Integration, Cointegration Analysis and Error-Correction Models: The Consumption Function re-visited

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CONTENTS

LIST OF FIGURES

I. Introduction

As Hendry (1986, p. 201) remarks, until " ... recently, the vast bulk of econometric theory has been based on the assumption that the underlying data processes are stationary and ergodic, despite the manifest non-stationarity of, say, the aggregate time series to which that theory was applied in economics." The usual result of this traditional ordinary least squares (OLS hereafter) assumption has been the proliferation of seemingly well-determined econometric models all of which are characterized by a strikingly high goodness-of-fit.

It took until the publication of the seminal paper by Granger and Newbold (1974) that econometricians realized that their theoretical inferences had been led astray by spurious regressions in non-stationary variables. One example for this development is the history of the modelling of consumer expenditure in the UK (see, for example, Spanos (1989) and Thomas (1989)). Whilst the early estimation of the empirical counterparts of both the absolute income and permanent income as well as the life-cycle hypothesis was characterized by the complete neglect of the issue of non-stationarity and its detrimental implications for the applicability of ordinary least squares, more recent contributions suffered from the neglect of valuable long-run information due to the ad hoc-adoption of the Box-Jenkins solution of differencing the data to achieve stationarity.

Davidson et al. (1978; DHSY hereafter) helped to overcome this dilemma by introducing the concept of error-correction mechanisms in consumption modelling. This approach has enabled econometricians to incorporate levels information in a non-integrated form into balanced models of consumption. As a consequence, although fragile and imprecise in its origin, the error-correction idea by DHSY has paved the way for the recent developments in the field of integration and cointegration analysis. These theoretical innovations have already proved indispensable in applied work. Accordingly, the corresponding theoretical considerations in chapter III and IV of this dissertation are followed by an empirical reconsideration of the DHSY study in the light of the integration and cointegration findings in chapter V.

Chapter II provides brief descriptions of the employed econometric methods and the development of the post-war UK consumption expenditure. Chapter III focuses on the stochastic nature of time series such as consumption and income and re-examines the Box-Jenkins methodology as the historical forerunner for the development of systematic unit root testing. This systematic testing procedure (and its weaknesses) is then explained to some extent on the basis of its use for the establishment of the orders of integration of economic time series (with particular emphasis on seasonal integration).

The following chapter extends this theoretical discussion to the concept of cointegration and the Engle-Granger two step estimator. It concludes with a relatively detailed description of the derivation of ECMs with particular reference to the Granger Representation Theorem.

These theoretical concepts and their implicit strengths and drawbacks are then illustrated by the reconsideration of the quarterly and annual versions of the DHSY model in chapter V. The corresponding empirical analyses are conducted for UK data on consumer expenditure on non-durables, personal disposable income and inflation for the period 1955(1) to 1993(2) and 1955 to 1992 respectively (see appendix III and IV) using PcGive Version 7 by Hendry and Doornik (1992).

On the basis of the empirical findings, the concluding chapter VI contains an attempt to assess the impact of both integration and cointegration analysis as well as error-correction modelling on consumption modelling in the UK. This final discussion finishes with a brief overview of those issues which remain to be settled by future research in this field of statistical analysis.

II. Employed econometric methods and development of UK consumption expenditure

II.a. Description of the employed econometric methods

Since the pioneering work by Granger (1981), it has become a widely accepted fact that the time series properties of the data under consideration and their econometric modelling can no longer be separated from each other. The various methods for the determination of orders of integration and cointegration, for example, are nowadays commonly presented in applied work as 'pre-tests' preceding the 'traditional' econometric model analysis and evaluation.
In accordance to this development, this section focuses on some distinctively econometric methods of model analysis and evaluation whereas chapter III deals with the theoretical and practical implications of the concepts of integration and cointegration.

The econometric treatment of the various regression equations introduced in due course is based on the use of OLS as standard regression technique. The validity of this regression technique depends on (among others) the assumption that the regression equation is 'statistically adequate' in the sense that it embodies a reasonable description of the underlying data generation process (DGP). Only if the model has been proved to be a convenient summary of the given sample information, can it be used reasonably for tests of impositions suggested by theory [cp. Spanos, A., 1989, p. 153]. In other words: "To test any theory requires a baseline, so first one must determine the extent to which that baseline satisfies the evaluation criteria. Thus, we are led to distinguish between the statistical model and the econometric model, where the former is the baseline and is judged on statistical criteria, and the latter is interpreted in the light of the economic theory ... " [Hendry, D. F. and Doornik, J. A., 1992, p. 19].

The level of statistical adequacy will be measured by using the following standard diagnostic tests:

a) Lagrange multiplier (LM) test for autocorrelated residuals

b) White's tests for heteroscedasticity and functional form misspecification

c) Jarque and Bera test for normality

d) RESET test for non-linearity

The first two test statistics proceed from the null hypothesis of either homoscedastic (simplified and original White tests) or non-autocorrelated residuals (LM test for lags 1 to 2 for annual data and for lags 1 to 4 in the case of quarterly data).

The LM test is based on the auxiliary regression of the retained residuals on all the regressors of the original equation and the residual lags 1 to 2 (or 1 to 4 respectively). From this regression the $n*R^2$ number is calculated. This final statistic can be shown to follow a Chi-squared distribution with two degrees of freedom and can hence be used as a test statistic for the null at a 5% significance level [cp. Thomas, R. L., 1992, pp. 79-81; for a qualification of the criterion of uncorrelated residuals as a measure for model adequacy see Davidson, J. E. H. and Hendry, D. F., 1981, pp. 190-191].

However, since the Chi-squared statistics might lead to unjustified rejections of the null more frequently than their F-Form counterparts, PcGIVE7 generally gives the F-Form equivalents along with the Chi-squared numbers.

The two tests for heteroscedasticity and functional form misspecification respectively are based on the one hand on the simplified and on the other hand on the original version of the test procedure put forward by White (1980). The first simplified White test for heteroscedasticity is based on the auxiliary regression of the squared residuals on the original explanatory variables and all their squares. Afterwards, the corresponding $n*R^2$ number, its F-Form equivalent, the auxiliary coefficients (only when necessary) and their t-values are presented. These values can then be examined for 5% significance [cp. Judge, G. G. et al., 1985, p. 453].

However, the second ('original') White test for heteroscedasticity also includes the cross products of the original explanatory variables on the right-hand side of the auxiliary regression equation. Since this statistic can be interpreted as a general test for functional form misspecification, its results are also given [cp. Maddala, G. S., 1992, pp. 204-205].

In addition to these tests, the RESET test (Regression Specification Test) for non-linearity of the explanatory variables will be presented as a diagnostic test for the omission of important explanatory variables [cp. Maddala, G. S., 1992, pp. 477-478].

Finally, the Jarque and Bera test for the null of normally distributed residuals will be stated along with the first four moments of the empirical distribution of the residuals.

These diagnostic tests are essentially related to the past of the process modelled and hence indicate the degree of specification/explanatory power achieved. However, " ... a model which predicts well historically may yield no insight into how a market will behave under some change in regulations, the implications of which will cause the model to mispredict" [Hendry, D. F. and Doornik, J. A., 1992, p. 14]. Consequently, some forecast/parameter constancy statistics are added to the individual model analysis. These 'statistics' include an examination of the 1-step forecasts produced by the model and both the forecast Chi-squared test and the Chow test for predictive failure (which follows a F-distribution) as tests for (out-sample) parameter constancy. The obtained results are illustrated by a graphical plot of the 1-step forecasts (and their 95% prediction intervals) along with the scaled residuals and the actual and fitted values against time.

