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Branch switching characteristics of coupled flutter instability

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ABSTRACT

As a bridge gets longer and slender, one of the main problems that have to be faced is the flutter instability of bridge girders such as the torsional flutter and the coupled flutter. Especially, the coupled flutter instability has become the most concerning issue in the design of long-span bridges, and the precise understanding on its generation mechanism is indispensable in order to effectively stabilize this divergent oscillation. In this study, the heaving/torsional 2-DOF flutter analyses and wind tunnel tests are carried out, focusing on the branch switching characteristics of the coupled flutter using B/D=20 rectangular section. Moreover, the branch co-existing characteristics in flutter onset velocity range is investigated in detail.

INTRODUCTION

Longer and slender bridge span leads to higher flexibility, lower structural damping, and lighter weight, being thus very vulnerable to wind excitations such as vortex-induced vibration, buffeting, and flutter. The flutter phenomenon is a self-excited oscillatory instability, which is induced by the interaction between the inertial forces, the damping forces, and the self-excited aerodynamic forces acting on structures. When a slender bridge is subjected to wind load of a certain critical wind velocity, i.e. flutter onset velocity, instead of dissipating energy, the aerodynamic forces start to feed energy into the oscillatory bridge structure, which make the bridge vibrate in a divergent and destructive manner, e.g. the collapse of the old Tacoma Narrows Bridge in 1940. Therefore, the flutter instability of long-span bridge decks has become a major concern for their design, and many countermeasures to stabilize this catastrophic oscillation have been proposed from the view points of aerodynamics and structural dynamics.

Generally, the streamlined box girders and the multi-box girders are adopted as deck system of long-span or super long-span bridges. These types of bridge girders are vulnerable to the coupled flutter instability which has appeared in the thin plate section or airfoil. In detail, this coupled flutter instability is defined as “stiffness-driven type” flutter, indicating that two coupling branches of heaving and torsional motions coalesce to a single flutter frequency (see Namini, A. et al., 1992). However, based on the results of the commonly used method, the complex eigenvalue analysis (CEVA), the predicted oscillation frequencies of different branches are not necessarily the same when flutter occurs since this approach is not fundamentally based on the flutter generation mechanism of each branch but only based on the mathematical calculation of 2-DOF dynamic instability (see e.g. Chen, X. and Kareem, A., 2003). On the other hand, Chen and Cai (2003) presented the results of time-domain flutter analysis for a long-span suspension bridge, in which the single flutter frequency is observed from the flutter onset velocity range because the modal coupling effects force all modes to respond to the oscillation frequency of the critical mode. Moreover, the authors reported the branch switching characteristics of coupled flutter instability from torsional branch (TB) to heaving branch (HB) at high wind velocity range through the Step-by-step analysis (SBSA) and wind tunnel tests for the rectangular section with the side ratio of B/D=20 (Matsumoto, M. et al., 2005).

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So far, the flutter analysis usually focused on finding the flutter onset velocity and the corresponding flutter frequency, and the generation mechanism and nature of transition of oscillation frequencies during the flutter initiation process has not been fully clarified. Thus, in this study, the heaving/torsional 2-DOF flutter analyses based on CEVA and SBSA and wind tunnel tests are carried out, focusing on the branch switching characteristics of coupled flutter instability using B/D=20 rectangular section model. Furthermore, the branch co-existing characteristics in flutter onset velocity range is examined in detail.

