Testing for the existence of a long-run relationship between UK consumption, income and inflation using Pesaran et al. bounds test procedure: A comparative study

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CONTENTS

I) Introduction

This paper provides an application of the bounds test procedure for testing for the existence of a long-run relationship suggested by Pesaran et al. (1997) using quarterly data on UK non-durable consumption, personal disposable income and inflation. Section II.a) briefly sets out the theory underlying the test procedure and section II.b) applies it to the time series under consideration. The test results are then checked against the conclusions following from the application of the conventional Engle-Granger (1987) two-step procedure (section III)) and the system-based rank tests suggested by Johansen (1988, 1991) (section IV)). Accordingly, sections III.a) and IV.a) respectively provide a brief discussion of the theoretical underpinnings of these test procedures while sections III.b), III.c) and IV.b) give the test results and conclusions obtained from applying them to the data. Section V) presents the overall conclusion drawn from the results in the light of the various conceptual problems encountered and indicates which long-run testing procedure seems currently preferable given the state of theoretical knowledge.

However, without discussing the partly still highly controversial theoretical issues concerning the 'appropriate' specification of consumption models in any detail, it should be clear that a fully specified UK consumption model will have to consist of a more complete set of regressors which takes into account the effect of, for example, assets/asset prices, credit restrictions and financial deregulation, demography, etc.[1] The analysis of these issues and the problematic modelling of a direct inflation effect,[2] however, are beyond the scope of this paper but will be considered when it comes to the interpretation of the various test results.

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[1] For an in-depth discussion of the various regressors generally to be considered, see, for example, Hadjimatheou (1987) and, more recently, Muellbauer and Lattimore (1994). See also Davis (1984) for a comprehensive survey of the development of some historically influential model specifications and Spanos (1989) for a critical discussion of some early 'stylized facts' in consumption modelling.

[2] A direct inflation effect might be rationalized along the lines of a 'classical' wealth, money or 'mass' illusion effect respectively. See Deaton (1977). However, Hendry and von Ungern-Sternberg (1981) argue convincingly in favour of an indirect rather than direct effect of inflation via wealth erosion and real income mismeasurement. In addition, illusion effects have early been rejected empirically. Finally, this issue gets more complicated when allowing for a potential role of the rate of interest. The modelling of a direct inflation effect is therefore somewhat controversial.

II) Testing for a long-run relationship between UK consumption, income and inflation using the Pesaran et al. bounds test procedure

II.a) The bounds test procedure suggested by Pesaran et al. (1997)

The emergence of the unit roots and cointegration literature as pioneered by Granger (1981), Granger and Weiss (1993), Engle and Granger (1987) and Johansen (1988) has undoubtedly encouraged and to some extent enabled applied economists to test for the existence of long-run relationships postulated by economic theory rather than taking them for granted.

On the other hand, however, the required pre-testing procedures for establishing the orders of integration of the underlying time series have introduced an additional element of uncertainty into the econometric analysis of time series data. In fact, as has been shown by (amongst others) Schwert (1989), Blough (1992) and Schlitzer (1994), conventional unit root tests such as the simple or augmented Dickey-Fuller (1979, 1981) tests (DF and ADF hereafter) or the stationarity test suggested by Kwiatkowski et al. (1992) (KPSS) suffer from considerable size distortions and generally lack power. This holds in particular when dealing with near-integrated processes[3] or (long) time series including structural breaks.[4] In addition, the various test statistics are highly sensitive to the parameterization chosen.

In the light of these drawbacks, Pesaran et al. (1997) suggest a bounds test procedure for the existence of a long-run relationship. They proceed from a general vector autoregessive (VAR) model in the (k+1)-vector $\mathbf{z}_t$:

$$\mathbf{z}_t=\mathbf{b}+\mathbf{c}t+\sum_{i=1}^{p}\phi_i\mathbf{z}_{t-i}+\varepsilon_t\ ,\ t=1, 2, \ldots, T, \tag{1.1}$$

with **b** representing a (k+1)-vector of intercepts, and **c** denoting a (k+1)-vector of trend coefficients. In the following, Pesaran et al. (1997, p. 2) assume seasonal and explosive roots to be absent and derive the unrestricted vector error-correction model (VECM) corresponding to (1.1):

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[3] See, for example, Cochrane (1991, p. 276) and Cavanagh et al. (1995, pp. 1133-1136).