However, when dealing with the issue of integration and cointegration, it is of particular importance to dispose of information concerning not only the stability of the parameters out of but also in the sample period. Such information can help to detect structural breaks in the data which otherwise might lead to incorrect conclusions in terms of the order of integration of a series.[1] Similarly, it " ... can be used to check the constancy of pre-tests such as (Augmented) Dickey-Fuller tests for the order of integration of a time series ... " [Hendry, D. F. and Doornik, J. A., 1992, p. 131].

Therefore, both the one-step residuals and the scaled one-step Chow tests resulting from the recursive estimation of the model under investigation are presented graphically as additional indicators for potential structural breaks and (in-sample)

[1] See section III.b.3. for more details on this.

parameter instability [cp. Charemza, W. W. and Deadman, D. F., 1992, pp. 67-72; cp. Hendry, D. F. and Doornik, J. A., 1992, pp. 17-18 and 129-130; cp. Banerjee, A. et al., 1993, pp. 194-195].
This final graphical analysis of the model considered will complete the individual evaluation section.

However, each of these sections is preceded by the following standard output which provides some rough indication of the quality of the estimated model:

a) The coefficient of multiple determination
b) The t-values of the regression coefficients
c) The partial R^2 for each regressor
d) The standard error of the residuals
e) The residual sum of squares (RSS)
f) The standard Durbin-Watson statistic for first order autocorrelated residuals

where the last test statistic can also be interpreted as a general misspecification test for omitted variables [cp. Thomas, R. L., 1992, pp. 142-143].
Furthermore, the Durbin-Watson test (henceforth DW) is likely to indicate the presence of any spurious correlation between the variables modelled [cp. Granger, C. W. J. and Newbold, P., 1974, pp. 110 and 116-118; cp. Newbold, P. and Davies, N., 1978, pp. 514-518; cp. Charemza, W. W. and Deadman, D. F., 1993, pp. 124-126]. For example, Granger and Newbold (1974) remark: "From our own studies we would conclude that if a regression equation relating economic variables is found to have strongly autocorrelated residuals, equivalent to a low Durbin-Watson value, the only conclusion that can be reached is that the equation is mis-specified, whatever the value of R^2 observed" [Granger, C. W. J. and Newbold, P., 1974, p. 117]. In fact, at the presence of spurious correlation, the DW statistic tends towards a value of zero as the sample size increases. Accordingly, as a rule of thumb any " ... regression for which $R^2 > DW$ ... " [Banerjee, A. et al., 1993, p. 81] should be treated as spurious.

Finally, a modified version of this test will prove useful for the evaluation of cointegrating regressions in the applied sections of this study (namely, the cointegration DW test).

However, if the evaluation of the diagnostic tests indicates serious misspecifications of the model employed, it can no longer be used for tests of theoretical impositions. Therefore, if the individual model under investigation fails to pass the misspecification tests, it seems reasonable to consider respecifications rather than different reparameterizations or transformations of the initial equation.
The empirical sections will reveal that in this context 'respecifications' primarily imply the inclusion of additional long-run determinants of consumption. "Fortunately, although economists do not have an exact idea of all the variables that have to be included in an equation, they do have an idea of what variables are likely to be very important and what variables are doubtful" [Maddala, G. S., 1992, p. 496].

II.b. The development of consumption expenditure in the UK (1955-1992)

It is apparent from figure II.1 that the volume of consumer spending in the UK exhibited a significant upward trend over the entire period from 1955 to 1992. In fact, until 1974 there was not a single year in which total consumer expenditure fell compared with the previous year. Even the severe 1980 recession only slowed down consumption rather than causing a breakdown as frequently observed in other European countries [cp. Morris, D., 1985, p. 48].

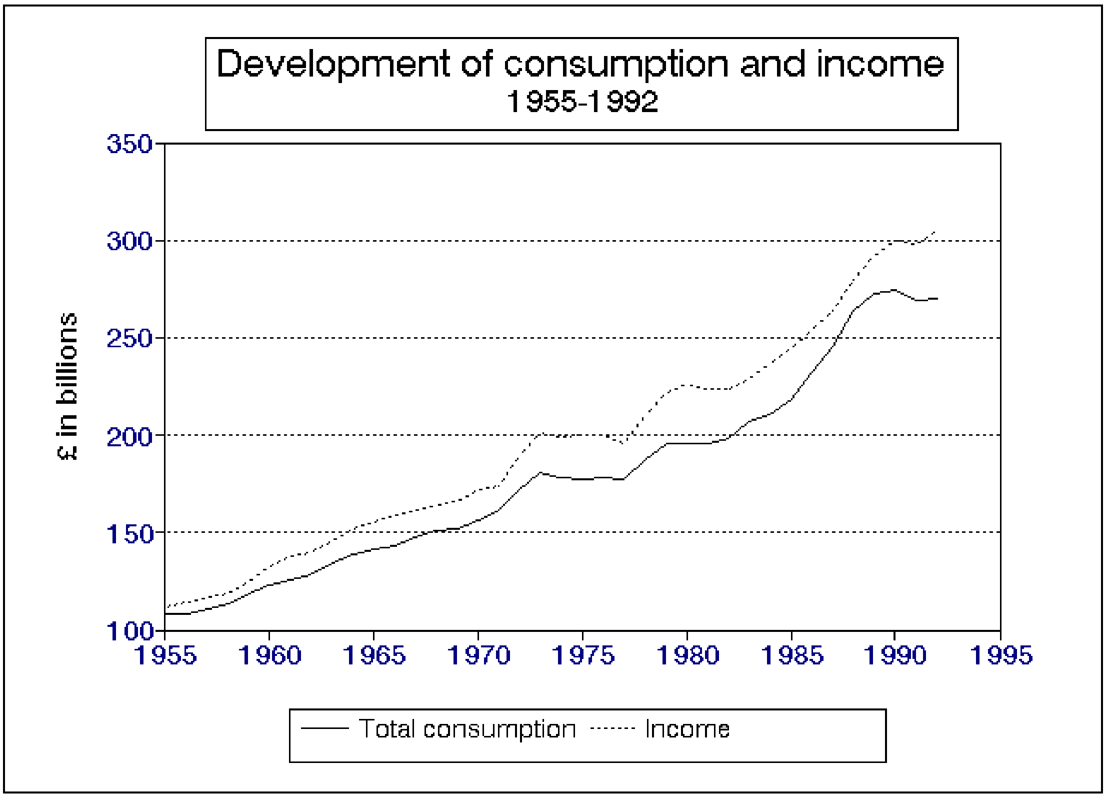

Figure II.1 Source: CSO

The examination of the ratio of total consumption to personal disposable income (the so-called average propensity to consume, APC) reveals some deeper insights into this development of UK consumption.

Not surprisingly (as shown in figure II.2), the huge early post-war demand caused the APC to reach a relatively high level of around 96% in 1955.

However, with the aftermaths of the second world war diminishing, the APC dropped during the following years down to its 1960 low. Remaining relatively stable from 1960 onwards to 1972/3, the apparent downward trend of the APC resumed by an extra-ordinary period of decline beginning in 1973 and lasting until 1980 with a record low of roughly 87% [cp. Hadjimatheou, G., 1987, pp. 13-15].

Obviously, this all-time low coincided with the deep 1980 recession which was characterized (among other factors) by high rates of inflation (caused by rapidly growing public sector deficits).

However, the 1980 low also meant a turning point in the trend pattern of the APC. Starting off from 1980 the APC showed a significant increase to its 1987/88 high reflecting the 'overheated' state of the UK economy during the 'Lawson boom'. The following high interest rate policy of the UK government increased the debt burden of the overborrowed private households and led to the collapse of consumer demand causing the UK economy to move from 'boom to bust'. Accordingly, the APC exhibited a sharp decline during the end of the 1980s/early 1990s reaching its low during the recent UK recession.

