$$

v: Speed of the bulk of fluid.

l: characteristic dimension of the solid body concerned.

For cylindrical pipes:

l = diameter.

If Re < 2200: Steady

~ 2200: Unstable [Critical velocity, v_c]

> 2200: Turbulent

Stokes' Law

Moving a sphere slowly (steady) in a fluid of infinite extend, the viscous retarding force:

$$F = 6\pi\eta v r$$

r: radius of sphere.

v: vel. of sphere

For a falling sphere in a fluid, the terminal velocity

$$v_t = \frac{2r^2 g(\sigma - \rho)}{9\eta}$$

σ: dens. of sphere.

ρ: dens. of fluid.

It only holds if $v_t < v_c$. If not, the drag force (阻力) will increase rapidly:

$$\text{Drag} = \frac{C\rho A v^2}{2}$$

A: cross-section area of the body $\perp$ velocity.

C: Drag coeff. In (0, 1).

Streamlining the body thus helps reducing drag.

Bernoulli's Equation

Along a streamline (For every point) in an incompressible inviscid (ideal) fluid,

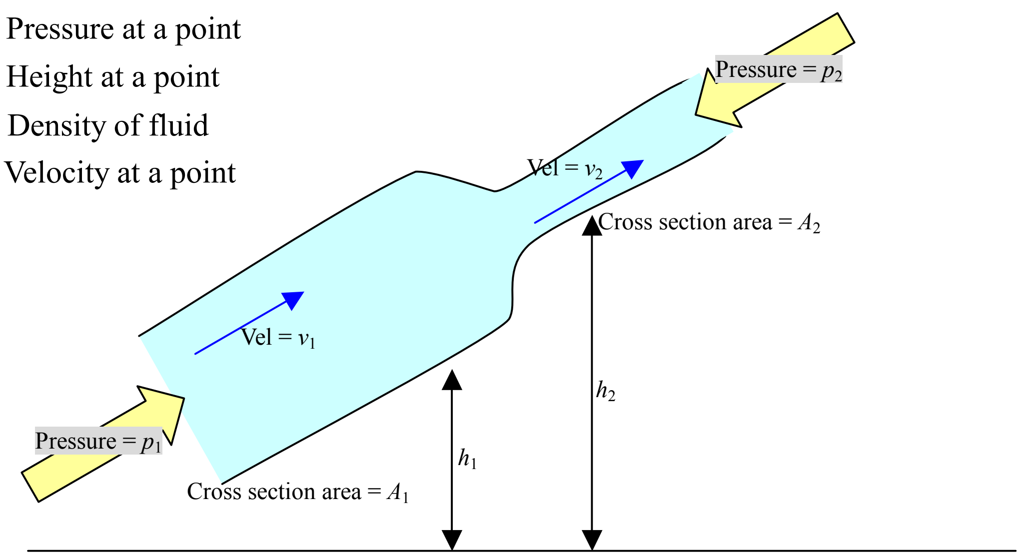
$$p + h\rho g + \tfrac{1}{2}\rho v^2 = \text{constant}$$

p: Pressure at a point

h: Height at a point

ρ: Density of fluid

v: Velocity at a point



Usually, fluid travels faster in a narrower tube.

Electrostatics

Coulomb's Law

For two points *A* and *B*, having charges Q_A C and Q_B C respectively, and are *r* m apart. Then the force F between them:

$$F \propto \frac{Q_A Q_B}{r^2}$$

Permittivity

The force between 2 charges also depends on what separates them; its value is always reduced when an insulating material replaces a vacuum. To take this into account a medium is said to have permittivity, denoted by ε.

A material with high permittivity is one which reduced noticeably the force between two changes compared with the vaccum value.

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_A Q_B}{r}$$

Unit of ε: F m^{-1}.

Permittivity of vacuum = $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}.

Permittivity of air as s.t.p. = $1.0005\varepsilon_0$.

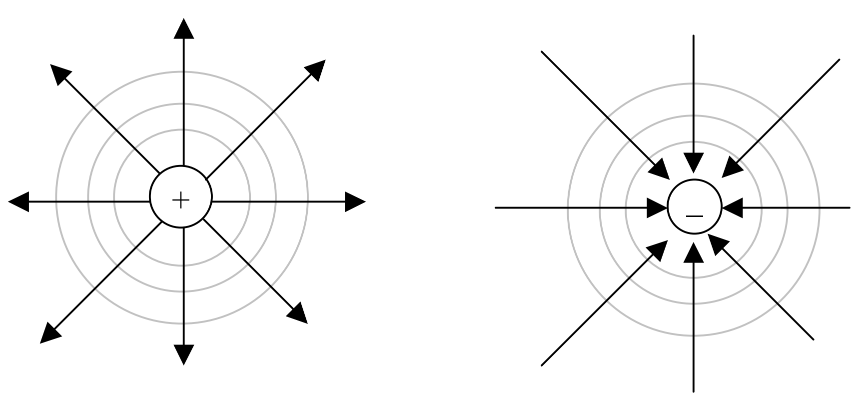
Electrical Potential

Like gravitational P.E., electrical P.E. at a point in a e^- field is defined as the energy req. to move unit +ve charge from "infinity" to that point. (Assume the charge doesn't affect the field.)

Unit: V.

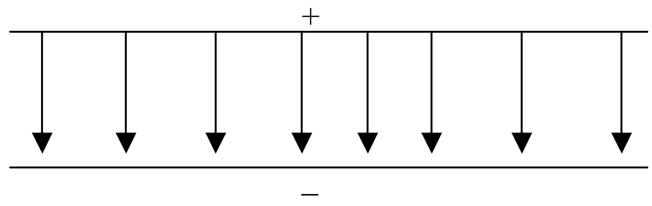
Electric Fields

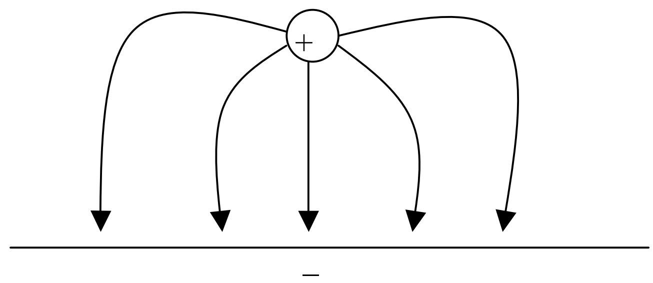
A vector field:



The arrows are called the "field lines". They never intersect each other.

The gray circles are called the "equipotentials" (等電位). Every point on that line has the same potential. The equipotentials are always perpendicular to the field lines.





For a point *A* with is *r* m from a charge with *Q* C in a medium with permittivity ε, its potential *V* V:

$$V = \frac{1}{4\pi\varepsilon}\frac{Q}{r}$$

For a charged sphere with radius *r*, its potential at surface is the same formula.

Potential Difference

P.D. between 2 points in e^- field is energy transformed when unit charge passes from one point to another.

$$W = QV$$

Potential Gradient

For a point charge *Q*, if the field strength is *E*, then the force act on *Q*:

$$F = EQ$$

If field strength inc., potential dec.

$$E = -\frac{dV}{dx}$$

dV/dx: Potential gradient in the *x*-direction.
Unit: V m^{-1}.

If e^- field is const, everywhere for a P.D. *V* V at separation of *d* m,

$$E = -\frac{V}{d}.$$

Gravity vs. Electricity

Gravitational force : e^- force = 1 : 10^{39}.

Electricity

Current

$Q = It$

I: current

t: time

Unit of I: A.

Current density

$J = I/A$

A: Cross-section area of conductor

Resistance

$R = V/I$

V: Voltage

Resistors in series: $R = \sum R$

Resistors in parallel: $\frac{1}{R} = \sum \frac{1}{R}$

Meters

Connect ammeters in *series*.

Connect voltmeters in *parallel*.

Resistance of ammeter should be very low (tends to 0).

Resistance of ammeter should be very high (tends to infinity).

Electromotive Force (電動勢)

The emf E of a source (battery, generator, etc.) is the energy transferred to electrical energy when unit charge passes through it.

Unit: V.

When a charge Q passes through source of emf E, the e^- energy supplied by source:

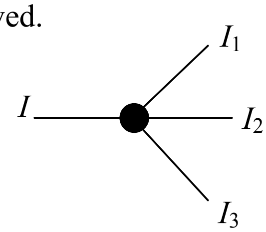
$$W = QE$$

Kirchhoff's Laws

At a junction in a circuit, the current arriving equals the current leaving.

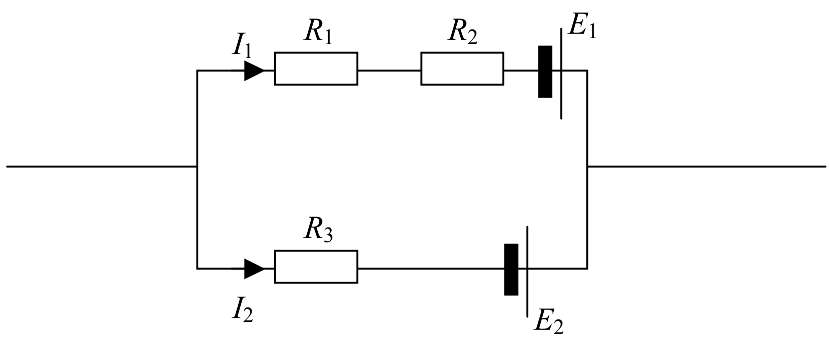
$$I = \sum I$$

That means, charge is conserved.