[4] See, for example, Rappoport and Reichlin (1989), Perron (1990), Hendry and Neal (1991) and Schlitzer (1994). For an in-depth treatment of the issue of unit root-testing and structural breaks, see Stock (1994). For a more general critique of using autoregressive approximations along with unit root testing in applied economics, see Harvey (1997, pp. 196-198).

$$\Delta \mathbf{z}_t = \mathbf{b} + \mathbf{c}t + \Pi\, \mathbf{z}_{t-1} + \sum_{i=1}^{p} \Gamma_i \Delta \mathbf{z}_{t-i} + \varepsilon_t\,,\ t=1, 2, \ldots, T, \qquad (1.2)$$

where the (k+1)×(k+1)-matrices $\Pi = -\mathbf{I}_{k+1} + \sum_{i=1}^{p} \Phi_i$ and $\Gamma_i = -\sum_{j=i+1}^{p} \Phi_j,\ i = 1,2,\ldots,p-1$ contain the long-run multipliers and short-run dyamic coefficients of the VECM. After suitable partitioning of (1.2) into a dependent variable y_t and a vector of 'forcing' variables $\mathbf{x}_t$ with $\varepsilon_t = (\varepsilon_{1t},\ \varepsilon'_{2t})'$, it is further assumed that there exists only one (non-degenerate) long-run relationship.[5] Along with the additional assumption that the error vector ε_t follows a multivariate identically and independently distributed zero mean process with a non-singular variance matrix, finite fourth-order moments and contemporaneous correlation between ε_{1t}, ε_{2t} of the form $\varepsilon_{1t} = \mathbf{w}'\varepsilon_{2t} + \xi_t$, where $\mathbf{w} = \sum_{22}^{-1} \sigma_{21}$ and $\{\xi_t\}$ represents an $\mathrm{iid}(0;\sigma_\xi^2)$-process uncorelated with $\{\varepsilon_{2t}\}$, Pesaran et al. (1997, p. 3) end up with the following 'unrestricted' ECM:

$$\Delta y_t = a_0 + a_1 t + \phi y_{t-1} + \delta' \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \psi_i \Delta y_{t-i} + \sum_{i=0}^{p-1} \varphi_i \Delta \mathbf{x}_{t-i} + \xi_t, \qquad (1.3)$$

for all t=1, 2, …, T.[6]

On the basis of this specification, they suggest a bounds test procedure for the existence of a long-run relationship between y_t and $\mathbf{x}_t$ in the form of a Wald/F-test statistic on the null of ($\phi=0$ and $\delta'=\mathbf{0}$) against ($\phi\neq 0$ and ($\delta'\neq\mathbf{0}$ or $\delta'=\mathbf{0}$)) thereby allowing for the possibility of a degenerate long-run relationship between y_t and $\mathbf{x}_t$ under the alternative. Since Wald statistics for restrictions on coefficients that cannot necessarily be written as coefficients on I(0) regressors have non-standard limiting distributions,[7] Pesaran et al. (1997, pp. 7-8) derive the asymptotic distribution of the proposed test statistic under the above null and its asymptotic distributions for the two polar cases of $\{\mathbf{x}_t\}$ being stationary and being I(1) respectively. The corresponding results provide the basis for the critical values tabulated in Pesaran et al. (1997, pp. 24-25) for the two polar cases which in turn represent bounds covering all possible classifications of the regressors into I(0), I(1), cointegrated and fractionally integrated processes.

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[5] In this case, this translates into y_{t-1} not entering the regression equation for $\Delta\mathbf{x}_t$.

[6] For ease of exposition the underlying parameter definitions which simply reflect the relationship between the parameters in (1.3) and their counterparts in the $\Delta\mathbf{x}_t$-specification after substitution of the ε_{1t}- and $\Delta\mathbf{x}_t$-equations into (1.3) are not presented. See Pesaran et al. (1997, pp. 3-4).

[7] See, for example, Watson (1994, pp. 2858-2860).

Accordingly, this bounds test procedure allows to make inferences irrespective the absence of any knowlegde concerning the actual order of integration of the series under investigation as long as the value of the test statistic falls outside the critical bounds. If the value of the test statistic is found to lie inside the critical bounds, however, it has to be fallen back on the problematic conventional unit root-testing discussed above.

II.b) **Test application and results**

Using quarterly seasonally adjusted UK data over the period 1955(1) to 1995(1),[8] the bounds test procedure was carried out on the basis of OLS regressions of the following unrestricted ECM specification:

$$\Delta y_t = a_0 + a_1 t + \phi y_{t-1} + \delta_1 x_{t-1} + \delta_2 x_{2t-1} + \sum_{j=1}^{p-1} \psi_j \Delta y_{t-j} + \sum_{j=0}^{q_1-1} \varphi_{1j} \Delta x_{1,t-j} + \sum_{j=0}^{q_2-1} \varphi_{2j} \Delta x_{2,t-j} + \xi_t\,, \quad (1.4)$$

with H_0: $\phi=\delta_1=\delta_2=0$ against H_1: $\phi\neq 0$ and ($\delta_1,\delta_2\neq 0$ or $\delta_1,\delta_2=0$).