A fully specified and statistically adequate consumption model can be expected to produce in-sample predictions close in line to the development of the post-war consumption pattern described above. This pattern, characterized by the distinct upward-movement of consumption, provides a simple justification for the extensive analysis of the integration properties of consumption series. The following sections will make clear that without such an analysis, the occurence of spurious regressions,
poor forecast performances as well as misspecified dynamics and long-run equilibria is largely inevitable.[2]

[2] See Spanos (1989, pp. 157-168) and Thomas (1989, pp. 142-144) for extensive reviews of the various econometric weaknesses of early consumption functions based on the absolute income, the life-cycle and the permanent income hypothesis.

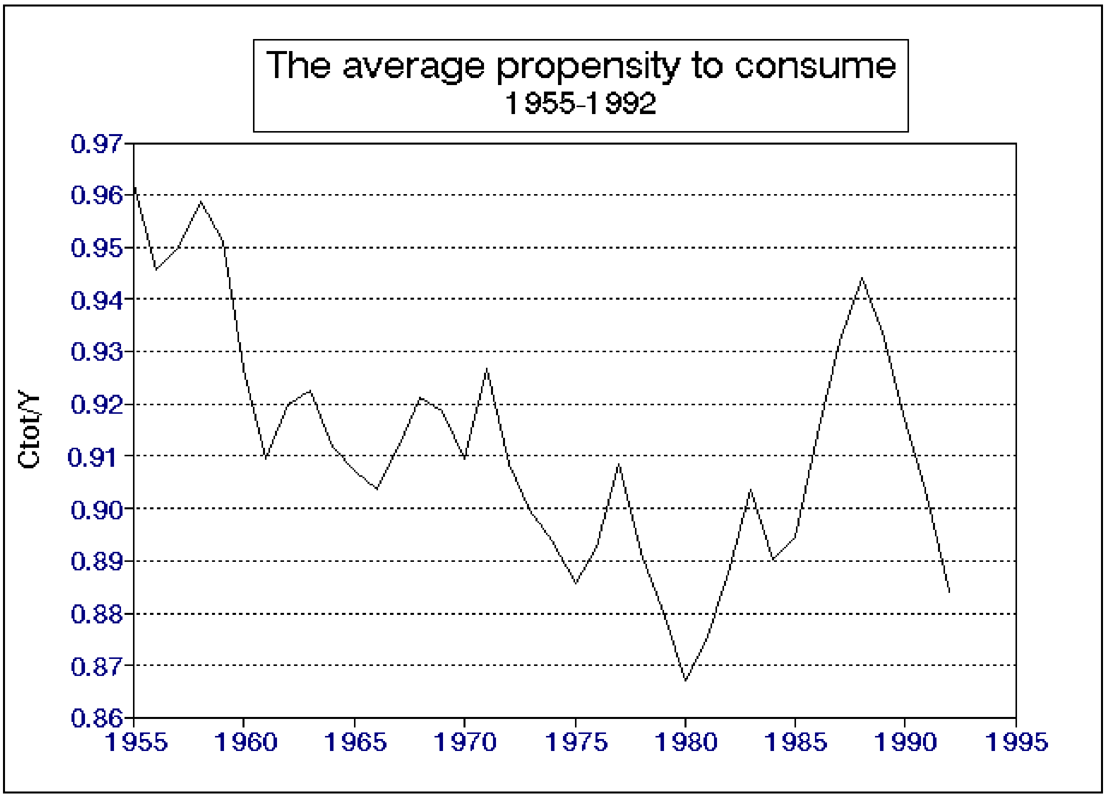

Figure II.2 Source: CSO

III. Stochastic processes and the concept of integration

III.a. Stationarity of stochastic processes and the Box-Jenkins approach

III.a.1. Stationary stochastic processes

The concept of stochastic processes derives its importance from the fact that time series such as quarterly or annual consumption for the UK described above can simply be regarded as examples for this type of process.[3] As a consequence, the theoretical properties of stochastic processes determine to a large extent the statistical limitations of the econometric modelling of time series.

Strictly speaking, a " ... stochastic process is an ordered sequence of random variables $\{x(s,t), s \in S, t \in T\}$, such that, for each $t \in T$, $x(\bullet,t)$ is a random variable on the sample space S and, for each $s \in S$, $x(s,\bullet)$ is a realization of the stochastic process on the index set T (that is, an ordered set of values, each corresponding to one value of the index set)" [Banerjee, A. et al., 1993, p. 10]. In other words, a stochastic process simply consists of a collection of random variables.

It should be clear, however, that the traditional OLS assumption of the non-stochasticity of the explanatory variables becomes untenanble when dealing with time series variables.[4] Accordingly, conventional distribution theory breaks down and spurious correlations or nonsense regressions might occur [cp. Thomas, R. L., 1993, p. 152; cp. Hendry, D. F., 1986, pp. 203-204]. Granger and Newbold (1974, p. 114), for example, show that regressions with stochastic/'non-stationary' time series imply the invalidity of the conventional F-test-distribution (due to the fact that the 'R^2-distribution' is no longer unimodal at the origin).

In other words and as will become clear further below, conventional statistical inference was obviously originally designed for variables which were assumed to be

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[3] There is still some discussion about the question whether stochastic and time series processes are the same or whether a time series represents a single realization of a stochastic process [see, for example, Charemza, W. W. and Deadman, D. F., 1992, p. 121]. However, for the purposes of this dissertation, this distinction is largely irrelevant.

non-stochastic/'stationary' and therefore permitted the application of standard distribution theory ignoring the detrimental implications of stochastic processes.[5]

However, due to the possibly rather peculiar statistical properties of such processes (e. g., interdependence of the realized values, time-heterogeneity, the availability of single realizations of the random variables only), some rather restrictive models for the statistical structure of stochastic processes/time series have to be imposed in order to ensure the (traditionally assumed) applicability of conventional distribution theory.

The property of time-heterogeneity in particular causes severe problems for the modelling of real phenomena. Due to this property, the parameters θ_t of the distribution function of stochastic processes, $F(x(t);\theta_t)$, would have to be estimated on the basis of a single observation, unless valid restrictions ensuring time-homogeneity can be imposed. Such a restriction holds when considering so-called stationary stochastic processes which in fact " ... can be used to model phenomena approaching their *equilibrium steady-state*, but continuously undergoing 'random' fluctuations" [Spanos, A., 1986, p. 137].

Hence, the statistical concepts of equilibrium and stationarity (of two or more time series) are obviously closely related to each other. In fact, " ... we say that an equilibrium relationship $f(x_1, x_2) = 0$ holds between two variables x_1 and x_2 if the amount $\varepsilon_t \equiv f(x_{1t}, x_{2t})$ by which actual observations deviate from this equilibrium is a median-zero process" [Banerjee, A. et al., 1993, p. 4].

With strict stationarity (of a stochastic process) given if " ... for any subset $(t_1, t_2, ..., t_n)$ of T and any real number h such that $t_i + h \in T$, $i = 1, 2, ..., n$, we have

$$F(x(t_1), x(t_2), ..., x(t_n)) = F(x(t_1 + h), x(t_2 + h), ..., x(t_n + h)),$$

where $F(\bullet)$ is the joint distribution function of the n values"[6] [Banerjee, A. et al., 1993, p. 11], it should be clear that the definition of an equilibrium relationship holds automatically for any combination of the stationary time series considered.[7]

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4 Thus, for example, the existence of the probability limits of ordinary least squares estimators and their consistency can no longer be taken for granted.