Round any closed circuit or loop the (signed) sum of the emf E = sum of $I * R$.

$$\sum E = \sum IR$$



(For this, take clockwise as +ve. Sum of $E = E_1 - E_2$. Sum of $IR = I_1R_1 + I_1R_2 - I_2R_3$)

Power

$P = IV$

Faraday constant

$F = 9.65 \times 10^4$ C mol^{-1}

This is the quantity of e^- charge which liberates 1 mol of any singly charged ion.

Ohm's Law

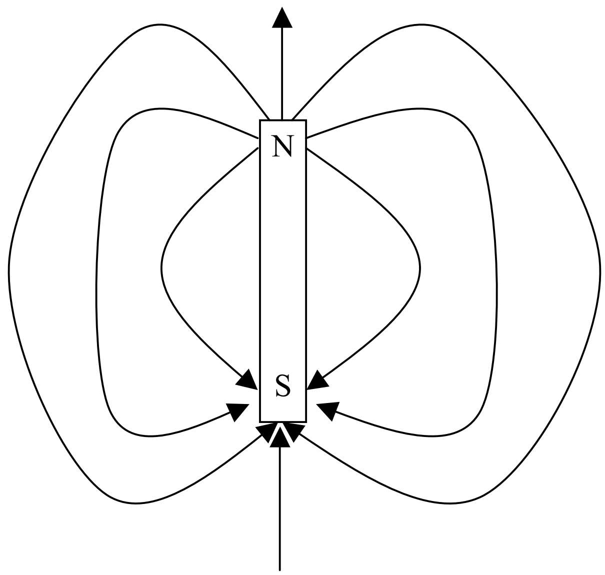
$V - E = IR$

[Or neglecting emf, $V = IR$]

Electromagnetism

Magnetic Field

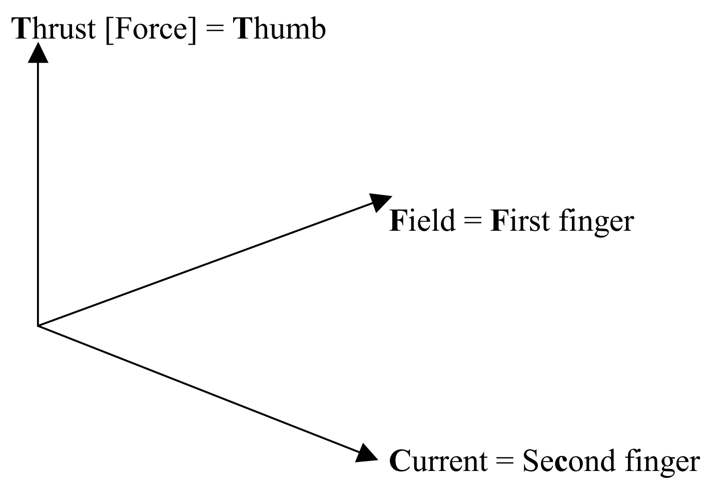
Similar to electric field



Force on Current in Magnetic Field (Lorentz Force)

Fleming's Left-Hand Rule:

"$\vec{T} = \vec{F} \times \vec{C}$"



Magnetic Flux Density (磁通密度)

Electric field strength E: Force / unit charge

Gravitation field strength g: Force / unit mass

Flux Density / Magnetic Induction B: Force / unit current length

B: The force acting per unit length on a conductor which carries unit current and is at right angles to the direction of the magnetic field.

$$B = \frac{F}{Il}$$

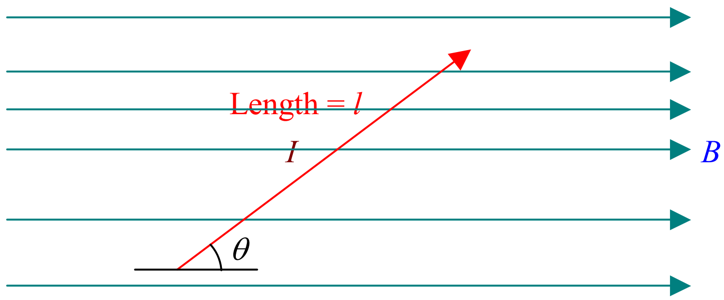
Unit: T

Type: Vector

If conductor and field are not at rt. ang., but an ang. θ with one another:

$$F = BIl \sin \theta$$

$$\vec{F} = \vec{B} \times I\vec{l}$$



Note: 1 T is quite strong already!

Permeability

Biot-Savart Law: For a very short length δ*I* of conductor, carrying a steady current I, the magnitude of the flux density δ*B* at a point *P* distance *r* from δ*I*:

$$\delta B \propto \frac{I \ \delta I \sin \theta}{r^2}$$

Where θ is the angle between δ*I* and the line joining it to *P*.

Permeability: Variation const. (over 4π) of the above eq.

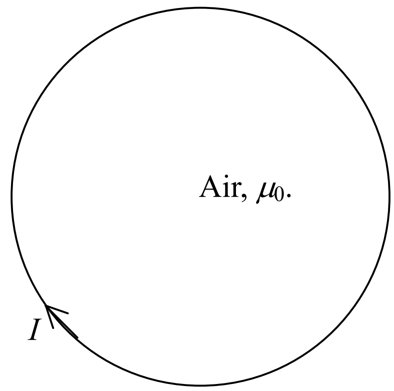
μ_0: Permeability of vacuum = $4\pi \times 10^{-7}$ H m^{-1}.

Air & most other materials (except ferromagnetics) have permeability ~ μ_0.

$$\delta B = \frac{\mu_0 I \ \delta I \sin \theta}{4\pi r^2}$$

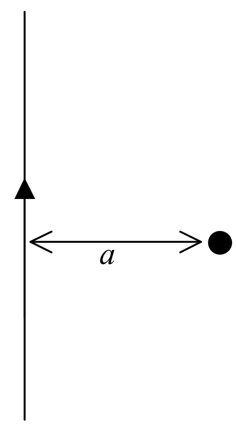
Note: $c^2 = \frac{1}{\mu_0 \varepsilon_0}$. c = speed of light in vacuum.

Flux Density Calculation



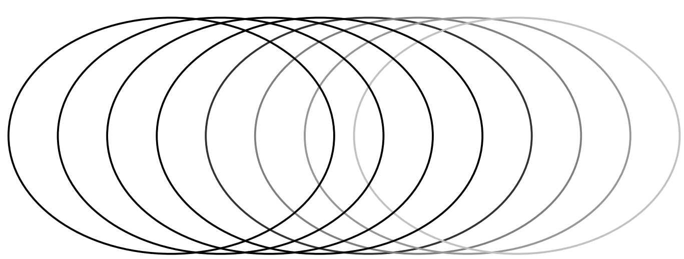
If radius = r, and there are N turns in the coil, the flux density at center of circle:

$$B = \frac{\mu_0 NI}{2r}$$



If the wire is very long and straight,

$$B = \frac{\mu_0 I}{2\pi a}$$



For a very long solenoid with N turns and length l,
The flux density at center of solenoid:

$$B = \mu_0 NI$$

At end of solenoid:

$$B = \frac{\mu_0 Nl}{2}$$

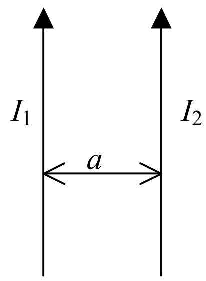
Force on a Charge in Magnetic Field

For a charged particle Q moving at a speed of v ms^{-1} in a conductor, which makes an angle of θ with the magnetic field of flux density B,

$$F = BQv \sin \theta$$

$$\vec{F} = Q\vec{v} \times \vec{B}$$

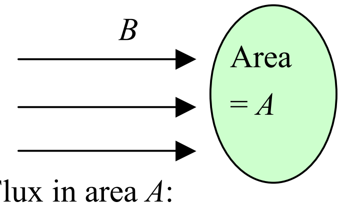
Force between two Currents



Length of length conductor = l.

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$$

Magnetic Flux (磁通量)



Magnetic Flux in area A:

$$\Phi = \vec{B} \cdot \vec{A}$$

Unit: Wb

Type: Scalar

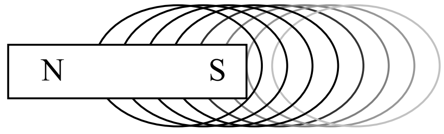
If Φ is the flux through the cross-section area A of a coil of N turns, the total flux through it, called the flux-linkage, is $N\Phi$ since the same flux Φ links each of the N turns.

Faraday's Law

The induced emf is directly proportional to the rate of change of flux-linkage or rate of flux cutting.

$$E = \frac{d}{dt}(N\Phi)$$

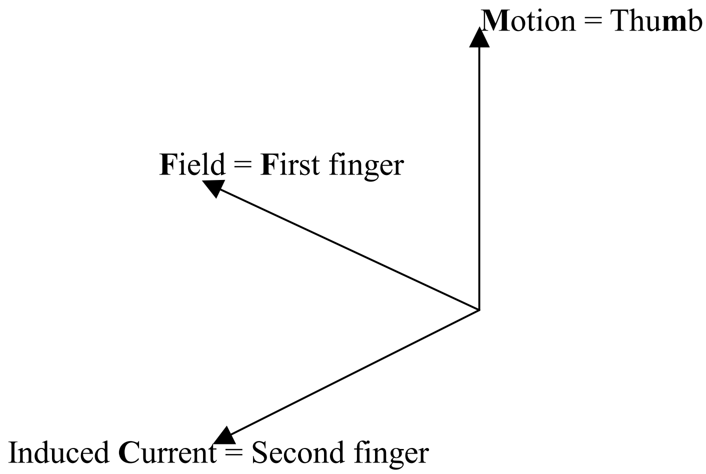
Unit of E: V.