Computation of the Wald statistic for testing for a zero value of the subset of regressors included in the null using specification (1.4), yielded the test results summarized in table II.1). These results cover the various sensible cases of $(y,x_1,x_2)=(c,y,\Delta_4 p)$, $(y,x_1,x_2)=(y,c,\Delta_4 p)$ and $(y,x_1,x_2)=(\Delta_4 p,c,y)$ respectively, with $p=q_1=q_2=2,3,4$ and with or without a trend included in the individual regression. Since the reported test results relate to the Wald statistic for testing a set of m linear constraints in its F(m,T-k)-form, the relevant critical values for the 5% significance level were taken from table B) in Pesaran et al. (1997, p. 25).

p,q_1,q_2	$(c \mid y, \Delta_4 p)$	$(y \mid c, \Delta_4 p)$	$(\Delta_4 p \mid c,y)$	Trend	Critical bounds
2,2,2	3.93	7.62	5.07	x	(4.90;5.87)
2,2,2	3.39	7.22	2.28	-	(3.79;4.86)
3,3,3	3.97	6.58	4.91	x	(4.90;5.87)
3,3,3	3.15	6.07	2.50	-	(3.79;4.86)
4,4,4	4.29	1.85	3.18	x	(4.90;5.87)

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[8] The quarterly seasonally adjusted data on UK real non-durable consumption (1990 prices), real personal disposable income of the personal sector (1990 prices) and the fourth difference of the logarithm of the implicit deflator of the consumption series (1990=100) used in this study were taken from the 'Economic Trends Annual Supplement'-database. As far as the econometric analyses in this paper are concerned, both the Pesaran et al. (1997) bounds test and the Engle-Granger two-step procedure were computed using 'PcGive 9.0' (Hendrik and Doornik, 1996) whereas the Johansen procedure was conducted using 'PcFiml 8.0' (Doornik and Hendry, 1994) with the results having been cross-checked with those produced by 'Microfit 4.0' (Pesaran and Pesaran, 1996).

4,4,4	1.66	1.74	1.84	-	(3.79;4.86)

Table II.1) Results of the bounds test procedure

Thus, rather surprising from a theoretical point of view, income and inflation are found not to be long-run forcing consumption in any of the test regressions considered while consumption and inflation seem to be strongly long-run forcing income for $p=q_1=q_2=2,3$ and irrespective of the trend specification chosen. However, testing for the significance of lags at $p=q_1=q_2=5$ using the Wald statistic for the testing of linear restrictions on a subset of regressors of the untransformed VAR specification[9] and the computation of a variety of VAR misspecification tests[10] actually revealed that $p=q_1=q_2=5$ might generally be the more appropriate setting. In this case, the test results for the case of a regression with and without a trend respectively changed to 2.19 and 2.31 thereby indicating that it might be more appropriate not to rejcet the null of no long-run relationship.

However, even more surprising, the test results concerning the potential long-run forcing effects of consumption and income on inflation are inconclusive for regressions including a trend and lag lengths $p=q_1=q_2=2,3$. Clear rejections of such a long-run relationship are in fact only obtained for $p=q_1=q_2=4$.[11] On the other hand, application of the Wald statistic and a variety of misspecification tests to the untransformed VAR model suggested that $p=q_1=q_2=5$ represents the more adequate specification. For this specification, however, the test results obtained were 2.53 (trend included) and 0.39 (trend excluded) respectively. Hence, tentatively, the null of no long-run relationship should again not be rejected.

Consequently, given the results of the application of the bounds test procedure reported above, the only clear-cut conclusion concerns the absence of any evidence on a long-run relationship between consumption and income and inflation running from income and inflation to consumption. The various other results, tentatively suggest the absence of any long-run forcing effects of consumption and income on inflation and of consumption and inflation on income. However, in the latter case, the evidence is somewhat mixed.

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[9] Elaborating upon the study by Sims, Stock and Watson (1990), Watson (1994, pp. 2858-2861) shows that any hypothesis which can be formulated as a restriction on zero mean stationary regressors within an appropriately transformed integrated VAR model (including an intercept) will have the usual large sample chi-squared distribution. Since the conventional unit root-testing carried out further below strongly indicates that Δc_t, Δy_t, $\Delta_4 p_t$ are all I(0), the application of the Wald statistic using standard critical values is justified (asymptotically).

[10] These tests consisted of the vector misspecification tests discussed in section IV.b).

[11] The insignificance of all test statistics for $p=q_1=q_2=4$ was generally found to carry over to regression specifications incorporating higher order and thus presumably over-specified lag lengths.

Thus, given the lack of clear-cut results, the following sections attempt to achieve some clarification by applying both the Engle-Granger and the Johansen procedure to the available data and subsequent comparison of the overall results.

III) Application of the Engle-Granger two-step procedure

III.a) Theoretical issues

The Engle-Granger single equation two-step approach to cointegration testing makes use of the fact that for a vector $\mathbf{y}_t$ with components assumed to be integrated to the same order, cointegrating vectors might exist which ensure that the components although being individually non-stationary show stationary co-movements over time. For the particularly interesting case of components being individually I(1) and cointegrating to I(0), 'Granger's Representation Theorem' ensures the existence of a ECM representation of the relationship (frequently extracted from a system) under investigation which considerably facilitates (OLS) estimation and interpretation of the short- and long-run parameters respectively.[12]

Correspondingly, in a first step, it has to be tested for both the presence of a common order of integration of the variables and the existence of a common long-run pattern among the variables applying conventional unit root tests[13] (but different sets of critical values) to the residuals from a static OLS regression reflecting the postulated economic long-run relationship. Given a large sample size, OLS will yield 'super-consistent'[14] estimates so that the omitted dynamics (and neglected endogeneity issues) will not matter asymptotically and can conveniently be ignored until the estimation of the corresponding ECM. Hence, the null of no cointegration can be tested by checking the retained residuals from the static regression for stationarity using, for example, the ADF test along with the critical values calculated from the responce surface provided by MacKinnon (1991, pp. 274-275).[15]

Provided the null can be rejected at conventional levels of significance, in the second stage of the Engle-Granger two-step procedure, the OLS estimates of the parameter coefficients are taken to be the true ones and the cointegrated relationship is transformed into a stationary ECM with the residuals from the static regression representing the extent of deviation from

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[12] See Engle and Granger (1987, pp. 255-256).