5 See, for example, Hendry (1986, pp. 202-204) for a historical review on this.

6 Thus, the distribution of any x(t) and all its moments are independent of t and depend " ... only upon the differences in their time subscripts" [Goldberger, A. S., 1991, p. 279].

This, however, is not the case when considering non-stationary series. Under the condition of non-stationarity, an equilibrium will hold if and only if the 'true' economic equilibrium relationship between the series can be determined (so that the statistical and economic equilibrium coincide).[8]

Hence, the concept of stationarity not only implies some favourable statistical properties but is also essential for the investigation of equilibrium relationships between economic time series. Accordingly, econometricians have to thoroughly investigate the time series properties of the variables they are considering in order to avoid any disregard or misspecification of potential equilibrium relationships.

However, the usefulness of the stationarity concept in the above strict sense appears rather limited in practice. First of all, the joint distribution of the x(t) is usually rather hard to determine and, secondly, the entire concept simply seems to be too rigid to be of any practical use [cp. Maddala, G. S., 1992, p. 528].

Therefore, the idea of strict stationarity is reduced in practice to the concept of lth-order stationarity (in particular in the form of second-order stationarity). According to Spanos (1986, p. 138), a " ... stochastic process $\{X(t), t \in T\}$ is said to be ***l*th-order stationary** if for any subset $(t_1, t_2, ..., t_n)$ of T and any τ, $F(X(t_1, ..., X(t_n))$ [sic!] is of **order *l*** and its joint moments are equal to the corresponding moments of $F(X(t_1+\tau), ..., X(t_1+\tau))$ [sic!], i. e.

$$E[\{X(t_1)\}^{l_1}\{X(t_2)\}^{l_2}, ..., \{X(t_n)\}^{l_n}] = E[\{X(t_1+\tau)\}^{l_1}, ..., \{X(t_n+\tau)\}^{l_n}],$$

where $l_1+l_2+...+l_n \leq l$;"

The order of stationarity most frequently referred to is the case of $l = 2$ so that:

1) $(l_1 = 1; l_2 = 0)$: $E[x(t)] = E[x(t+\tau)] = \mu_1 < \propto$
2) $(l_1 = 2; l_2 = 0)$: $E[\{x(t)\}^2] = E[\{x(t+\tau)\}^2] = \mu_2 < \propto$
3) $(l_1 = 1; l_2 = 1)$: $E[\{x(t_1)\}\ \{x(t_2)\}] = E[\{x(t_1+\tau\}\ \{x(t_2+\tau)\}] = \mu_{12} < \propto$

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7 However, it should also be clear that these combinations meet the statistical equilibrium conditions but not necessarily their counterparts in economic theory.

8 This, in fact, represents the basic idea behind the concept of cointegration which is discussed to some extent in the following chapter.

with μ_1, μ_2, μ_{12} constant over t [cp. Spanos, A., 1986, p. 138; cp. Banerjee, A. et al., 1993, p. 11].
The popularity of this second-order stationarity (henceforth also called weak stationarity) is partly due to the fact that the concepts of weak and strict stationarity coincide when dealing with random variables which follow a (multivariate) normal distribution. This follows from the fact that the latter is completely characterized by the first and second order moments only [cp. Maddala, G. S., 1992, pp. 528-529; cp. Spanos, A., 1986, p. 139].

However, it should be clear at this stage that non-stationarity does not necessarily imply trend behaviour (which in turn might result in spurious regressions). For example, if $E(x_t)$ is constant over all t (and $E(x_t) < \propto$ holds) but the variance and covariance conditions for second-order stationarity do not hold, the series is not even weakly stationary but nonetheless unlikely to exhibit any deterministic or stochastic trend [cp. Thomas, R. L., 1993, p. 158; cp. Charemza, W. W. and Deadman, D. F., 1992, pp. 122-123]. Hence, simple graphical plots of time series have actually to be interpreted with care when the absence of any trend seems to be obvious and thus the series to be stationary in the levels. In fact, the visual examination of the correlograms of the series (as discussed below) represents the more powerful graphical tool.
However, the distinction between deterministic and stochastic trends itself is of vital importance. For example, provided the series under investigation can be shown to exhibit a stochastic (or difference-stationary) trend, it can be made stationary by differencing and is said to be integrated[9] whereas in the case of a deterministic trend a regression on time represents the appropriate technique for its removal.[10]

Hence, after all, it seems desirable to dispose of some powerful methods for the detection and classification of any potential non-stationarity of the series under

[9] For a more thorough treatment of the concept of integration and various tests for difference-stationarity, see section III.b.1. and III.b.2.

[10] See Maddala (1992, pp. 260-262), Cochrane (1991, p. 275) and Nelson and Plosser (1991, p. 143) for the consequences of regressions on time when dealing with a difference-stationary series and vice versa. For ease of exposition, the case of mixed determinisitic-stochastic trends is ignored.

consideration. So-called unit root testing represents such a kind of method and is therefore introduced on the basis of its time series origin in the following section.

III.a.2. The Box-Jenkins approach

Ignoring the question of non-stationarity for a moment, the above definition of lth-order stationarity indicates that in the case of weak stationarity the covariance at two different points in time depends only on the difference between these two points. Thus, the following relationship can be derived:

$$\gamma(h) = E[(x(t)-\mu)(x(t+h)-\mu)]$$
$$= \gamma(-h) \text{ because of stationarity,}$$

where $\gamma(h)$ denotes the hth autocovariance. Standardization yields the hth autocorrelation, p(h):

$$p(h) = \frac{\gamma(h)}{\gamma(0)}$$
$$= p(-h) \text{ because of stationarity,}$$

where $\gamma(0)$ represents the variance of the process under investigation [cp. Goldberger, A. S., 1991, pp. 278-279; cp. Maddala, G. S., 1992, p. 529; cp. Perman, R., 1991, p. 8].

Plotting p(h) against h, where h denotes the order of lags included, results in the correlogram of the series which is used as a mean to determine orders of integration within the Box-Jenkins approach towards time series modelling.

According to this traditional time series method, (difference) stationarity can conveniently assumed to be present if the correlogram of the series dampens as the order of lags increases.

The correlogram of an adequately differenced and thus stationary series can then also be used for the derivation of the order of the autoregressive and moving average components of the process under investigation. The resulting model can be estimated, diagnostically checked and, if found to be adequate, can be used for its main purpose of producing reliable forecasts.

Hence, the basic steps of the Box-Jenkins approach can be summarized as follows: " ... (1) differencing the series so as to achieve stationarity, (2) identification of a tentative model, (3) estimation of the model, (4) diagnostic checking (if the model is found inadequate, we go back to step 2), and (5) using the model for forecasting and control" [Maddala, G. S., 1992, p. 542; see also, Hylleberg, S., 1986, pp. 172-173].

This Box-Jenkins methodology, which apparently does not refer to any findings of economic theory, was considered as being superior to standard econometric models until the mid 1970s, at least when focusing on the forecast performance. This comparison in terms of forecast results " ... brought out a very important defect in the empirical macro-models; the macro-modellers, preoccupied with simultaneity, were largely ignoring the temporal structure of the time-series data, at their peril ... " [Spanos, A., 1988, p. 340]. In other words, econometric modelling at that time was characterized by the neglect of both the issues of stochastic processes and stationarity as well as the modelling of dynamic structures thereby producing rather poor forecast results.

One of the turning points in this development was the paper published by DHSY. DHSY put forward a reconciliation of short-run and long-run aspects within a generalized econometric framework. Indirectly, DHSY also contributed to the formalization of the concept of stationarity (and the development of the related issues of integration and cointegration) and its incorporation into relatively powerful econometric models.