Lenz's Law

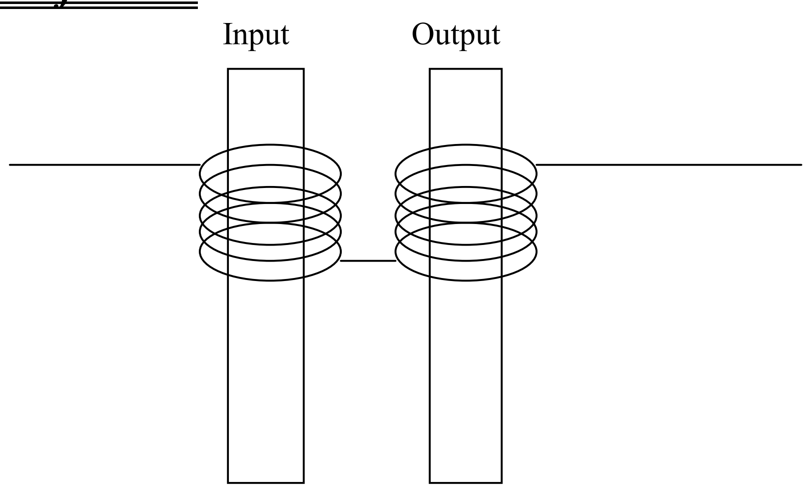
The direction of the induced emf is such that it tends to oppose the flux change causing it, and does oppose it if induced current flows.

Fleming's right-hand rule:



So, $E = -\frac{d}{dt}(N\Phi)$

Transformers



If voltage of input (primary) = V_P, number of turns = N_P;
Voltage of output (secondary) = V_S, number of turns = N_S:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

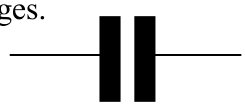
Also:

$$V_S I_S = V_P I_P$$

Electrical Devices

Capacitor (電容)

To "store" charges.

Symbol: 

Capacitance (靜電容量): Charge-storing capacity. Charges an obj can store before break down occurs

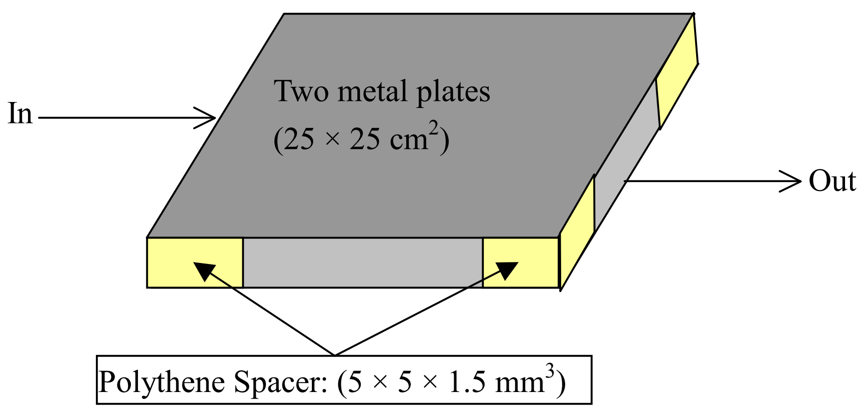
$$C = \frac{Q}{V} = \frac{I}{fV} = \frac{A\varepsilon}{d}.$$

f: Switching freq. of A.C. supply.
A: Area of capacitor plate (see below)
d: "Height" between 2 plates.
ε: permittivity of space btn 2 plates

Unit of C: F
Usually C is const.

For a sphere with permittivity ε and radius r, $C = 4\pi\varepsilon r$.

Capacitor:



Inserting an insulator between the plates of capacitor increases its capacitance.
Practical capacitor is smaller, of course.

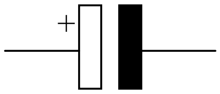
Connecting Capacitors

In parallel: $C = \sum C$ [The P.D. across each capacitor are the same]

In series: $\frac{1}{C} = \sum \frac{1}{C}$ [The charge are the same]

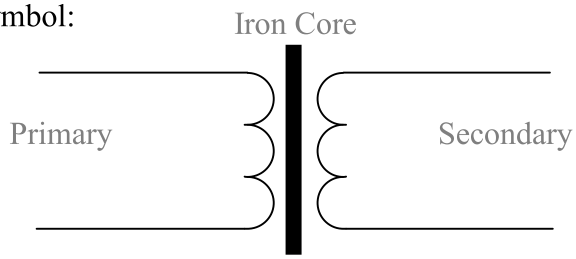
Electrolytic Capacitor

Similar to usual capacitor, but very high capacitance (~ 100 mF).

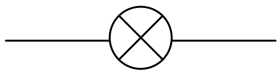
Symbol: 

Transformer

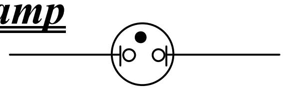
Symbol:

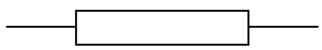


Lamp

Symbol: 

Neon Lamp

Symbol: 

Variable Resistor 

Inductor (感應器)

The flux deu to current in a coil links that coil and if the current changes the resulting flux induces an emf in the coil itself. This changing-magnetic-field type of EM induction is called **self-induction** (自感), and the coil is said to have **self-inductance**, or simply inductance, L. (因電流通過電路時的變化，而在電路中產生電壓)

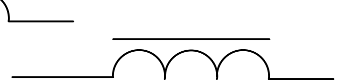
The induced emf obeys Faraday's law.

$$L = -\frac{E}{\frac{dI}{dt}}$$

E: emf.

Unit of L: H.

Symbol: 

(With magnetic material core: )

Solenoid Inductor

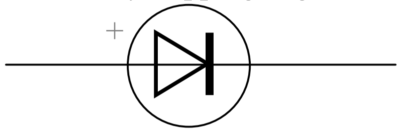
If the inductor is a solenoid without core and with N turns, length l and cross-section area A,

$$L = \frac{\mu_0 A N^2}{l}$$

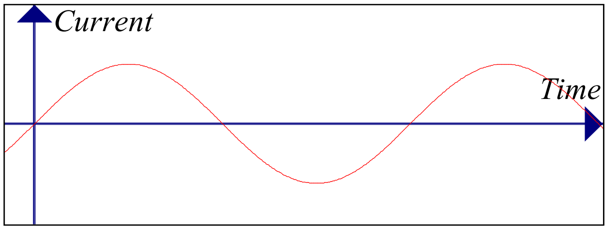
Rectifier (整流器)

Convert A.C. to D.C. [by trapping negative currents]

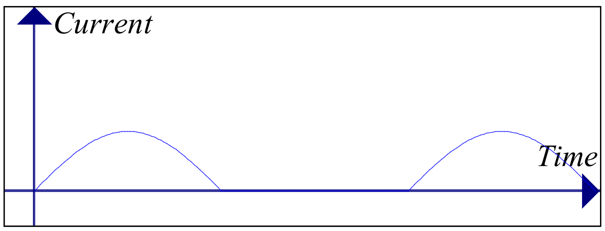
Symbol:



Original current:



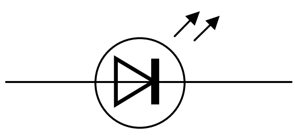
After passing through rectifier



Diode (二極管)

Symbol: same as rectifier

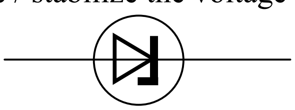
LED



Zenor Diode

To regulate / stabilize the voltage output of a power supply.

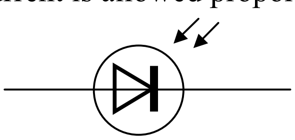
Symbol:



Photodiode

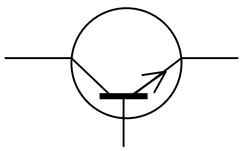
Reverse current is allowed proportional to light intensity.

Symbol:

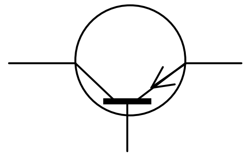


Transistors (電晶體)

n-p-n type:



p-n-p type:



The left wire is the collector C, the right is the emitter E, and the bottom is the base B.

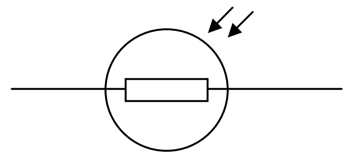
Usage:

Switch. (Current will not flow from C to E unless there is current in B.)

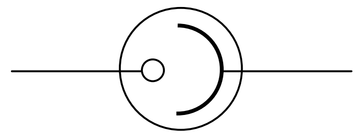
Voltage Amplifier.