[13] It should be pointed out that, strictly speaking, the formulation 'conventional unit root tests' refers to unit root tests applied to original rather than estimated series. Since the static regression estimates the (usually unknown) cointegrating vector, the term 'conventional' is to some extent inappropriate.

[14] Stock (1987) shows that the OLS estimates converge at the rapid rate of T rather than the conventional rate of $\sqrt{T}$ to its probability limits. Consequently, although OLS estimates might be biased in small samples, 'super-consistency' ensures that this bias diappears rather rapidly with a growing sample size.

[15] Due to the fact that the ADF test is carried out on the estimated rather than original series of disturbances, it can be shown that the standard DF distribution does not hold. In addition, the distribution under the null depends on the number of regressors. See Phillips and Ouliaris (1990, pp. 173-176).

the long-run equilibrium path. Standard OLS estimation and inference can then be applied to this representation of the original relationship.[16]

III.b) Unit root testing

Trying to establish the order of integration of the c_t, y_t, $\Delta_4 p_t$ series first, the ADF(k) test[17] (including a constant) was used with the appropriate lag-length k determined by a sequence of Lagrange Multiplier (LM) tests for residual correlation from lag one to four with the number of lags increasing with each step to ensure an adequate approximation of the autocorrelation in the residuals. Once k was determined, the ADF test was started off with I(2) assumed to be the highest order of integration of the series and was then reiterated until the null of non-stationarity was rejected following the 'testing down'-procedure suggested by Dickey and Pantula (1987, pp. 458-461) for preventing over-rejections of the null.

With the condition of asymptotically white noise errors being checked by the respective LM test and under the null of I(2) or I(1) respectively, the critical values for the ADF t-test have to be taken from Fuller (1976, p. 373)[18] whereas the critical values for the F-test for the significance of a deterministic trend in the test regression are tabulated by Dickey and Fuller (1981, p. 1063).[19]

Tables III.1) and III.2) summarize the results of the various 'testing down'-procedures, the relevant critical values for the 5% level of significance at a sample size of 100 observations and the LM statistics for the preferred specification in their F-form.

	ADF test result	Trend	Critical value	LM statistic
c_t	-5.51	included	-3.45	1.26
y_t	-4.64	excluded	-2.89	0.71
$\Delta_4 p_t$	-5.31	included	-3.45	0.46

Table III.1) ADF test results on the null of I(2) against I(1)

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[16] According to the Engle-Granger theorem, OLS estimation of the ECM will in fact yield the maximum likelihood estimates of the parameter coefficients. See Engle and Granger (1987, pp. 262-283).

[17] Since it had to be dealt with higher order residual autocorrelation, the simple DF test was inappropriate. For Monte Carlo evidence in favour of the ADF test when compared to the non-parametric adjustment procedures put forward by Phillips (1987) and Phillips and Perron (1988), see Schwert (1989, pp. 148-149).

[18] See Dickey and Fuller (1981, pp. 1065-1066).

	ADF test result	Trend	Critical value	LM statistic
c_t	-0.58	excluded	-2.89	1.16
y_t	-1.05	excluded	-2.89	0.75
$\Delta_4 p_t$	-1.42	excluded	-2.89	0.47

Table III.2) ADF test results on the null of I(1) against I(0)

Hence, the null of integration of order two is strongly rejected for all the series under consideration whilst the null of I(1) against stationarity cannot be rejected at the 5% significance level. In the case of $\Delta_4 p_t$ this implies that the implicit deflator of consumption in levels is integrated to the order two which does not represent an unusual test conclusion.
Thus, all series are found to be I(1) so that meaningful static regressions and subsequent cointegration testing can be carried out within the Engle-Granger two-step framework.

III.c) Static regressions and cointegration tests

Following the recommendation by Engle and Granger (1987, pp. 269-270), the ADF test for the stationarity of the residuals from the various static regressions was used alongside the critical values tabulated by MacKinnon (1991). In line with Banerjee et al. (1993, p. 208), the ADF test results were checked for 'consistency' by computing the test statistic for various presumably over-specified regressions.[20] In addition, it should be noted that due to the fact that I(1) represents the highest order of integration assumed to be present when testing for cointegration within the Engle-Granger two-step approach, the Dickey-Pantula 'testing down' approach boils down to the 'standard' first step of an ADF test for the null of a unit root. Finally, diagnostic tests are not reported since the performance of the various static models is anyway certain to be poor.