However, in order to illustrate this development from 'simple' visual examinations of correlograms to thorough statistical testing for stationarity, it seems reasonable to follow the lines of Banerjee et al. (1993) and to reconsider the usual autoregressive-moving average (ARMA(p,q))-result of the Box-Jenkins procedure [cp. Hylleberg, S., 1986, p. 161]:

$$x_t = \sum_{i=1}^{p} \alpha_i * x_{t-i} + \sum_{j=0}^{q} \theta_j * \varepsilon_{t-j},$$

where $\theta_0 = 1$ and $\{\varepsilon_t\}$ is a white-noise process.

Using the lag operator L[11], this relationship is equivalent to:

$$\alpha(L)x_t = \theta(L)\varepsilon_t,$$

with $\alpha(L) = 1 - \sum_{i=1}^{p} \alpha_i * L^i$ and $\theta(L) = 1 + \sum_{j=1}^{q} \theta j * L^j$.

These lag polynomials can be rewritten as:

$$\alpha(L) = \prod_{i=1}^{p} (1 - \lambda i * L)$$

$$\theta(L) = \prod_{j=1}^{q} (1 - \delta j * L).$$

Provided all potential common factors are deleted, if there is any $\lambda_i = 1$, " ... then the process is said to contain a *unit root*" [Banerjee, A. et al., 1993, p. 13] and is non-stationary (difference-stationary).[12]

Hence, the concept of unit root testing which is firmly based on statistical theory (and represents the basic instrument for standard integration and cointegration testing) replaces the somehow fuzzy Box-Jenkins concept of the visual examination of correlograms.[13]

In spite of this, the development of unit root tests reflects also the growing tendency towards a unifying methodological framework in empirical modelling. In opposite to the state of empirical modelling twenty years ago, the findings of time series theory and econometrics nowadays tend to be complementing in nature rather than excluding [cp. Spanos, A., 1988, pp. 345-360].

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[11] See, for example, Judge et al. (1988, pp. 679-680) for the definition of L.

[12] This analysis is restricted to the AR part of the process only since MA processes can be shown to be generally stationary whereas the time path of the AR component will be stable if and only if all roots are greater than unity in absolute value. In spite of the stationarity property of MA processes, an analogous condition has to hold for the MA process to ensure that the ARMA-representation is invertible [cp. Judge, G. G. et al., 1985, p. 658; cp. Banerjee, A. et al., 1993, p. 13]. Only if these two conditions are met, conventional distributional results concerning the coefficient estimates will be applicable [cp. Hendry, D. F. and Doornik, J. A., 1992, p. 30].

[13] Such a visual examination, however, still represents a useful instrument for the confirmation of the results of unit root tests. In other words, the " ... tests simply make the procedure less subjective without eliminating visual inspection as a useful tool" [Dickey, D. A. and Pantula, S. G., 1987, p. 455]. It will therefore be referred to it in the applied sections of this dissertation (see also Hall, S. G., 1986, p. 238).

However, as indicated above, the concept of unit root testing represents the baseline for the conventionally used methods for the detection and classification of non-stationarity, namely integration and cointegration tests. The power of these unit root tests and their relationship to the theory of integration/cointegration as well as the potential subsequent derivation of error-correction models are therefore explored to some extent in the following sections.

Such an extensive discussion appears reasonable, since the detection of unit roots obviously has severe statistical and economic implications. On the statistical side, unit root testing provides the justification for the adoption of particular trend removal methods and non-standard distribution theory. Similarly, from an economic point of view, the simple existence of a unit root can contribute to the rejection of an economic theory on empirical grounds [cp. Maddala, G. S., 1992, pp. 581-582].

III.b. The concept of integration and unit root testing

III.b.1. Orders of integration and unit root tests

It seems reasonable at this point to define more thoroughly the concept of orders of integration casually introduced in the previous section. According to Banerjee et al. (1993), a " ... series with no deterministic component and which has a stationary and invertible autoregressive moving average (ARMA) representation after differencing d times, but which is not stationary after differencing only d-1 times, is said to be integrated of order d, denoted x_t~I(d)" [Banerjee, A. et al., 1993, p. 84].

Not surprisingly and as already mentioned above, series which are I(d) ($d \geq 1$) show statistical properties which deviate significantly from those of time series which are I(0), and thus stationary. A series I(d) is characterized by (amongst others) an unbounded variance, an infinite expected time between crossings of y=0 and a permanent memory, etc. [cp. Dickey, D. F. and Fuller, W. A., 1979, p. 427; cp. Granger, C. W. J., 1986, pp. 214-215].

The additional problem that conventional convergence theorems concerning sample moments break down for integrated series is particularly detrimental. Under these circumstances, sample moments can in fact be shown to converge to random variables rather than constants. "Analytical results concerning limiting distributions must therefore be based on an extended asymptotic theory" [Banerjee, A. et al., 1993, p. 86; cp. Banerjee, A. and Hendry, D. F., 1992, pp. 226-233]. As a consequence, 'non-standard' critical values have to be derived for the various test procedures which are usually based on the null of non-stationarity.

However, assuming any deterministic component to be absent[14], the consideration of the simplest data generation process (namely, an AR(1)-process) helps to highlight the intuitive idea behind unit root testing for the orders of integration:

$$y_t = p * y_{t-1} + \varepsilon_t \qquad (3.1)$$

where $\varepsilon_t \sim IID(0;\sigma^2)$.

At first glance, testing for a unit root simply seems to amount to testing for p=1 when using the conventional approach with ε_t being white noise. However, as indicated above, under the null of non-stationarity " ... the distribution of the test statistic $[\frac{\hat{p} - p_0}{SE(\hat{p})}]$ is not asymptotically normal, or even symmetric" [Banerjee, A. et al., 1993, p. 100]. In fact, the OLS estimator of p is generally downwardly biased. Accordingly, the inclusion of any drift and/or trend terms in the regression model in order to make allowance for potential nuisance parameters in the DGP leaves this bias unchanged [cp. Dickey, D. A. and Fuller, W. A., 1979, pp. 428-429].[15]

However, given the simple AR(1) data generation process considered above, Dickey and Fuller (1979) derive the limiting distributions of the OLS estimator of p and the corresponding t-statistic under the null of non-stationarity.

[14] It can relatively easily be made allowance for potential deterministic components in the DGP when using the Dickey-Fuller 'F-test' [cp. Dickey, D. A. and Fuller, W. A., 1981, pp. 1058-1064]. See also Banerjee et al. (1993, pp. 113-119), Maddala (1992, pp. 259-260) and Thomas (1993, pp. 162-163) for expositions on this. The DF F-test will be employed in the empirical sections of this dissertation to test for the presence of any deterministic trend.

They show that the properties of the distribution of the estimators and the t-tests for a simple model with or without intercept and with or without time trend correspond to those of the distribution tabulated by Fuller (1976, p. 373). Using Monte Carlo simulation, they conclude that their test procedure outperforms the traditional time series test based on the autocorrelation function (namely, the Box-Pierce test).
Hence, the use of standard OLS for the estimation of:

$$\Delta y_t = p^* * y_{t-1} + u_t \qquad (3.2)$$

where $p^* = p - 1$,
along with the critical values for a t-test on p^* provided by Fuller (1976) leads to a valid test decision. This **simple Dickey-Fuller test** therefore represents a relatively straightforward test for the null of $p^* = 0$ against $p^* < 0$.[16]
The test derives its validity from the fact that the critical values tabulated by Fuller (1976) are shifted to the left and thus make allowance for the negative skewness of the t-test-distribution with most of its mass below zero so that overrejections are prevented [cp. Banerjee, A. and Hendry, D. F., 1992, p. 235; cp. Charemza, W. W. and Deadman, D. F., 1992, p. 132] .
On the other hand, it has to be remembered that the results of the underlying simulation technique depend significantly on the structure of the model used. Therefore the tabulated critical values have to be treated with care if doubts about the adequacy of the regression model exist. Such doubts might be present in particular when applying the rather simple Dickey-Fuller model introduced above.