Light-Dependent Resistor (LDR)

The resistance (e.g. CaS) decrease as intensity of light increase

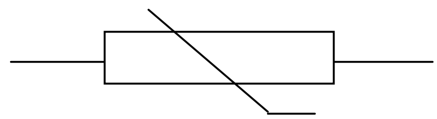


Photocell (光電池)



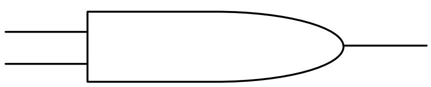
Thermistor

Resistance of it will decrease when temperature increase

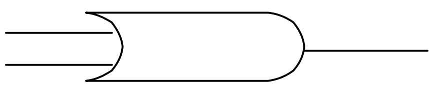


Logic Gates

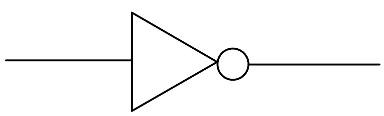
And:



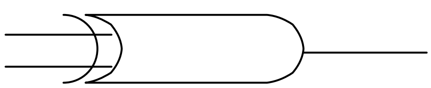
Or:



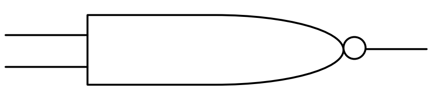
Not:



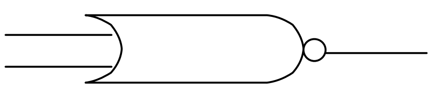
X-Or:



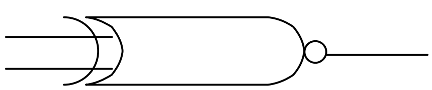
N-And:



N-Or:



XN-Or



Operational Amplifier (Op Amp)

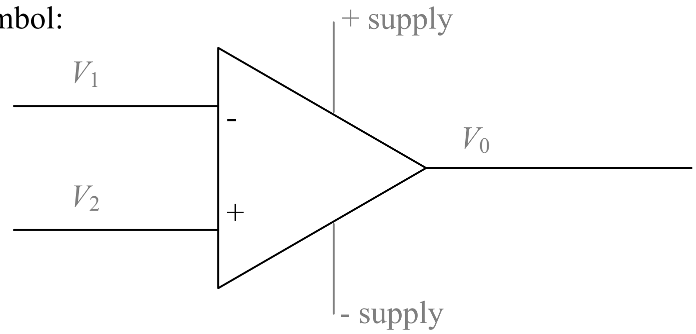
It can perform electronically mathematical operations such as +, × and ∫.

It's also used widely as a high-gain amplifier of D.C. & A.C. voltages and as a switch.

It has a very high voltage gain, high input resistance and low output resistance.

The voltage gain is called the open-loop gain A_0, usually 10^5 for D.C.

Symbol:



"+": Non-inverting input

"–": Inverting input

Supplies: should be numerically equal, range ±5 V to ±15 V.

$$V_0 = A_0 (V_2 - V_1)$$

Waves

Mechanical Wave

Produced by disturbance (e.g. a vibrating body) in a material medium and are transmitted by the particles of the medium oscillating to and fro.

Such waves can be seen or felt and include waves on a spring, water waves, waves on stretched strings (e.g. in musical instruments) and sound waves in air and in other materials.

Electromagnetic Wave (EM Wave)

Consist of a disturbance in the form of varying electric and magnetic fields. No medium is necessary and they travel more easily in a vacuum than is matter.

Speed, Frequency and Wavlength

$$v = f\lambda.$$

v: Speed of wave

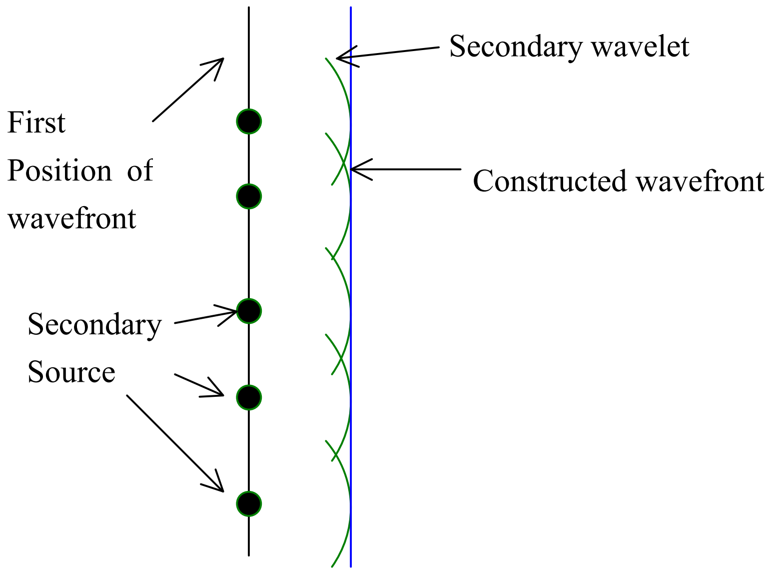
f: Freq. of wave

λ: wavelength of wave.

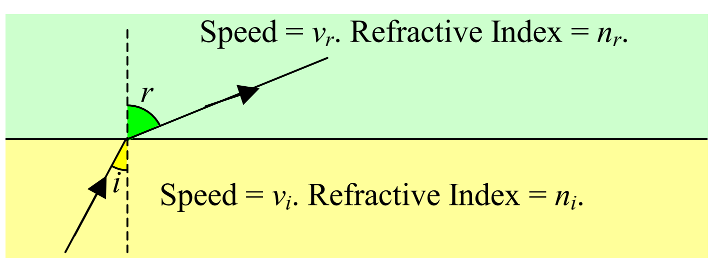
Huygens' Construction

Note: Ray ⊥ Wave-fronts

Every point on a wavefront may be regarded as a source of secondary spherical (circular in 2D) wavelets which spread out with the save speed. The new wavefront is the envelope of these secondary wavelets, that is, the surface which touches all the wavelets.



Snell's Law



$$\frac{v_i}{v_r} = \frac{\sin i}{\sin r} = \text{constant}$$

$$n_i \sin i = n_r \sin r$$

If $v_i > v_r$ (The ray slowed down), it bends **towards** the normal.

If $v_i < v_r$ (The ray fasten up), it bends **away** from the normal.

Wave Speed

Transverse waves on a taut string or spring:

$$v = \sqrt{\frac{T}{\mu}}$$

T: Tension; μ: Mass / unit length

Longitudinal waves along masses (e.g. trolleys) linked by springs:

$$v = x\sqrt{\frac{k}{m}}$$

x: Spacing between mass centers; *k*: Spring Constant; *m*: One mass

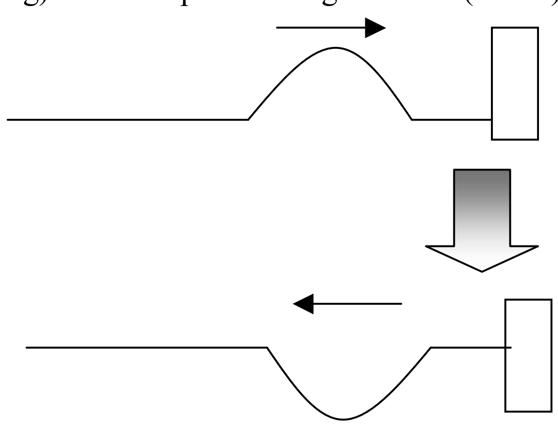
Short wavelength ripples on surface of deep water:

$$v = \sqrt{\frac{2\pi\gamma}{\lambda\rho}}$$

γ: Surface Tension; λ: Wavelength; ρ: Density.

Reflection and Phase Changes

When a transverse wave on a spring is reflected at a “denser” medium (e.g. a fixed end or a heavier spring) there is a phase change of 180° (or $\lambda/2$)



Equation of Wave

For waves traveling left to right:

$$y = a\sin(\omega t - kx)$$

$k = 2\pi / \lambda$. $\omega = 2\pi f$.

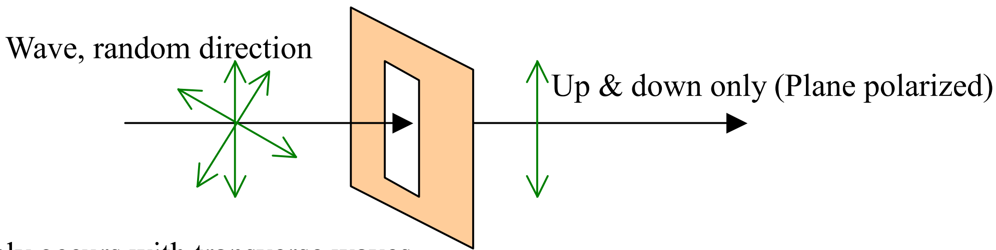
If traveling right to left, use “+ *kx*” instead.

Principle of Superposition

Pulses & waves pass through each other unaffected.

When they cross, the total disp. is the vector sum of the indiv. disp. due to each pulse at that pt.

Polarization

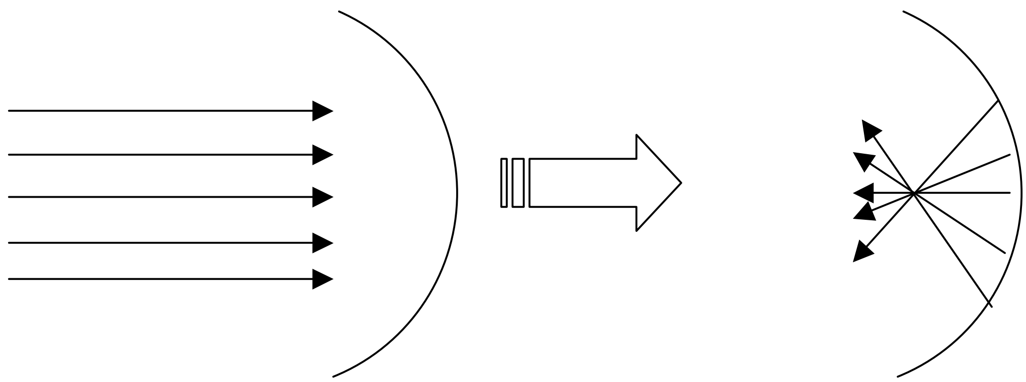


Only occurs with transverse waves.