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[19] For ease of exposition, apart from the ADF F-test conclusion on the significance of a deterministic trend in the preferred specification, the actual F-test results and the relevant critical values are not reported in tables III.1) and III.2).

[20] Since Hansen (1992) using Monte Carlo experiments shows that inclusion of a time trend will generally result in a loss of power irrespective whether or not the true disturbances contain a deterministic trend, a deterministic trend was excluded from the test regressions. Similarly, due to the fact that a constant was incorporated into the static regressions, the retained residuals could be expected to have a zero mean. Hence, no deterministic components were included in the various test regressions.

The various static regressions for the period 1956(1) to 1995(1) of both c_t on y_t and $\Delta_4 p_t$ and y_t on c_t and $\Delta_4 p_t$ as well as $\Delta_4 p_t$ on c_t and y_t are listed below:

1) Static regression of c_t on y_t and $\Delta_4 p_t$

Variable	Coefficient	Std.Error	t-value	Partial R^2
Constant	1.0010	0.067163	14.904	0.5906
y_t	0.89494	0.0061272	146.059	0.9928
$\Delta_4 p_t$	-0.21646	0.039466	-5.485	0.1634

$R^2 = 0.993044$; $F(2, 154) = 10992$ [0.0000]; $\sigma = 0.0222162$
RSS = 0.07600789328 for 3 variables and 157 observations

2) Static regression of y_t on c_t and $\Delta_4 p_t$

Variable	Coefficient	Std.Error	t-value	Partial R^2
Constant	-1.0320	0.082106	0.085824	0.5064
c_t	1.1094	0.0075955	146.059	0.9928
$\Delta_4 p_t$	0.24975	0.043622	5.725	0.1755

$R^2 = 0.993144$; $F(2, 154) = 11154$ [0.0000]; $\sigma = 0.0247352$
RSS = 0.09422172201 for 3 variables and 157 observations

3) Static regression of $\Delta_4 p_t$ on c_t and y_t

Variable	Coefficient	Std.Error	t-value	Partial R^2
Constant	0.51091	0.19165	2.666	0.0441
c_t	-0.75494	0.13765	-5.485	0.1634
y_t	0.70269	0.12273	5.725	0.10905

$R^2 = 0.199726$; $F(2, 154) = 19.217$ [0.0000]; $\sigma = 0.0414897$
RSS = 0.2650951487 for 3 variables and 157 observations

The ADF test results on the basis of the residuals retained from these regressions are presented in table III.3) alongside the relevant critical value:

	Lag length k[21]	ADF test result	Critical value[22]
$(c_t \mid y_t, \Delta_4 p_t)$	4	-2.79	-3.38
$(y_t \mid c_t, \Delta_4 p_t)$	4	-2.92	-3.38
$(\Delta_4 p_t \mid c_t, y_t)$	8	-2.15	-3.38

Table III.3) ADF test results for cointegration on the retained residuals

Hence, from table III.3) it is apparent that contrary to the results obtained from applying the bounds test procedure, the null of no cointegration cannot be rejected irrespective which normalization is adopted. In particular, the existence of a potential long-run relationship running from consumption and inflation to income is strongly rejected. Similarly, the ADF test on the residuals retained from the static regression of inflation on consumption and income yields the clear-cut decision that the null of no cointegration cannot be rejected at the 5% level. Thus, the rather tentative conclusions obtained in section II.b) are to some extent confirmed.

However, as shown by the Monte Carlo evidence presented in Banerjee et al. (1986, pp. 260-265) and Banerjee et al. (1993, pp. 225-230), for the sample size of 165 considered in this study the finite-sample bias in the static regressions can be of considerable size. In fact, Banerjee et al. (1993, p. 220) argue that even a sample size of 200 might be too small to ensure that 'super-consistency' emerges. Since T is restricted to 161 for the above regressions, the finite-sample bias can be expected to be large. Furthermore, this bias is generally found to decline at a rate faster than $\sqrt{T}$ but not as fast as T for sample sizes common in practice. Hence, the above test conclusions have again to be treated with care and the mixed test results reported in section II.b) are not clarified entirely.These setbacks, however, can be alleviated to some extent by either using (over-parameterized) dynamic rather than static regressions (thereby trying to increase the precision of the estimates in finite-samples)[23] or by applying

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[21] The various results were found to carry over to presumably over-specified regressions and thus appear consistent.

[22] For comparison, the corresponding but less precise critical value taken from Engle and Yoo (1987, p. 158) is given by -3.62.

[23] See, for example, Harris (1995, pp. 60-62).

likelihood ratio tests suggested by Johansen (1988). The next section therefore applies the Johansen (1988) procedure to the data under investigation.[24]

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[24] The dynamic regressions carried out by the author confirmed the test results reported in table III.3). However, the system-based approach introduced by Johansen (1988) can generally be regarded as the more powerful test procedure and is anyway based on a VECM thereby including the model dynamics. See, for example, Watson (1994, pp. 2885-2887) and Gonzalo (1994) for simulation evidence on this.