FLUTTER ANALYSIS METHOD

For 2DOF fundamental sections, the equations of motion in heaving and torsional directions are expressed using eight aerodynamic derivatives proposed by Scanlan as follows (Scanlan, R. H., et al., 1974);

$$m\ddot{\eta}+c_\eta\dot{\eta}+k_\eta\eta=\rho b^2\omega_F H_1^*\dot{\eta}+\rho b^2\omega_F{}^2H_4^*\eta+\rho b^3\omega_F H_2^*\dot{\phi}+\rho b^3\omega_F{}^2H_3^*\phi \tag{1}$$

$$I\ddot{\phi}+c_\phi\dot{\phi}+k_\phi\phi=\rho b^3\omega_F A_1^*\dot{\eta}+\rho b^3\omega_F{}^2A_4^*\eta+\rho b^4\omega_F A_2^*\dot{\phi}+\rho b^4\omega_F{}^2A_3^*\phi \tag{2}$$

where η and ϕ are the heaving and torsional displacements, m and I are the mass and mass inertia per unit length, c_η and c_ϕ are the damping coefficients of heaving and torsional vibrations, k_η and k_ϕ are the heaving and torsional stiffness, ρ is the air density, b is the half chord length, ω_F is the circular frequency, H_{i}^* and A_{i}^* (i=1~4) are the aerodynamic derivatives, k is the reduced frequency (=$b\omega_F/V$), and V is the wind velocity.

The complex eigenvalue analysis (CEVA) method is widely used to solve the 2-DOF equations of motions, given in expressions (1) and (2). The main characteristics of flutter instability, such as the logarithmic damping decrement (δ), frequency (f_F), heaving/torsional amplitude ratio (η_0/φ_0) and the phase difference between heaving and torsional response (Ψ), are obtained for the torsional branch (TB) and the heaving branch (HB) by resolving the oscillation frequencies into the initial values of the corresponding natural branches. The expression (3) is the deterministic equation of CEVA. This approach is not fundamentally based on the flutter generation mechanism of each branch but only based on the mathematical calculation of 2-DOF dynamic stability problem. Thus, it is difficult to exactly identify the coupled flutter branch, especially in the flutter onset velocity range in which the heaving and torsional frequency may approach each other.

$$\begin{aligned}&\lambda^4+(2\varsigma_\phi\omega_\phi+2\varsigma_\eta\omega_\eta)\lambda^3+(4\varsigma_\phi\varsigma_\eta\omega_\phi\omega_\eta+\omega_\phi{}^2(1-\varsigma_\phi{}^2)^{1/2}+\omega_\eta{}^2(1-\varsigma_\eta{}^2)^{1/2}-(\rho b^2/m)(\rho b^3/I)\omega_F{}^2H_2^*A_1^*)\lambda^2\\&+(\omega_\phi{}^2(1-\varsigma_\phi{}^2)^{1/2}+\omega_\eta{}^2(1-\varsigma_\eta{}^2)^{1/2}-(\rho b^3/m)\omega_F{}^2H_3^*-(\rho b^2/I)\omega_F{}^2A_4^*)\lambda+(\omega_\phi{}^2\omega_\eta{}^2(1-\varsigma_\phi{}^2)^{1/2}(1-\varsigma_\eta{}^2)^{1/2}\\&-(\rho b^3/m)\omega_F{}^2H_3^*-(\rho b^2/I)\omega_F{}^2A_4^*)=0\end{aligned} \tag{3}$$

On the other hand, the coupled flutter characteristics of HB and TB can be identified using Step-by-step analysis (SBSA), which is based on the 1-DOF forced vibration analysis considering the coupling terms of aerodynamic derivatives between the heaving and torsional motions, i.e. coupled flutter generation mechanism (Matsumoto, M., et al., 1995). In SBSA, the branch identification depends on the governing equation of motions which characterizes the oscillation frequency and damping. Namely, if the final equation of SBS analysis characterizing the oscillation frequency and damping is that of heaving motion, the branch is defined as HB, and on the contrary case, it is defined as TB. Through the iteration of oscillation frequency under a certain wind velocity, the frequency and logarithmic damping for TB and HB are given as expressions (4) and (5), respectively.