Optics

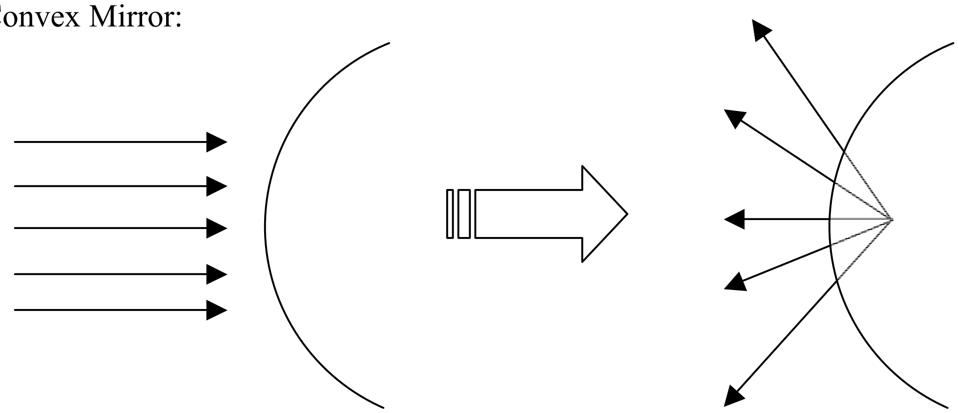
Curved Mirrors

Concave Mirror:



The light converges. The point of convergent in called the "principal focus". This focus is "real" because the light actually passes through it.

Convex Mirror:



The light diverges. There is a *virtual* focus *behind* the mirror.

If the incident angle is not large:

$$f = \frac{r}{2}$$

f: Focal Length (length from focus to the mirror).

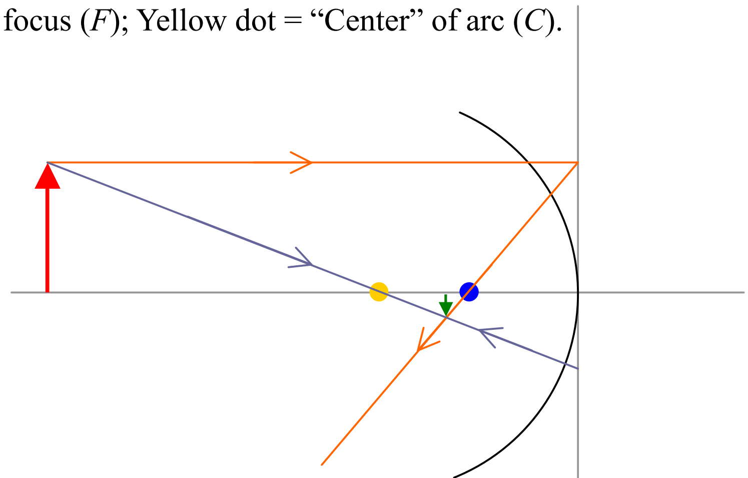
r: Radius of curvature, i.e., "radius" of the arc.

Ray Diagram for Spherical Mirrors

Red arrow = obj; Green arrow = img.

Orange & Purple lines: rays

Blue dot = focus (*F*); Yellow dot = "Center" of arc (*C*).



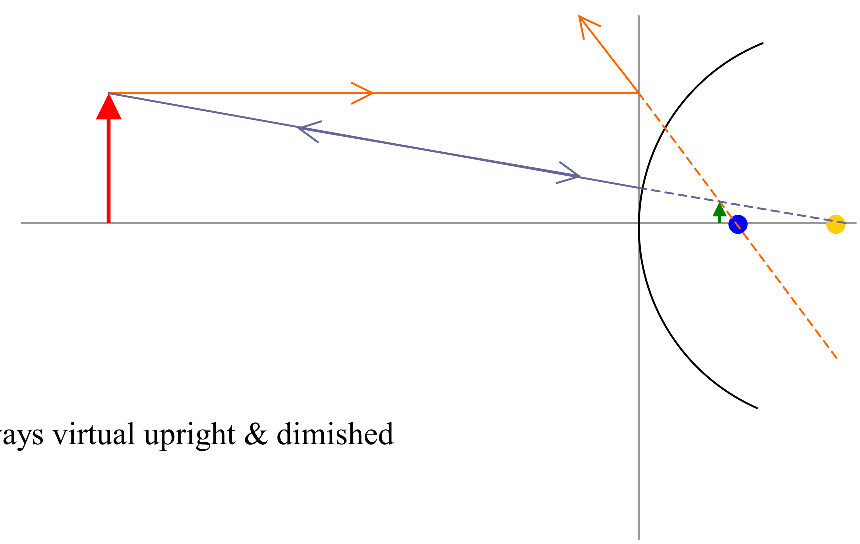
If obj. behind *F* and *C*: img inverted, diminished and real. [Between *F* and *C*]

If obj. on *C*: img inverted, same size and real. [On *C*]

If obj. between *F* and *C*: img inverted, magnified and real. [Beyond *C*]

If obj. on *F*: img at infinity.

If obj. after *F*: img upright, magnified and virtual [Behind Mirror]



Img: always virtual upright & dimished

Mirror Formula

$$\frac{1}{d_{\text{Image}}} + \frac{1}{d_{\text{Object}}} = \frac{1}{f}$$

d_{xxx}: Distant of mirror from “xxx”.
f: Focal length

These values are +ve if real (in front of mirror), -ve if virtual (behind mirror)

Magnification

$$m = \left| \frac{d_{\text{Image}}}{d_{\text{Object}}} \right|$$

Refraction of Light

Refractive index of vacuum = 1. Refractive index of air $\gtrsim$ 1.

Refractive index of this medium = $\dfrac{\text{Real Depth}}{\text{Apparent Depth}}$

Total Internal Reflection

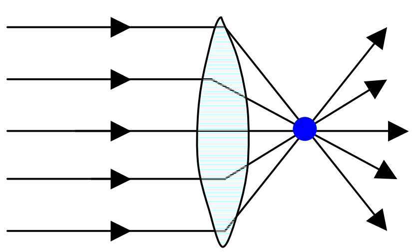
Only when leaves from denser medium to lighter medium (e.g., glass to air)
Occur if incident angle > critical angle.

$$c = \sin^{-1} \tfrac{1}{n} = \csc^{-1} n$$

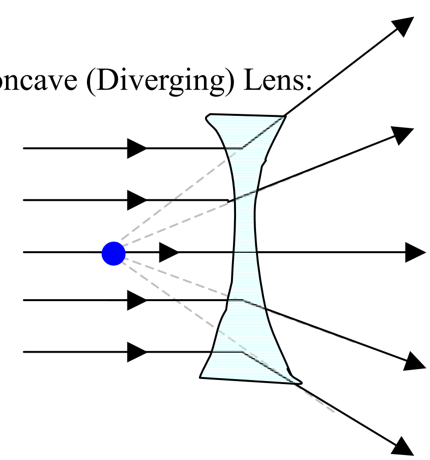
n: Refractive index of medium

Thin Lenses

Convex (Converging) Lens:



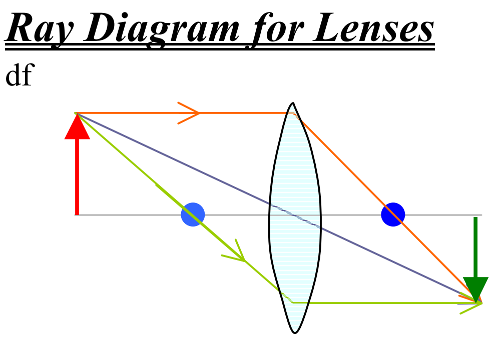
Concave (Diverging) Lens:

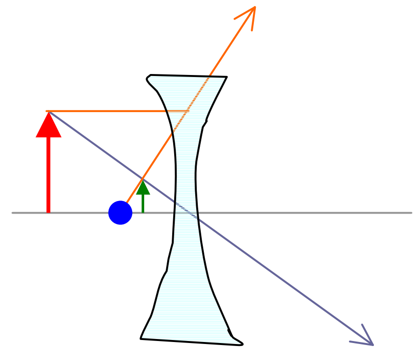


Blue dot: Focus (F).

Ray Diagram for Lenses

df





For Convex Lenses:

Obj. Pos	**Reality**	**Size**	**Rotation**
Behind 2*F*	Real	Diminished	Inverted
2*F*	Real	Same	Inverted
Btn 2*F* and *F*	Real	Magnified	Inverted
F	Real	Infinity	Inverted
After *F*	Virtual	Magnified	Erect

For Concave Lenses:

Always virtual, diminished and erect.

Lenses Formula

$$\frac{1}{d_{\text{Image}}}+\frac{1}{d_{\text{Object}}}=\frac{1}{f}$$

(Converging Lens: f = +ve. Diverging Lens: f = -ve)

$$m=\left|\frac{d_{\text{Image}}}{d_{\text{Object}}}\right|$$

(Unsigned)

Full Lenses Formula

For a lens with refractive index n, if r_L is the radius of curvature on the left of the lens, r_R on the right, its focal length:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_L}+\frac{1}{r_R}\right)$$

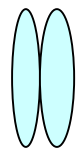
It is signed!