IV) Application of the Johansen procedure

IV.a) Theoretical issues

Proceeding from an unrestricted VAR(p) model in the (k+1)-vector of jointly determined variables $\mathbf{z}_t$,

$$\mathbf{z}_t=\mathbf{A}_1\mathbf{z}_{t-1}+\mathbf{A}_2\mathbf{z}_{t-2}+\ldots+\mathbf{A}_p\mathbf{z}_{t-p}+\varepsilon_t \qquad (1.5)$$

if $\varepsilon_t\sim IN(0;\Sigma)$ (and serially uncorrelated) and $\mathbf{A}_i$ represents a (k×k)-matrix of parameters, (1.5) can be transformed into the following VECM form:

$$\Delta\mathbf{z}_t=\Gamma_1\Delta\mathbf{z}_{t-1}+\Gamma_2\Delta\mathbf{z}_{t-2}+\ldots+\Gamma_{p-1}\Delta\mathbf{z}_{t-p+1}+\Pi\mathbf{z}_{t-p}+\varepsilon_t \qquad (1.6)$$

with $\Gamma_i=-\mathbf{I}+\mathbf{A}_1+\mathbf{A}_2+\ldots+\mathbf{A}_i$, i=1, 2, …, p-1 and $\Pi=-\mathbf{I}+\mathbf{A}_1+\mathbf{A}_2+\ldots+\mathbf{A}_p$. Given $\mathbf{z}_t$ consists of variables integrated to the order unity, $\Pi\mathbf{z}_{t-p}$ has to be stationary for ε_t to be white noise. In addition, for $\Pi\mathbf{z}_{t-p}$ to contain the long-run error-correction relations and thus to be white noise itself, Π has to be of reduced rank, i. e. $r(\Pi)\leq k-1$.[25]

If in fact the 'long-run impact matrix' Π is found to be of reduced rank, Π can be factorized into matrices $\alpha\beta'$ (each of full column rank) with the (k×r)-matrix α consisting of the adjustment coefficients corresponding to the row vectors in the (r×k)-matrix β, which are spanning the cointegration space.[26]

Thus, cointegration testing in this framework amounts to determining the number of the linearly independent columns in the long-run impact matrix. Johansen (1988) suggests a reduced rank regression yielding eigenvalues $\hat{\lambda}_1>\hat{\lambda}_2>\ldots>\hat{\lambda}_k$ and their corresponding eigenvectors $\hat{\upsilon}_1,\hat{\upsilon}_2,\ldots,\hat{\upsilon}_k$ which can be shown to include the r cointegration vectors (r=0, 1, …, k-1) so that $\hat{\beta}=\left(\hat{\upsilon}_1,\hat{\upsilon}_2,\ldots,\hat{\upsilon}_r\right)$. However, for the $\hat{\upsilon}_i'\mathbf{z}_t$, i=1, 2, …, r, to be cointegrating relationships, they have to be highly correlated with the stationary $\Delta\mathbf{z}_t$ in (1.6), the degree of which can in fact be shown to be captured by the magnitude of the corresponding eigenvalues $\hat{\lambda}_i$.

Accordingly, given r cointegrating relationships in the variables under consideration, the eigenvalues corresponding to i=r+1, …, k have to be zero so that testing for the presence of r cointegration vectors amounts to testing for H_0: $\hat{\lambda}_{r+1}=\hat{\lambda}_{r+2}=\ldots=\hat{\lambda}_k=0$ against H_1: $\hat{\lambda}_i\neq 0$, i=1, 2,

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[25] Clearly, if $r(\Pi)=k$, the variables in $\mathbf{z}_t$ are I(0) whereas if $r(\Pi)=0$, no cointegrating relationship exists. See, for example, Johansen and Juselius (1990, p. 170).

[26] See Johansen and Juselius (1990, p. 170) and Johansen (1988, p. 232). Similar to the cointegration space spanned by the β vectors, the vectors in α span the so-called adjustment space.

…, k (full rank).[27] Imposition of this restriction and calculation of a likelihood ratio (LR) test yields the so-called 'trace statistic':[28]

$$\lambda_{\text{trace}}=-T\sum_{i=r+1}^{k}\log\left(1-\hat{\lambda}_i\right),\ r=0,\ 1,\ \ldots,\ k-1.$$

Alternatively, the so-called maximal-eigenvalue statistic can be used:

$$\lambda_{\max}=-T\log\left(1-\hat{\lambda}_{r+1}\right),\ r=0,\ 1,\ \ldots,\ k-1$$

with the null hypothesis that there exist r cointegration vectors against the alternative that there are (r+1) cointegration relationships.[29] Since the distributions of these LR statistics are non-standard under the null,[30] the critical values tabulated by Osterwald-Lenum (1992) have to be used to obtain valid test decisions.

In this context, it should be pointed out that the asymptotic distributions of the estimators and test statistics are very sensitive to the model formulation chosen so that different model specifications imply the use of different sets of critical values. In fact, the dynamics of the VECM should be modelled 'adequately' with the lag-length p determined in such a way that the residuals are Gaussian[31] but that the adjustment matrix can still be interpreted properly. For the same reason, within the extended Johansen (1991) framework, the deterministic components in the model have to be chosen properly according to the characteristics of the data.