$$\begin{aligned}\omega_\phi&=\sqrt{\omega_{\phi0}{}^2-(X/\pi)\omega_F{}^2A_3^*-(XY/\pi)\omega_F{}^2\{A_1^*|H_2^*|\sin\theta_1+A_1^*|H_3^*|\sin\theta_2+A_4^*|H_2^*|\cos\theta_1+A_4^*|H_3^*|\cos\theta_2\}}\\\delta_\phi&=-XA_2^*-XY\{A_1^*|H_2^*|\cos\theta_1+A_1^*|H_3^*|\cos\theta_2-A_4^*|H_2^*|\sin\theta_1-A_4^*|H_3^*|\sin\theta_2\}\end{aligned} \tag{4}$$

where

$$X=\pi\rho b^4/I,\ \ Y=(\rho b^2/m)\cdot(\omega_F/\omega_\eta^*)^2/\sqrt{(1-(\omega_F/\omega_\eta^*)^2)^2+4\zeta_\eta^{*2}(\omega_F/\omega_\eta^*)^2}$$

$$\begin{aligned}\omega_\eta&=\sqrt{\omega_{\eta0}{}^2-(X/\pi)\omega_F{}^2H_4^*-(XY/\pi)\omega_F{}^2\{H_2^*|A_1^*|\sin\theta_1+H_2^*|A_4^*|\sin\theta_2+H_3^*|A_1^*|\cos\theta_1+H_3^*|A_4^*|\cos\theta_2\}}\\\delta_\eta&=-XH_1^*-XY\{H_2^*|A_1^*|\cos\theta_1+H_2^*|A_4^*|\cos\theta_2-H_3^*|A_1^*|\sin\theta_1-H_3^*|A_4^*|\sin\theta_2\}\end{aligned} \tag{5}$$

where

$$X=\pi\rho b^2/m,\ Y=(\rho b^4/I)\cdot(\omega_F/\omega_\phi^*)^2/\sqrt{(1-(\omega_F/\omega_\phi^*)^2)^2+4\zeta_\phi^{*2}(\omega_F/\omega_\phi^*)^2}$$

BRANCH SWITCHING CHARACTERISTICS

Figure 1 shows the coupled flutter characteristics of thin plate, the logarithmic damping and the flutter frequency, based on CEVA and SBSA. As shown in fig. 1, both flutter characteristics by two different analysis methods show fairly good agreement in low wind velocity range. However, a drastic difference is observed at higher wind velocity range. In the conventional CEVA, the frequency curve of TB never crosses with that of HB, and the flutter frequency is continuously characterized for TB from the natural torsional frequency. The coupled flutter instability is observed also in TB at high wind velocity range. On the other hand, in the SBSA, both frequency curves cross each other at the vicinity of flutter onset velocity and the flutter frequency after flutter onset is characterized by the HB. Moreover, the HB damping curve suddenly decreases and approaches to the TB damping curve at the vicinity of flutter onset velocity. These tendencies in SBSA mean the branch switching of coupled flutter instability form TB to HB, that is, the coupled flutter instability at high wind velocity range occurs in HB but not in TB. By the SBSA, These switching characteristics in coupled flutter instability are also observed for other sections, such as rectangular sections with the side ratio B/D of 12.5, 15, 20, and the separated 2-box girder section with the side ratio B/D of 5 (Matsumoto, M et al., 2005). It should be noted that the solutions obtained from CEVA are similar to those obtained from SBSA at low and high wind velocity range, despite the difference of branches.

The different solutions between CEVA and SBSA may come from the different definition of coupled flutter branches, TB and HB as follows. 1) In the CEVA, the classifications of branches are not substantial from the physical viewpoint but mathematical judgement from the frequency property whether which curve in two frequencies curves, corresponds to the original starting point at initial wind velocity. 2) In the SBSA, the coupled flutter branches are divided into the HB and the TB according to the governing motions which characterize the frequency and the damping.