If the t-statistic turns out to be smaller than the appropriate critical value, the null of a unit root has obviously to be rejected in favour of I(0). However, if $[\frac{\hat{}p - p_0}{SE(\hat{}p)}]$ exceeds the critical value[17], the null of non-stationarity is maintained and

[15] Such an extension of the model usually serves the purpose of ensuring the similarity of the test statistic. Similarity is given if the regression model contains more parameters than the underlying DGP [cp. Banerjee, A. et al., 1993, p. 105].

[16] Obviously, $p^* < 0$ implies $p < 1$ and thus stationarity.

[17] See Charemza and Deadman (1992, p. 132) for a set of critical values constructed differently from the conventionally used table by Fuller (1976). Since the use of simulation techniques inevitably involves some

the additional possibility of d>1 has to be considered before any valid conclusion can be drawn. If, for example, " ... two roots are present then ΔY_t as well as Y_t is a non-stationary process. (...) This implies that, having found a series Y_t to be non-stationary, rather than assume the first difference is stationary we must apply the Dickey-Fuller test to ΔY_t" [Thomas, R. L., 1993, pp. 161].
Standard textbooks, in fact, suggest to iterate this procedure until the null of non-stationarity is rejected [see, for example, Charemza, W. W. and Deadman, D. F., 1992, p. 133 and Thomas, R. L., 1993, p. 161]. Dickey and Pantula (1987, p. 455), however, point to the fact that the conventional Dickey-Fuller tests take the absence of unit roots as the alternative hypothesis. Thus, strictly speaking, the standard iteration procedure is invalid and overrejections are likely to occur. Dickey and Pantula (1987, pp. 458-461) suggest to reverse the testing procedure and to start with the largest number of unit roots under investigation rather than the smallest in order to maintain the nominal test size chosen. This 'testing down'-procedure has then again to be iterated until H_0 is rejected (see, for example, Osborn, D. R. et al., 1988, pp. 364-370).[18]

However, apart from the more general problem of deriving a valid test decision within the Dickey-Fuller procedure, there exists another more specific problem. The original Dickey-Fuller test is based on the rather rigid assumption that the DGP corresponds to a simple AR(1) process[19] which implies the ignorance of potential nuisance parameters and dynamics [cp. Dickey, D. F. and Fuller, W. A., 1979, p. 428; cp. Charemza, W. W. and Deadman, D. F., 1992, p. 135]. As a consequence, the original Dickey-Fuller test will usually be affected by residual autocorrelation due to omitted variables. Under these circumstances, the u_t can no longer be assumed to be white noise. Thus, the application of OLS will yield inefficient estimates of p^* and the test procedure breaks down. Dickey and Fuller (1981, pp. 1065-1066) succeed in overcoming this drawback and derive the limiting distributions of the OLS estimator of p and various t- and F-type statistics thereby extending their 1979 study.

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error, they derive upper and lower limits for the actually unknown critical value following the approach by Durbin and Watson.

18 Hence, the problem of testing for exactly one unit root usually represents the last step in a Dickey-Pantula testing sequence rather than the first.

19 Potentially including an intercept or a deterministic trend [cp. Dickey, D. A. and Fuller, W. A., 1979, p. 428].

They show that proceeding from the general autoregressive process of the form:

$$y_t = \sum_{i=1}^{p} p_i * y_{t-i} + \varepsilon_t, \tag{3.3}$$

the error terms in the following regression model will be asymptotically white noise if it includes an appropriate number of lagged dependent variables:[20]

$$\Delta y_t = p^* * y_{t-1} + \sum_{i=1}^{k} \theta_i * (y_{t-i} - y_{t-i-1}) + u_t. \tag{3.4}$$

The autocorrelation in the residuals will simply be approximated by the lagged dependent variables if k is determined adequately.

With the condition of white noise errors being met, the asymptotic distribution of ^p will be identical to that of $\hat{p}^*$ and the critical values tabulated by Fuller (1976) can be used to derive a valid test decision for the null of p=1 [cp. Dickey, D. A. and Fuller, W. A., 1981, p. 1066].

This result gives rise to the immediate question how to determine the appropriate number of lagged dependent variables in (3.4) in practice. However, in applied work, k will best be approximated by a sequence of LM tests for residual autocorrelation with the number of lags increasing with each step (whilst observing the degrees of freedom problem) [cp. Thomas, R. L., 1993, p. 161; cp. Osborn , D. R. et al., 1988, p. 365; cp. Banerjee, A. et al., 1993, p. 107].

Once k is determined, this **augmented Dickey-Fuller test** has also to be iterated until the null of non-stationarity is rejected when adopting the Dickey-Pantula approach. However, even without adoption of the Dickey-Pantula approach, Monte Carlo evidence indicates that the ADF test represents the most efficient test statistic of the class of simple, parametric unit root tests.

This conclusion extends to the case including non-parametric tests such as the **Phillips-Perron statistics** provided the ADF approach is generalized by allowing for a ARMA(p,q)[21]-process as put forward by Said and Dickey (1984).

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20 The regression model can again be extended to make allowance for a potential intercept and/or a deterministic time trend in the DGP.

Phillips and Perron argue in favour of simple adjustments of the t-statistic on ^p after estimation rather than augmenting the model by lagged dependent variables (in order to avoid the recalculation of critical values each time a different DGP is considered).[22] The corresponding t-statistics follow conventional Dickey-Fuller distributions and can be shown to include the Dickey-Fuller tests as special case [cp. Banerjee, A. et al., 1993, pp. 109-112].

However, as far as the power of these tests in comparison to the extended ADF test is concerned, the Phillips-Perron statistics " ... are more likely to reject the null of a unit root, *whether or not* it is false; for errors with strong negative MA components, the difference is quite large" [Banerjee, A. et al., 1993, p. 113]. Hence, although Phillips-Perron statistics seem to be preferable at the presence of positive MA or identically and independently distributed errors, the unit root tests in the applied sections of this dissertation will be based on the Dickey-Fuller procedures only.

III.b.2. Seasonal integration

So far the analysis of unit root testing and integration has been restricted to the case of unit roots at the zero frequency only. However, when using seasonally unadjusted data, as given in the DHSY study discussed further below, this is no longer appropriate and the possibility has to be taken into account " ... that it may be necessary to difference, seasonally average, or seasonally difference the data to make it stationary" [Ilmakunnas, P., 1990, p. 79].

In fact, a time series can be integrated at any of the seasonal frequencies $\omega_S = 2*\Pi*\frac{i}{s}$, $i = 1, ..., \frac{s}{2}$, where s denotes the number of times the data is recorded in a year [cp. Engle, R. F. et al., 1989, p. 49]. In the case of quarterly seasonal unit root processes, for example, in total four roots of modulus unity can be identified as might become clear when considering the relationship $(1-L^4) = (1-L)*(1+L)*(1-i*L)*(1+i*L)$ [cp. Banerjee, A. et al., 1993, p. 121; cp. Hylleberg, S. et al., 1990, p. 221]:

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[21] With p and q unknown.

- one at the zero frequency as tested for by the conventional unit root tests
- one at the half-yearly frequency
- two at the annual frequency.