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[27] This is equivalent to testing the α_i, i=r+1, r+2, …, k for being insignificantly small.

[28] See Johansen (1988, pp. 233-236) and Johansen (1991, p. 1555).

[29] The Monte Carlo evidence presented by Cheung and Lai (1993) indicates that the λ_{trace}-statistic is generally more robust than the $\lambda_{\max}$-statistic and thus is to be preferred. Nevertheless, to check for the consistency of the test decisions both statistics are reported below.

[30] See Johansen (1991) and Johansen (1995, pp. 151-163) for the derivation of the asymptotic distributions.

[31] The LR test statistics are explicitly derived under the assumption of Gaussian errors. See Johansen (1988, p. 231) and Johansen (1991, p. 1566).

IV.b) Test application and results

For the LR statistics reported below, a deterministic trend was either excluded from the model (case I)) or was restricted to enter the long-run model only (case II)) reflecting the two sensible cases in which there is either no linear or no quadratic trend in the levels of the series. As far as the intercept is concerned, however, for case I) the intercept was restricted to enter the long-run relationship only (thereby representing an 'autonomous growth factor') whereas for case II) the long-run intercept was cancelled out by its short-run counterpart leaving only the latter in the model.[32]

Finally, every model specification was diagnostically checked using various vector misspecification tests including a F-test for the significance of the unrestricted regressors and the multivariate equivalent of the single equation portmanteau statistic which is asymptotically $\chi^2(k^2(s-m))$, where s represents the chosen lag length of the VAR model and m denotes the lag length of the k dependent variables. In addition, a LM test for vector autocorrelation up to lag r is reported which is asymptotically $\chi^2((s-r+1)k^2)$-distributed under the null of no autocorrelation. Finally, the results of a Jarque-Bera-type vector test for Gaussian distributed errors and a White-type-vector heteroscedasticity test for the null of homoscedastic residuals including all error variances and covariances of the original regressors and their squares in the multivariate auxiliary regression are presented. If applicable, all test results are presented in both their χ^2- and F-form approximation respectively.[33]

The results of these misspecification tests for the preferred specification are reported below alongside the values of the λ_{trace}- and λ_{max}-statistics and the relevant critical values for the 5% significance level.

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[32] For a detailed discussion of the various possible specifications of deterministic components, see, for example, Pesaran and Pesaran (1996, pp. 433-437).

[33] For more detail on the various test statistics reported, see Doornik and Hendry (1994, pp. 214-217).

F-test against unrestricted regressors:
F(84,353)=3285.7 [0.0000]**

Vector portmanteau statistic for 12 lags and 148 observations: 69.37

Testing for vector error autocorrelation from lags 1 to 4:
Chi^2(36) = 60.753 [0.0061]** and F-Form(36, 313) = 1.4441 [0.0535]

Vector normality test for residuals:

Skewness	0.9101	1.938	0.8003
Excess kurtosis	2.214	2.335	3.086

Vector normality Chi^2(6) = 25.1 [0.0003]**

Testing for vector heteroscedasticity using squares:
Chi^2(324) = 338.21 [0.2822] and F-Form(324, 367) = 0.72663 [0.9984]

Table IV.1) Misspecification tests for VAR(9) and case I)

H_0: r	$-T\log(1-\hat{\lambda}_{r+1})$	CV_{max}	$-T\sum\log(1-\hat{\lambda}_i)$	CV_{trace}
0	15.68	22.0	29.6	34.9
1	10.79	15.7	13.91	20.0
2	3.121	9.2	3.121	9.2

Table IV.2) Values of the λ_{trace}- and λ_{max}-statistics for VAR(9) and case I)

F-test against unrestricted regressors: F(84,350) = 221.54 [0.000]**

Vector portmanteau statistic for 12 lags and 148 observations: 65.22

Testing for vector error autocorrelation from lags 1 to 4:
Chi^2(36) = 59.633 [0.0079]** and F-Form(36, 310) = 1.3931 [0.0731]

Vector normality test for residuals:

Skewness	0.8799	0.7902	1.810
Excess kurtosis	1.748	3.037	2.582

Vector normality Chi^2(6) = 23.619 [0.0006]**

Testing for vector heteroscedasticity using squares:
Chi^2(336) = 339.3 [0.4394] and F-Form(336, 349) = 0.66748 [0.9999]

Table IV.3) Misspecification tests for VAR(9) and case II)

H_0: r	$-T\log(1-\hat{\lambda}_{r+1})$	CV_{max}	$-T\sum\log(1-\hat{\lambda}_i)$	CV_{trace}
0	22.3	25.5	35.93	42.4
1	10.76	19.0	13.63	25.3
2	2.869	12.2	2.869	12.2

Table IV.4) Values of the λ_{trace}- and λ_{max}-statistics for VAR(9) and case II)

Thus, from the results reported in tables IV.2) and IV.4) respectively and with the exception of the (less robust) λ_{max}-statistic for H_0: r=0 in table IV.4), the existence of a cointegrating vector is strongly rejected by both the λ_{trace}- and λ_{max}-statistic when compared to their 95% critical values thereby confirming the results obtained in section III.c) and the tentative conclusions on the absence of any 'long-run forcing effects' of consumption and income on inflation and of consumption and inflation on income reported in section II.b).