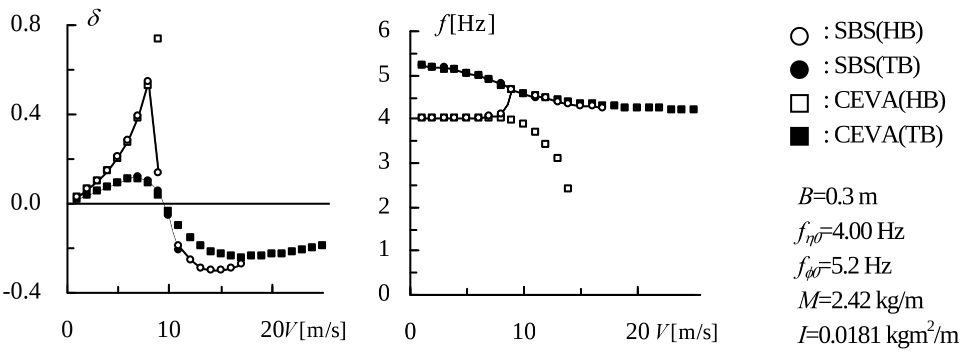


Fig. 1 Comparison of coupled flutter characteristics of thin plate by CEVA and SBSA

In order to verify the result of SBSA, the heaving and torsional 2-DOF free vibration wind tunnel tests are carried out using the B/D=20 rectangular section. The switching characteristics of coupled flutter branches predicted by SBSA are certainly confirmed by the direct measurements of damping and frequencies under smooth flow as shown in Figure 2, in which the identical dynamic parameters are used in flutter analyses.

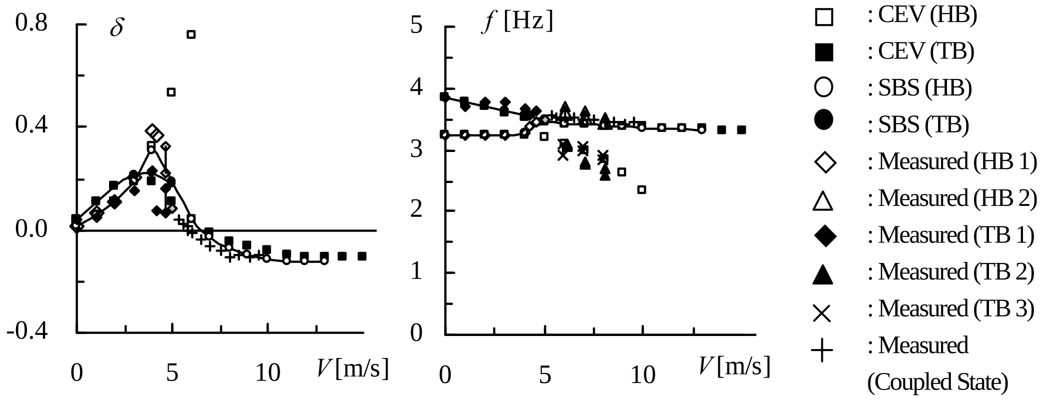
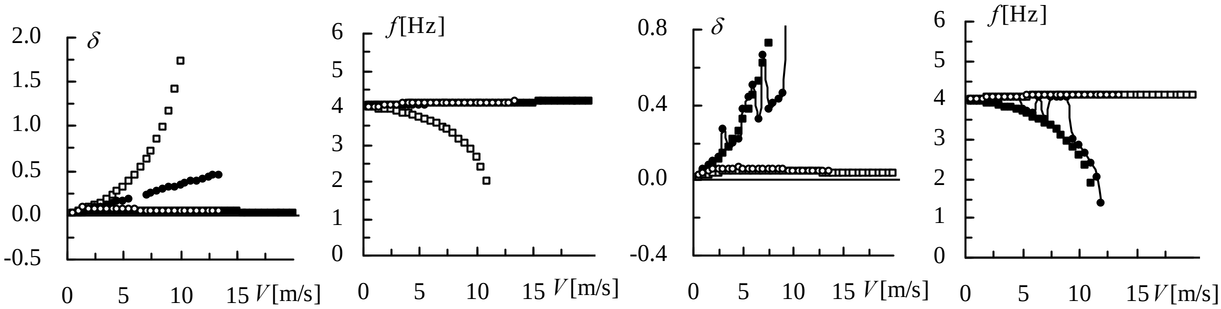