If refractive index of surrounding materials is n':

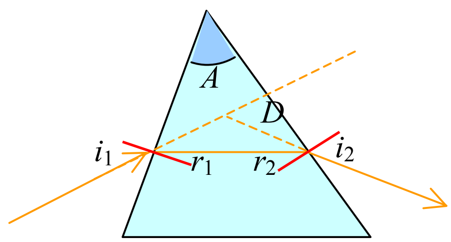
$$\frac{1}{f} = \left| \frac{n'}{n} - 1 \right| \left(\frac{1}{r_L} + \frac{1}{r_R} \right)$$

Focal Length of two Thin Lenses in Contact

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$



Prisms



$$A = r_1 + r_2$$

The angle "D" is called the **deviation** of the prism. It is minimum when $i_1 = i_2$.

$$D_{\min} = 2i - A$$

If n is the refractive index of the prism, then:

$$n = \frac{\sin\left(\frac{A + D_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

If A is small (< 6° or 0.1 rad):

$$D = (n - 1)\, A$$

Being a mixture of light of different colors, white light will disperse while passing through a prism. Since red light is slowest while purple is fastest in the prism, the red light will bend the most while purple the least. The result is the spectrum of light:

Types of EM Waves

γ-ray > X-ray > UV > Visible light > IR > Microwave > Radio wave

More Energy → Lower Energy

Shorter Wavelength → Longer Wavelength

Interference of Light (Young's Double Slit Experiment)

If the distance from the source is d, the distance between the two sources (slits) is a, and the distance between two "same" fringes is y, then:

$$\lambda = \frac{ay}{d}$$

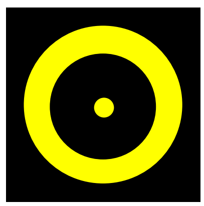
Optical Path Length

If light traveled l m in a medium of refractive index n, it is optically equivalent to length nl m in a vacuum.

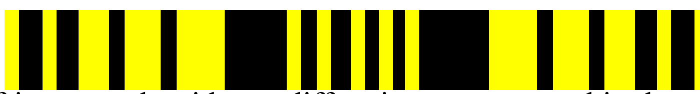
Diffraction Pattern of Light

Straight Edge: (Placed on the left)

Circular obstacle:



Straight Narrow obstacle (e.g. Pin) (Placed in the middle)



Note: The fringes on the side are diffraction patterns, and in the middle is interference pattern.

When light passing through a gap, the minima (dark fringes) occurs when it diffracts at a angle of $\sin^{-1}\frac{n\lambda}{a}$, where $n \in \mathbb{Z} \setminus \{0\}$, a is the gap width and λ is the wavelength.

Polarized Light

To produce polarized light, one can use

- Polaroid
- Reflection.

  When a light is reflected by a medium of refractive index n, and the incident ray is $\tan^{-1} n$ (The polarizing angle), the reflected ray is totally plane polarized.

Polarized light can be used for

- Reducing glare.
- Stress analysis
- LCD

Infrared Radiation (IR)

At low temperature, IR is emitted by a body.

At 500°C, red light is emitted as well. (Red-hot)

After that, orange, yellow, … violet will be shown.

At 1000°C: White-hot

After that, UV will be emitted.

Absorption of IR → Warm.

Can be detected by:

- Special photographic films, which is sensitive to IR.
- Very sensitive photoelectric devices.
- Thermo-detector, includes:
  - Thermometer
  - Thermopile (熱電堆), which consists of many thermocouples (熱電偶) in series.
  - Bolometer (幅射熱測定器)

Ultraviolet Radiation (UV)

Fluorescent (螢光) materials absorb UV and re-radiate visible light.

X-Ray

- Travel in st. lines
- Readily penetrate matter.

  Penetration is least in materials containing elements of high density and high atomic number. E.g. sheet of Pb 1 mm think.
- Not deflected by electric or magnetic fields.
- Eject e^- from matter by photoelectric effect, so:
  - Ionize a gas, permitting it to conduct.
  - Cause cetain substances, e.g. Ba-platinocyanide, to fluoresce
  - Affect a photographic emulsion in a similar manner to light.

Heat & Thermodynamics

Absolute Zero

0 K = -273.15 °C

[K = °C + 273.15]

Molar Heat Capacity

To most solids, it needs 25 J to heat up a mole of substance for 1°C.

Molar Heat Capacity ~ 25 J mol^{-1} K^{-1} for most solids.

Cooling Laws

$$\text{Rate of loss of heat} \propto \left(T - T_0\right)^{5/4}$$

(For cooling in still air by natural convection)

$$\text{Rate of loss of heat} \propto \left(T - T_0\right)$$

(Under forced convection, e.g. wind)

Gas Laws

For ideal gas:

$$\frac{pV}{T} = \text{constant}$$

p: Pressure. *V*: volume. *T*: temperature in K.

$$\frac{pV}{nT} = R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$$

n: Number of moles in the gas = $\dfrac{\text{mass of gas in kg}}{\text{molar mass (kg mol}^{-1}\text{)}}$

Pressure

$$pV = \frac{1}{3} nm\overline{v^2}$$
$$p = \frac{1}{3}\rho\overline{v^2}$$

p: Pressure.

V: volume.

n: # of moles

m: Mass of gas

$\overline{v^2}$: Mean speed square. $= \dfrac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_n^2}{n}$ (v_k: speed of k^{th} molecule)

ρ: density

For air, $\sqrt{\overline{v^2}} = 485 \text{ m s}^{-1}$

Laws of Thermodynamics

Zeroth Law:

If bodies A and B are each separately in thermal equilibrium (no net flow of energy) with body C, then A and B are in thermal equilibrium with each other.

E.g.: If C is a thermometer and reads the same when in contact with A and B, then both of them are at the same temperature.

First Law:

$$\Delta Q = \Delta U + \Delta W$$

ΔQ: Heat supplied to a mass of gas

ΔW: External work done by it.

ΔU: Increase of internal energy.

ΔQ: +ve if heat supplied to the gas. –ve if transferred from it.

ΔW: +ve if expand. –ve if compress.

Second Law:

Heat cannot be transferred continually from one body to another at a higher temperature unless external work in done.

若無外影響，由高溫至低溫的方向是不可逆的.

Work Done by Expanding Gas

$$W = \int_{V_1}^{V_2} p\, dV = p\left(V_2 - V_1\right)$$

p: Pressure.

V_1, V_2: Initial/Final volume.

Also applies for compressing.

Expansion of Solids

If a solid of length l increases in length by δl owing to a temperature rise δT,

$$\alpha = \frac{\delta l}{l} \cdot \frac{1}{\delta T}$$

α: Linear Expansivity.

Unit: K^{-1}.

If the original length of a solid is l_0, after rising for T K, the length is:

$$l_T = l_0\,(1 + \alpha T)$$

If a solid of c.s. area A increases by δA owing to a temp. rise δT,

$$\beta = \frac{\delta A}{A} \cdot \frac{1}{\delta T}$$

β: Superficial Expansivity.

Unit: K^{-1}.

If the original c.s. area of a solid is A_0, after rising for T K, the area is:

$$A_T = A_0\,(1 + \beta T)$$

For a given material, $\beta \sim 2\alpha$.

Cubic Expansivity:

$$\gamma = \frac{\delta V}{V} \cdot \frac{1}{\delta T}$$

Usually, $\gamma \sim 3\alpha$.

Thermal Conductivity

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

Q: Heat. t: Time. A: c.s. Area. T: temperature. x: Length.

k: Thermal conductivity of the material.

Unit: W m^{-1} K^{-1}.

Fourier's Law

In a conductor of length x, c.s. area A and thermal conductivity k, where the temperatures at two ends are T_2 and T_1 ($T_2 > T_1$), the quantity of heat Q passing any point in time t when the lines of heat flow are // and steady state has been reached:

$$\frac{Q}{t} = kA\left(\frac{T_2 - T_1}{x}\right)$$

Charles' Law and Pressure Law for Gas

For gases,

$$V = V_0 (1 + \alpha T)$$

$$p = p_0 (1 + \beta T)$$

$$\alpha \approx \beta \approx \frac{1}{273} \approx 0.00366\,\text{K}^{-1}$$

Indicator Diagrams

An indicator diagram is a graph showing how the pressure p of a gas varies with its volume V during a change. (y-axis = p, x-axis = V.)

Principal Heat Capacities of Gas

Molar Heat Capacity at Const. Vol (C_V) is the heat req. to produce unit rise of temp. in 1 mol of gas when vol. is kept const.

Molar Heat Capacity at Const. Pressure (C_p): Similar to C_V, but pressure is const.

$$C_p - C_V = R.$$

R: 8.31 J mol^{-1} K^{-1}.

For ideal monatomic gas, $C_V = 12.5$, $C_p = 20.8$.

Atomicity	$\boldsymbol{\gamma = C_p / C_V}$
Monatomic	1.67
Diatomic	1.40
Polyatomic	1.30

Heat Processes

Isovolumetric

$$\Delta W = 0$$

$$\Delta Q = \Delta U = C_V(T_2 - T_1)$$

Ind. Diag: a vertical st. line

Isobaric (Const Pressure)

$$\Delta Q = \Delta U + \Delta W$$

$$C_p\,\Delta T = C_V\,\Delta T + p_1\,\Delta V.$$

Ind. Diag: a horizontal st. line

Isothermal

$$pV = \text{const}$$

Ind. Diag.: Part of $xy = k$. (Hyperbola)

Adiabatic (隔熱)

$$\Delta Q = 0$$

$$\Delta U + \Delta W = 0$$

$$\frac{p^{\gamma-1}}{T^{\gamma}}, pV^{\gamma}, TV^{\gamma-1} \text{ are const.}$$

Ind. Diag.: Curve.