Accordingly and applying the definition of seasonal integration put forward by Osborn et al. (1988, p. 362), a " ... non-deterministic series x_t is said to be integrated of order (d,D), denoted x_t~I(d,D), if the series has a stationary, invertible ARMA representation after one-period differencing d times and seasonally differencing D times."[23]

However, the properties of seasonally integrated series, although not being exactly identical to those of I(1) series, are similar to the properties of non-seasonally integrated processes. In fact, their variances are increasing with time, they have permanent memory, etc. [cp. Hylleberg, S. et al., 1990, pp. 218-219].

On the other hand, since seasonally integrated series contain multiple roots of modulus unity, the seasonal observations might develop differently over time so that first differencing will usually not suffice to achieve stationarity.

Furthermore, the consideration of seasonal integration is of vital importance for the analysis of cointegration between series due to the fact that the 'super-consistency'[24] property of OLS estimates in the cointegrating regression might be lost when dealing with seasonally integrated series [cp. Engle, R. F. et al., 1989, pp. 50-51].

However, despite of these implications, testing " ... for a unit root at a seasonal frequency has much in common with testing for unit roots at the zero frequency" [Banerjee, A. et al., 1993, p. 122].

Testing for seasonal integration is probably most easily pursued by applying the test procedure suggested by Dickey, Hasza and Fuller (1984; DHF hereafter). DHF proceed from the simple seasonal time series model [cp. Dickey, D. A. et al., 1984, p. 355]:

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22 I. e., Phillips and Perron replace the extension of the regression model proposed by Dickey and Fuller (1981) and Said and Dickey (1984) by an adjustment of the corresponding t-statistics.

23 For a critique of this definition in comparison to the slightly more general one provided by Engle et al. (1989, p. 49) see Ilmakunnas (1990, pp. 79-80).

24 The 'super-consistency' property is formally introduced in section IV.a.

$$y_t = \alpha_d * y_{t-d} + e_t \tag{3.5}$$

where $e_t \sim IID(0,\sigma^2)$ and s=4 for quarterly data, etc. From this simple representation, they develop a t-test for the OLS estimate of the parameter β in:[25]

$$\Delta_s y_t = \beta * z_{t-s} + \sum_{i=1}^{p} \beta_i * \Delta_s y_{t-i} + e_t \tag{3.6}$$

under the null of $\beta=0$ (corresponding to H_0: $\alpha_d=1$ since $\beta=\alpha_d-1$) against $\beta<0$; the regressor z_{t-s} is derived as follows [cp. Dickey, D. A. et al., 1984, p. 360; cp. Charemza, W. W. and Deadman, D. F., 1992, pp. 137-138]:

1) Estimate $\Delta_s y_t = \sum_{i=1}^{p} \lambda_i * \Delta_s y_{t-i} + \xi_t$ using OLS (3.7)

2) Estimate $z_t = y_t - \sum_{i=1}^{p} \hat{\lambda}_i * y_{t-i}$ using OLS (3.8)

The conventional LM test for residual autocorrelation can be employed to determine the appropriate size of p [cp. Osborn, D. R. et al., 1988, p. 365]. Comparison with the critical values tabulated by DHF yields a test decision on the null hypothesis.
However, a " ... major drawback of this test is that it doesn't allow for unit roots at some but not all of the seasonal frequencies and that the alternative has a very particular form, namely that all the roots have the same modulus" [Hylleberg, S. et al., 1990, p. 221]. Consequently, Hylleberg, Engle, Granger and Yoo (1990; henceforth HEGY) develop a seasonal unit root test which explicitly allows for the possibility of unit roots at each individual frequency.
Proceeding from a quarterly process of the form:

$$\varphi(L)x_t = \varepsilon_t \tag{3.9}$$

where $\varepsilon_t \sim IID(0,\sigma^2)$ and $\varphi(L)$ represents a fourth-order lag polynomial, HEGY use:

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[25] The dependent variable $\Delta_s z_t$ originally included by DHF is replaced by $\Delta_s y_t$ following the modification put forward by Osborn et al. (1988, p. 365).

$$\varphi(L) = (1-L)*(1+L)*(1+L^2) \qquad (3.10)$$

to develop the equation:

$$\begin{aligned}\varphi(L) = {} & \lambda_1*L(1+L)*(1+L^2)+\lambda_2*(-L)*(1-L)*(1+L^2) \qquad (3.11)\\ & +\lambda_3*(-iL)*(1-L)*(1+L)*(1-iL)\\ & +\lambda_4*(iL)*(1-L)*(1+L)*(1+iL)\\ & +\varphi*(L)(1-L^4)\end{aligned}$$

where $\lambda_k = \frac{\varphi(\theta_k)}{\prod_{j\neq k}\delta_j(\theta_k)}$ with θ_k representing the various frequencies and $\delta_k(L)=1-\frac{1}{\theta_k}*L$.

After simplification and setting $\Pi_1=-\lambda_1$, $\Pi_2=-\lambda_2$, $2\lambda_3=-\Pi_3 + i*\Pi_4$ and $2\lambda_4=-\Pi_3-i*\Pi_4$, they end up with:

$$\begin{aligned}\varphi(L) = {} & -\Pi_1*L*(1+L+L^2+L^3) \qquad (3.12)\\ & - \Pi_2*(-L)*(1-L+L^2-L^3)\\ & - (\Pi_4+\Pi_3*L)*(-L)*(1-L^2)\\ & + \varphi*(L)(1-L^4).\end{aligned}$$

This relationship can be used to replace $\varphi(L)$ in (3.9) so that HEGY eventually obtain the following regression equation:

$$\varphi*(L)\Delta_4 x_t = \Pi_1*y_{1t-1} + \Pi_2*y_{2t-1} + \Pi_3*y_{3t-2} + \Pi_4*y_{3t-1} + \varepsilon_t \qquad (3.13)$$

where $y_{1t} = (1+L+L^2+L^3)*x_t$
$y_{2t} = -(1-L+L^2-L^3)*x_t$
$y_{3t} = -(1-L^2)*x_t$.

(3.13) can be estimated by OLS (possibly with added lags of the dependent variable to whiten the errors) [cp. Hylleberg, S. et al., 1990, pp. 221-223].
From this lengthy derivation, it follows that:

- testing for a unit root at zero frequency corresponds to testing for $\Pi_1=0$
- testing for a unit root at half-yearly frequency is equivalent to testing for $\Pi_2=0$ and
- testing for a unit root at annual frequency boils down to testing $\Pi_3=\Pi_4=0$ using a F-type statistic with the critical values tabulated by HEGY (1990, p. 227).

Hence, there are no seasonal unit roots if Π_2 and either Π_3 or Π_4 are unequal to zero. The rejection of all of these tests, however, implies that the series under investigation is stationary.

Apart from the critical values for the F-statistic, HEGY also derive critical values for the t-tests on Π_1 and Π_2 using Monte Carlo techniques which can be used to obtain relatively reliable test decisions. This reliability follows from the similarity property of the test statistic and goes in line with its robustness against specific alternatives [cp. Hylleberg, S. et al., 1990, p. 224]. This latter feature in particular contrasts the properties of some frequently used tests for unit roots discussed in section III.b.3.

However, apart from the widely used DHF and HEGY test, the ADF seasonal integration test (also ADFSI hereafter) will be employed as an additional indicator for seasonal integration in the applied sections of this dissertation.

The ADFSI regression follows directly from the DHF equation by replacing the term z_{t-s} by y_{t-s}, i. e.:

$$\Delta_s y_t = \beta * y_{t-s} + \sum_{i=1}^{p} \beta_i * \Delta_s y_{t-i} + e_t \tag{3.14}$$

where $e_t \sim IID(0,\sigma^2)$.