Fig. 2 Comparison of calculated coupled flutter characteristics for *B/D*=20 rectangular section with the result of free vibration test (*B*=0.3 m, *M*=2.98 kg/m, *I*= 0.0198 kgm^2/m, $f_{\eta0}$=3.26 Hz, $f_{\phi0}$=3.84 Hz, δ_η=0.0124, δ_ϕ=0.0413)

Around the transition wind velocity range of the branch switching, the solutions of HB and TB co-exist. This branch co-existing characteristics cannot be obtained from CEVA because this is a transient phenomenon. For further investigation of the branch co-existing characteristics, SBSA are carried out for thin plate with every 0.01 frequency ratio, $f_{\phi 0}/f_{\eta 0}$, from 1.00 to 1.10. In the case of frequency ratio 1.02 to 1.10, the solutions co-exist only at particular velocity range as many other cases, but in the case of frequency ratio 1.01, the solutions co-exist through wide velocity range as shown figure 3(a). It should be noted that the solutions of frequency, amplitude ratio and phase difference between the heaving displacement and the torsional displacement, are almost the same as the solutions of another branch, but that the solutions of damping show no agreement. Furthermore, in the case of frequency ratio 1.00, the solutions of TB of SBSA diverge between two solutions as shown in figure 3(b). These results imply that the solutions of SBSA, in particular TB, have some stable values even though they are unstable in real phenomenon. Then, SBSA are carried out using the solutions of CEVA as initial values. Results of these analyses imply that SBSA, in particular TB of SBSA, has some unstable solutions, which are affected by initial values. To verify this, 2DOF free vibration test is carried out. As a result, the existence of these unstable solutions is verified. Although these unstable solutions cannot be obtained from CEVA, the meaning of the existence of these solutions in the transition wind velocity range is important.



(a) $f_{\eta 0}$=4.00 Hz, $f_{\phi 0}$=4.04 Hz

(b) $f_{\eta 0}$=4.00 Hz, $f_{\phi 0}$=4.00 Hz

Fig. 3 Comparison of coupled flutter characteristics of thin plate with frequency ratio around 1.00
(B=0.3 m, M=2.42 kg/m, I=0.0181 kgm^2/m)

CONCLUSION

Based on the results of Step-by-step analysis, the flutter instability mechanism is much more clarified than before. The conclusions of this study can be summarized as follows.

(1) From the SBSA and the free vibration wind tunnel tests for the rectangular section of B/D=20, the branch switching of coupled flutter instability from TB to HB at high wind velocity range are confirmed.
(2) The existence of unstable solutions is verified by SBS analysis and wind tunnel test, and these unstable solutions of SBSA are affected by initial values.
(3) For future works, the difference between the SBSA and the CEVA at high velocity range and the physical meaning of the CEVA should be precisely examined.

REFERENCES

Chen, X. and Kareem, A. "Curve veering of eigrnvalue loci of bridges with aeroelastic effects" , *Jour. Eng. Mech. Division, ASCE., Vol.129,No. 2, pp. 146-159,* 2003.

Chen, S. and Cai, C. "Evolution of long-span bridge response to wind-numerical simulation and discussion", *Computers and Structures, Vol. 81, pp. 2055-2066,* 2003.

Matsumoto, M., et al,: "Flutter mechanism and its stabilization of bluff bodies", *Proc. of 9th ICWE*, 1995.

Matsumoto, M.,: "Flutter instability of structures", *Proc. of 4th EACWE*, 6-11, 2005.

Namini, A. et al. "Finite element-based flutter analysis of cable-supported bridges", *Jour. Struct. Eng. Division, ASCE., Vol.118, No. 6, pp. 1509-1526,* 1992.

Scanlan, R. H., et al, "Indicial aerodynamic functions for bridge decks", *Jour. Eng. Mech. Division, Proc. ASCE., Vol.100, EM4*, 1974.