Saturation Vapor Pressure (SVP)

The svp of a substance is the pressure exerted by the vapor in equilibrium with the liquid.

A liquid boils when its svp equals the external pressure.

Van der Waal's Equation

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

a: const for effect of attractive intermolecular forces.

b: const for effect of repulsive intermolecular forces.

Entropy (熵)

A quantum of energy = the energy which is simple integral multiple of a certain minimum.

(Pl. of Quantum = Quanta)

1 Quantum of energy
2 Quanta of energy
3 Quanta of energy
4 Quanta of energy…

df

Entropy: Measure of "disorder" in a system.

Change of entropy (ΔS):

$$\Delta S = k\Delta(\ln W) = \frac{\Delta U}{T} = \Delta\left(\frac{Q}{T}\right)$$

k: 1.38 × 10-23 J K^{-1}.

W: number of ways which q quanta can be distributed in n atoms (?).

U: Internal energy

T: Temperature.

Q: Total Energy

The second law of thermodynamics can be re-stated as:
In a closed system,

$$\Delta S > 0.$$

Nuclear Physics

Radioactivity

Type	Alpha ray / particle	Beta ray / particle	Gamma ray
Symbol	α	β	γ
Actual Identity	Helium Nucleus (2 Proton + 2 Neutron)	Electron	EM Wave (Gamma ray)
Range in Air	Few cm	Several m	Very long
Stopped by	Thick sheet of paper	Few mm of Aluminum	None. It can penetrate several cm of Lead
Ionization Power	Intense	Less intense	Weak
Mass	High	Light	None
Charge	+2	-1	0
Speed	5~7% of c	99% of c	c
Energy	4 ~ 10 MeV	0.025 ~ 3.2 MeV	1.2 ~ 1.3 MeV

Decay of Atom

If too much proton / nucleus to heavy, do α decay. E.g.:

$$^{226}_{88}\text{Ra} \to\ ^{222}_{86}\text{Rn} +\ ^{4}_{2}\text{He}$$

If too much neutron, do β decay. E.g.:

$$^{14}_{6}\text{C} \to\ ^{14}_{7}\text{N} +\ ^{0}_{-1}\text{e} + \bar{\nu}_\text{e}$$

$\bar{\nu}_\text{e}$: Antineutrino. Will be introduced later.

After decay → Too much energy → Release by gamma ray.

Decay Law

$$N = N_0 e^{-\lambda t}$$

N: Number of undecayed nuclei now.

N_0: Initial number of nuclei.

t: Time from initial state.

λ: Decay constant. Unit: Bq (s^{-1})

e: 2.718281828459045…

$$\lambda = -\frac{1}{N}\frac{dN}{dt}$$

Half Life

由開始時至剩半變衰變原子核需時.

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

ln 2 = 0.693147180559945309417232121458l8…

Instrument

Can be measured by GM tube.

Usage

- The radioisotopes of an element can be "tracers" in medicine, agriculture & biological research, as they are chemically identical.
- Carbon-14 Dating ($^{14}N + n \rightarrow {}^{14}C + {}^{1}H$)
  Half-life = 5700 years
- Check thickness & density of material (by β)
- γ from Co-60 Radiotherapy: Replace X-Ray, as X-Ray is more $$$.
- Sterilization of food.
  Meat can be preserved in fresh for 15 days instead of 3.
- Smoke detector.

Hazard

- Immediate damage to tissue
- Radiation burn
- Radiation sickness
- Loss of hair
- Death (Extreme)
- Cancer, Leukemia (白血病), Eye cataracts (Delayed Effects)
- Hereditary defects (生天缺憾) (Due to Genetic Damage)

Damage to body cells due to creation of ions which upset or destroy them.

Susceptible (易受影響) parts =

- Reproductive organs
- Blood-forming organs (e.g. liver)
- Eye

{Hazard from α is slight, unless the source enters the body}

Absorbed Dose D = Energy absorbed unit mss of irradiated material.
Unit: Gy.

Dose Equivalent H = Effect that a certain dose of a particular kind of ionizing radiation has on a person.
Unit: Sv.

Relative Biological Effectiveness (RBE):

$$\text{RBE} = H \times D.$$

For X-ray & γ, RBE ~ 1.
For α, proton & fast neutron, RBE ~ 20.

A year dose from natural bg radiation ~ 0.0015 Sv.
A dose from a chest X-ray ~ 0.0003 Sv.
Dose from experimental source in school = very small
Dose for Radiation Worker should < 0.05 Sv a year
5 Sv to every part of body ➔ Kill > 50% of those receiving it in 2~3 months

Particle Physics

Energy of EM Wave

$$E = hf$$

h: Planck constant = 6.63×10^{-34} J s.

Wave-Particle Duality

Matter and radiation have both wave-like and particle-like properties.

E.g.: Electrons (e^-) has interference pattern.

"Wavelength" of a particle:

$$\lambda = \frac{h}{p}$$

h: Planck's const. p: momentum of particle.

Mass vs. Energy

When a particle with mass m kg is totally "broken down" to energy, then:

$$E = mc^2$$

Disintegration Energy

E.g.: ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$.

Atomic mass of ^{226}Ra = 226.0254; ^{222}Rn = 222.0176; ^{4}He = 4.002602.

Mass difference in reaction = 226.0254 – 222.0176 – 4.0026 = 0.0052

Energy carried away by γ = 0.0052 × 931 = 4.84 MeV.

Particles

Proton. Symbol = p.

Neutron. Symbol = n.

Electron. Symbol = e^-.

Antiparticles

Particle which has the same property of its corresponding "particle", except the charge and spin (which is opposite).

Particle + Antiparticle → Energy

E.g. $e^+ + e^- \rightarrow \gamma + \gamma$. [$Q$ = 1.02 MeV]

Spin

Angular Momentum, but quantified. (Thus spin must be conserved)

0: Same when you look from every position. Like the letter "O".

1: Same when you rotate 360°. Like the letter "Q".

2: Same when you rotate 180°. Like the letter "S".

1/2: Same when you rotate 720° (2 cycles).

Particles with non-integral spins: Makes up matters. Called "Fermions".

Particles with integral (整數) spins: Force carriers. Called "Bosons".

- Spin = Even number (0, 2, 4,…): Carries Attractive Force (e.g. gravity)
- Spin = Odd number (1, 3, 5,…): Carries Repulsive Force (e.g. Strong force)

Lepton (輕子)

Particle	Symbol	Charge	Antiparticle	Mass (MeV/c^2)
Electron	e^-	-1	e^+	0.5
Electron neutrino (電中微子)	ν_e	0	$\bar{\nu}_e$	0
Muon (μ介子)	μ^-	-1	μ^+	106
Muon neutrino	ν_μ	0	$\bar{\nu}_\mu$	0
Taon (τ介子)	τ^-	-1	τ^+	1780
Taon neutrino	ν_τ	0	$\bar{\nu}_\tau$	0

All leptons have lepton number (L) = +1. Spin = ±1/2. Anti-lepton: L = -1.

Lepton # for e, μ, τ must be conserved.

Quarks (夸克)

Protons and neutrons are not fundamental particles. They are built-up from quarks.

Particle	Symbol	Strangeness	Charge	Mass (MeV/c^2)
Up	u	0	+2/3	5
Down	d	0	-1/3	10
Strange	s	-1	+2/3	200
Charm	c	0	-1/3	1500
Top	t	0	+2/3	180000
Bottom	b	0	-1/3	4300

All quarks spin = 1/2. Baryon # = 1/3.

3 Quarks = Baryon (重子)

Quark + Anti-Quark = Meson (介子)

E.g.: Baryons:

Particle	Symbol	Charge	Strangeness	Structure
Proton	p	+1	0	u-u-d
Neutron	n	0	0	u-d-d
Lambda	Λ^0	0	-1	
Sigma	Σ^+	+1	-1	u-u-s
	Σ^0	0	-1	
	Σ^-	-1	-1	
Xi	Ξ^0	0	-2	
	Ξ^-	-1	-2	

Mesons:

Particle	Symbol	Charge	Strangeness	Structure
Pion	π^+	+1	0	
	π^0	0	0	
Kaon	K^+	+1	+1	d-$\overline{s}$
	K^0	0	+1	
Eta	η	0	0	

Forces

Gravitational Force.

Force between masses. Extremely weak.

Range = infinite

Carrier = Graviton (?)

Electromagnetic Force.

Force acts between charged particles.

Range = infinite

Carrier = "Virtual" Photon (光子)

Weak Force

Responsible for radioactive decay when β^- are emitted.

Range = 10^{-17} m

Carrier = Z^+, W^0, Z^- [These are very heavy].

Strong Force

Holds quarks together. Holds neutron & protons together.

Attractive Range = $1.2 \times 10^{-15} \sim 3 \times 10^{-15}$ m.   Repulsive Range = 10^{-15} m.

Carrier = Gluon (膠子)

Special Relativity

Frame of Reference

Two observers are in different frame of ref. if they are traveling in diff. vel.

Inertial Ref. Frame = frame which Newton's 1st Law holds, i.e., the observer is not accelerating.

Postulates

The laws if physics are the same for all observers in all inertial reference frames.

The measured velocity of light in vacuum, *c*, is the same in all inertial frames and is independent of the motion of the light source or the observe.