However, even though the lag length p=9 seems over-specified,[34] there are strong signs of non-normally distributed residuals as reported in table IV.1) whilst the χ^2-form of the LM test on residual autocorrelation strongly rejects and its F-form approximation only weakly non-rejects the presence of non-autocorrelated residuals at the 5% level.

These results remain virtually unchanged when turning to cointegration estimation and testing on the basis of the VAR(9) and case II)-specification. The results reported above in fact were also found to hold for regressions computed for various other lag lengths indicating that in this case increasing the value of p does not represent a solution to the problem of non-Gaussian residuals. Since checks for outliers did not resolve the problem either, this suggests that the misbehaved residuals are caused by the omission of important explanatory variables and that enlarging the information set by additional regressors might be the correct approach to improve the model performance. In fact, omitted variables generally hinder cointegrating relationships from emerging thereby affecting the residuals from an OLS estimation of the (underspecified) VAR model and thus explaining the poor misspecifciation results obtained above. As already mentioned in section I), a fully specified consumption model showing cointegrating relationships will simply have to consist of various additional explantory variables to be well-determined.

As a consequence, the test results summarized in tables IV.2) and IV.4) have once again to be interpreted with care. However, given the strong indications against the presence of any long-run relationship in the data, the (tentative) conclusions from the application of the Pesaran et al. (1997) bounds test and the Engle-Granger two-step procedure are tentatively confirmed.

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[34] The F-tests on the retained regressors not reported in table IV.1) indicated weak additional explanatory power for p=7,8,9. However, for VAR specifications with p<9, the misspecification test results deteriorated considerably. Apart from this, the Monte Carlo evidence presented by Cheung and Lai (1993) indicates that the cointegration rank tests are relatively robust to over-parameterizations whereas too small a lag length affects the size of the tests.

V) Conclusion

From the test results reported above, it should be clear that the omitted variable effects present in the consumption model under investigation undermine to some extent the interpretation and applicability of all the tests for the existence of long-run relationships discussed in this paper.

In addition, the use of seasonally adjusted data as a consequence of the assumed absence of seasonal integration within the Pesaran et al. (1997) framework implies that the time series properties of the raw data set have been altered by the adjustment filter thereby potentially affecting the cointegration test results.[35] As, for example, pointed out by Maravall (1993) for the case of vector autoregressions, given the underlying structural model contains unobservable components with unit roots, seasonal adjustment will introduce non-invertible moving average components in the available data, which then can no longer be approximated by a (vector) autoregressive model. As a consequence, the interdependencies included in the data will be left undetected and wrong test conclusions result.

This criticism has recently been extended by Harvey (1997, pp. 199-200) who argues that VAR approximations in the multivariate and autoregressive approximations in the univariate case almost always represent poor approximations to the underlying data generation process irrespective the number of parameters included and the presence of seasonality.

In addition, as far as the role of data quality is concerned, Hendry (1994, pp. 89-94) shows that past data revisions had a major impact on the consumption-income ratio reflected by a correlation of only 0.92 between the original and the revised series. Accordingly, the potential cointegration characteristics of the data have been affected or even removed.

However, when ignoring these problems (which affect all the test procedures employed), for the rather 'small-scale' model specification considered, the overall test results tentatively suggest the absence of any cointegrating relationship within the utilized data. With the Engle-Ganger two-step and the Johansen system-based procedures strongly supporting this finding, the Pesaran et al. bounds test procedure yields somewhat mixed results

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[35] On the other hand, when using seasonally unadjusted data, it has to be checked for seasonal integration and cointegration, the methods for which are still unsatisfactory, especially when applied on the basis of autoregressive approximations. See, for example, Harvey and Scott (1994). For evidence on the sensitivity of cointegration test results to the kind of data considered, see, for example, Davidson and MacKinnon (1993, pp. 687-696).

concerning the the existence of a long-run relationship running from consumption and inflation to income on the one hand and running from consumption and income to inflation on the other hand. However, while the finite-sample weaknesses of the Engle-Granger and Johansen procedures are well-known,[36] the finite-sample properties of the Pesaran et al. bounds test remain to be explored by future research. Therefore, as long as the superior power of the bounds test in finite samples cannot be taken as given, the validity of the unusual bounds test results mentioned above seems doubtful.

Consequently, given both the lack of evidence concerning the finite-sample properties of the bounds test procedure and the 'peculiar' test results reported in this study as well as the simulation evidence on the superior power of the Johansen procedure relative to other more traditional cointegration tests,[37] the application of the Johansen procedure seems still preferable.

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[36] See, for example, Campbell and Perron (1991) and Cheung and Lai (1993) respectively.

[37] See footnote 24).

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