Time Dilation

The "time" in static is faster than the "time" in moving objects. If the "time" elapsed in the moving place is t_p ("proper time"), then for the static one:

$$t_p = \gamma t$$

Note: $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}; \beta = \dfrac{v}{c}$. v = speed of moving obj.

Length Contraction

The observer on the moving object measures a length, it will be shorter than measuring it in static.

$$l_p = \gamma l$$

Mass Increase

Rest mass (m_0): unchanged whenever how faster an object moves

Relativistic mass (m): can be changed. More if moving faster.

$$m = \frac{m_0}{\gamma}$$

Momentum, energy and mass

$$E^4 = m_0^{\,2}\,c^4 + p^2\,c^2$$

Astrophysics

Gravity between two Objects

For two objects with mass m_1 kg and m_2 kg, where their distance is r m,

$$F = G\frac{m_1 m_2}{r^2}$$

$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.

Kepler's Laws

Each planet moves in an ellipse which has the sun at one focus.

The line joining the sum to the moving planet sweeps out equal area in equal times.
[i.e., the planet moves slower away the sun, and faster near the sum]

If t = time for a revolution, r = the mean distance from the planet to the sun,

$$r^3 \propto T^2$$

Length Measurements

ly = Light year = 9.45×10^{15} m

pc = parsecs = 3.26 ly.

AU = mean Earth-Sun dist = 1.496×10^{11} m

Brightness Measurements

Absolute Luminosity, *L*:

$$L = A\sigma T^4 = 4\pi r^2 \sigma T^4$$

A: Surface area of star

r: Radius of star

$s = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}.

T: Temperature (In K)

Apparent Brightness / Luminosity, *l*:

$$l = \frac{L}{4\pi d^2}$$

d: Distance from observer (Earth) to star.

Apparent Magnitude, *m*:

$$m = \text{constant} - 2.5 \log_{10} l$$

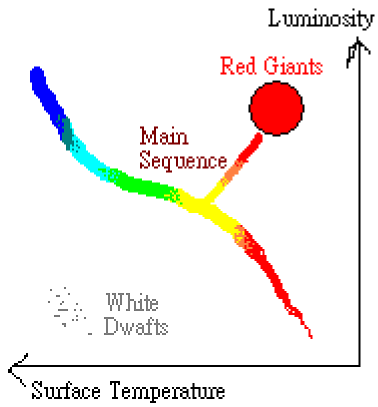
Absolute Magnitude, *M*:

$$M = m - 5\log\left(\frac{d}{10}\right)$$

d: in parsecs.

The brighter the star is the lower *M* is.

Hertzprung-Russell Diagram



The surface temperature of a star can be estimated by:

$$l_{max}\, T = 2.9 \times 0\text{-}3 \text{ m K}$$

Stellar Spectral Classes

O-stars: 40000 ~ 30000 K

A-, B-stars: 20000 ~ 10000 K

F-, G-stars: 7500 ~ 5500 K

K-, M-stars: 4500 ~ 3000 K

Our sun is a G-star.

Hubble's Law

If a star moves toward us, the "color" of the star shifts to blue. Otherwise, it shifts to red.

As the universe is expanding, all stars are moving away from us. (Red shift)

The recession speed *v*:

$$v = H_0 d$$

d = distance from earth (observer)

H_0 = Hubble constant = 23 ± 3 km s^{-1} Mly^{-1}.

1 / H_0 is the estimation of the age of universe.

Fusion Reaction and Fate of Stars

PP cycle:

Fusion Reaction	Energy Released (MeV)
${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu$	0.4
${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma$	5.5
${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H$	12.9

Q = 24.7 MeV for one cycle.

With carbon (CNO cycle):

$$
\begin{aligned}
{}^{12}_{6}C + {}^{1}_{1}H &\rightarrow {}^{13}_{7}N + \gamma \\
{}^{2}_{1}N &\rightarrow {}^{13}_{6}He + e^{+} + \nu \\
{}^{13}_{6}C + {}^{1}_{1}H &\rightarrow {}^{14}_{7}N + \gamma \\
{}^{14}_{7}N + {}^{1}_{1}H &\rightarrow {}^{15}_{8}O + \gamma \\
{}^{15}_{8}O &\rightarrow {}^{13}_{7}N + e^{+} + \nu \\
{}^{15}_{7}N + {}^{1}_{1}H &\rightarrow {}^{12}_{6}C + {}^{4}_{2}He
\end{aligned}
$$

Net result of these: p + p + p + p → He.

CNO cycle dominates in stars with temperature > 2×10^{7} K, if C is avail.

The energy released is first used to counteract the gravity, preventing the core collapsing. Then release as heat + light to surroundings.

If H is used up (The star is about to "die"):

- Gravity dominates
- Core contracts
- Gravitational P.E. → K.E.
- Core Hotter
- Faster burn-up of remaining H envelope
- Expansion and cooling of outer surface gas
- Become "Red Giant".
- Helium Burning Starts:

$$
\begin{aligned}
&{}^{4}_{2}He + {}^{4}_{2}He \leftrightarrow {}^{8}_{4}Be + \gamma \\
&{}^{8}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + \gamma
\end{aligned}
$$

- Ejection of material from H envelope

- For small stars (< 1.4 mass of sun):
  - Core → Carbon
  - Core contract
  - Material lost from outer envelope forms o planetary nebula (星雲).
  - The core shrinks to a white dwarf.
  - Radiates hear unit it cooled to a back dwarf.
- For more massive stars
  - Carbon fuses, producing O, Si, … Fe (For mass > 8 suns)
  - Core → Layered "Onion" Structure
  - Energy cannot be extracted from fusion of elements heavier than Fe, so such reactions do not fuel.
  - Core → Iron
  - Density, Temperature: very high
  - $e^- + p \rightarrow n$
  - Collapses catastrophically (災難地) until density of neutron is so high that resisting further contraction
  - Core "bounces back" and a shock wave is generated which blows off the outer layers of star in a giant supernova (超新星) explosion.
  - Core become neutron star or black hole.

Birth of Universe

- 0 s,∞ K:

  Big Bang.

  The 4 forces are the same.
- 10^{-43} s, 10^{32} K:

  GUT Era

  Gravity separates from the 4 forces.
- 10^{-35} s, 10^{27} K:

  Quark Era

  Inflationary scenario: Expansion was exponential.

  Leptons & Quarks were formed from radiation.
- Short time later (10^{-12} s):

  Strong force separate.

  Protons & Neutrons formed.

  Matter + Antimatter → Energy
- 10^{-4} s, 10^{14} K:

  Lepton Era

  Electroweak force broken up.

- 10 s, 10^{10} K:
  Radiation Era
  Continue Matter + Antimatter → Energy
- 1 min ~ 20 min
  Nucleosynthesis
  p + n → light neuclei
  H : He = 3 : 1 (till now)
- 300 000 year ~ now:
  Matter era
  Atoms formed
  Stars formed, galaxies, …

Black hole

Schwarzchild radius of black hole:

$$R_S = \frac{2GM}{c^2}$$

M: mass of black hole.
G: 6.7×10^{-11} N m^2 kg^{-2}

R_S is the radius of the spherical event horizon of the black hole.
(We cannot see events within the event horizon)

Materials

Terms

Strength = How great an applied force a material can withstand before breaking
Stiffness = Opposition a material set up to being distorted by having its shape and/or size changed. (Stiff = Not Flexible. Totally stiff = rigid)
Ductility = The ability of the material to be hammered / pressed / bent / rolled / cut / stretched into useful shapes
Toughness = Not brittle.

Stress σ = Force acting on unit c.s. area. $\sigma = \frac{F}{A}$. [Unit = Pa]

Strain ε = Extension of unit length. $\varepsilon = \frac{e}{l}$ [e = Extended length. l = Original length]

Deformation

Elastic deformation:

$\sigma \propto \varepsilon$.

It returns to its original length when stress is removed. No extension remains

Plastic deformation:

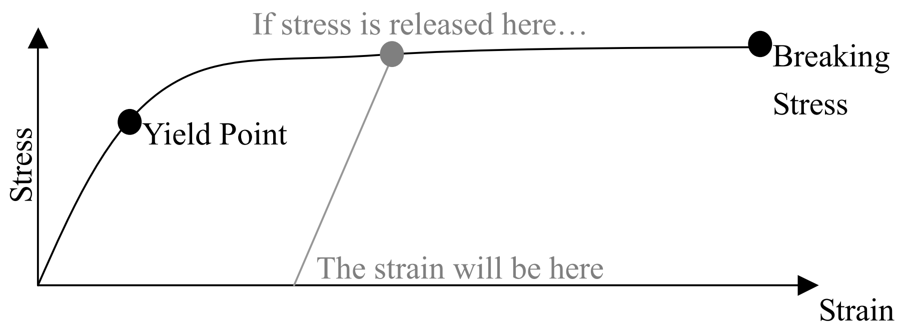
After a certain strain, called "yield point", a permanent/plastic deformation starts.

Recovery is incomplete after removing the stress.

Breaking Stress:

The greatest stress a material can bear.

After that → Break.



Hooke's Law, Young Modulus

$$E = \frac{\sigma}{\varepsilon} = \frac{Fl}{Ae}$$

E: Young Modulus.

Unit: Pa.

E measures elastic stiffness.

If *E* is large, it resists elastic deformation strongly.

Material	***E*** **(10^{10} Pa)**
Steel	21
Copper	13
Glass	7
Polythene	0.5
Rubber	0.005