The critical values for the ADFSI test including an intercept (as given in the empirical sections of this study) for the null of $\beta=0$ against $\beta<0$ can be taken from the table provided by Fuller (1976, p. 373) [cp. Osborn, D. R. et al., 1988, p. 363].

In contrast to this convenient property, the ADF test for seasonal integration suffers from a lack of reliability and should therefore be interpreted with care [cp. Charemza, W. W. and Deadman, D. F., 1992, p. 139].

However, the application of the various test statistics for seasonal integration introduced above raises again the question after the appropriate testing sequence. It seems reasonable in this context to follow the approach put forward by Ilmakunnas (1990). Ilmakunnas argues in favour of the incorporation of the Dickey-Pantula 'testing down'-approach towards non-seasonal unit root testing into a testing sequence for seasonal integration.[26] The adoption of this innovation, again helps to preserve the nominal test size chosen [cp. Ilmakunnas, P., 1990, p. 80].

As indicated in figure III.1 and table III.1, the " ... basic idea is that when the maintained hypothesis is that there is a unit root at lag 1 or at a seasonal lag, the available test statistics are modified so that appropriately differenced (Δ or Δ_4, respectively) data is used when running the test regression" [Ilmakunnas, P., 1990, p. 80; see also Dickey, D. A. and Pantula, S. G., 1987, p. 458]. Since the HEGY test takes account of units roots at each individual frequency, this implies that before any further tests are carried out, Π_1 has to be restricted to zero if a 'zero frequency root' is maintained whereas $\Pi_2=\Pi_3=\Pi_4=0$ has to be imposed on the regression equation if 'seasonal frequency roots' are maintained. Similarly, a test for both zero and seasonal frequency unit roots implies testing of all the parameters.

Accordingly, $\Pi_1\neq0$ or $\Pi_2\neq0$, $\Pi_3\neq0$, $\Pi_4\neq0$ should hold if the nulls of zero or seasonal frequency respectively have been rejected previously [cp. Ilmakunnas, P., 1990, p. 81].

The adoption of this elaborate testing sequence rather than the traditional textbook approach should help to obtain a relatively reliable test decision on the

[26] See figure III.1 for the case of a quarterly series with I(1,1) assumed to be the maximum order of seasonal integration.

order of integration of a seasonal time series despite of the general weaknesses of unit root tests discussed in the following section.

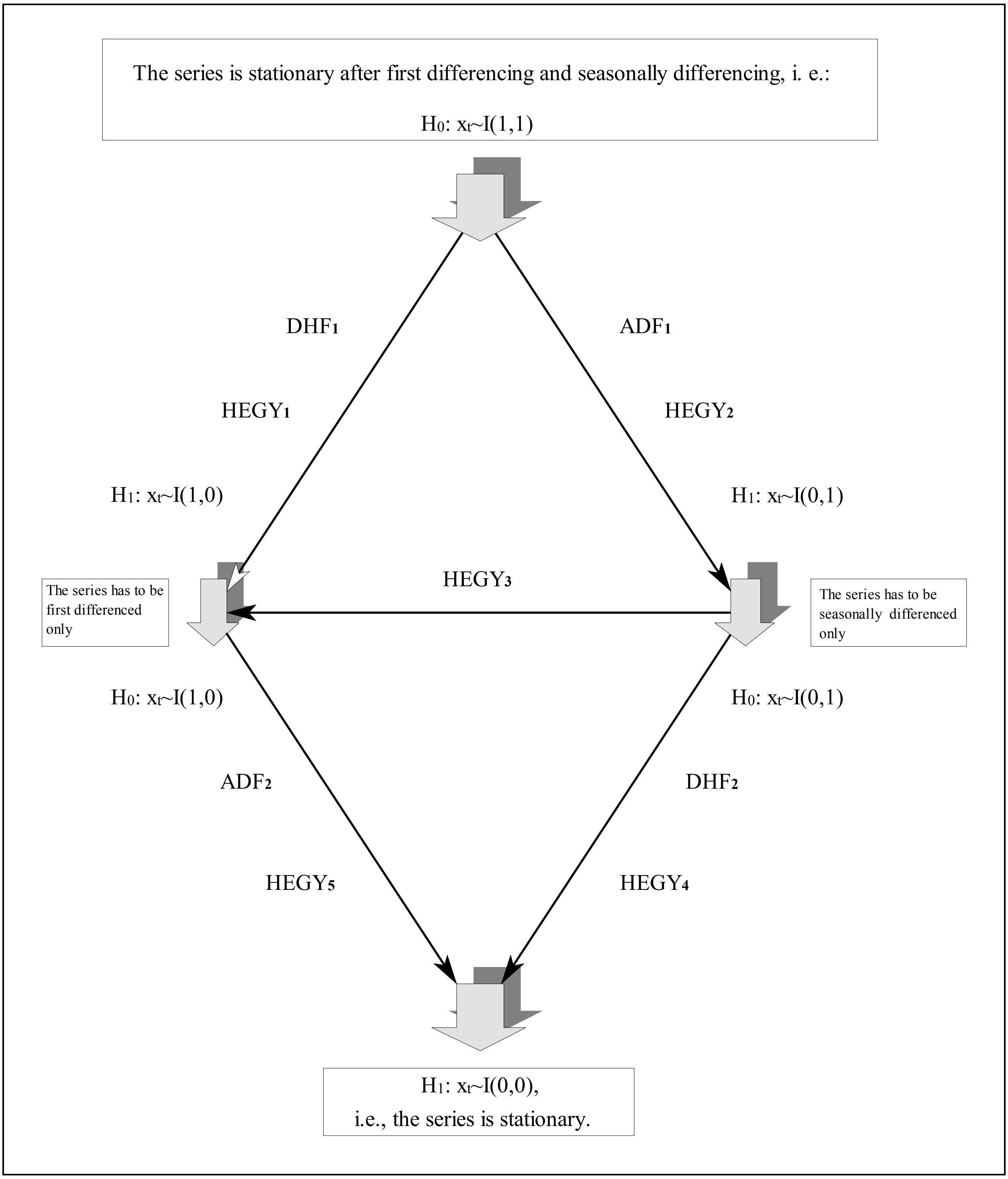

Figure III.1: Testing for the order of seasonal integration of a quarterly series

	Test regression	Seasonal frequencies tested
ADF_1 (3.15)	$\Delta\Delta_4 x_t = \beta * \Delta_4 x_{t-1} + \sum_{j=1}^{p} \alpha_j * \Delta\Delta_4 x_{t-j} + u_t$	
ADF_2 (3.16)	$\Delta x_t = \beta * x_{t-1} + \sum_{j=1}^{p} \alpha_j * \Delta x_{t-j} + u_t$	
DHF_1 (3.17)	$\Delta\Delta_4 x_t = \beta * z_{t-4} + \sum_{j=1}^{p} \alpha_j * \Delta\Delta_4 x_{t-j} + u_t$	
DHF_2 (3.18)	$\Delta_4 x_t = \beta * z_{t-4} + \sum_{j=1}^{p} \alpha_j * \Delta_4 x_{t-j} + u_t$	
$HEGY_1$ (3.19)	$\Delta\Delta_4 x_t = \Pi_1 * z_{1,t-1} + \Pi_2 * z_{2,t-1} + \Pi_3 * z_{3,t-2} + \Pi_4 * z_{3,t-1} + \sum_{j=1}^{p} \alpha_j * \Delta\Delta_4 x_{t-j} + u_t$	with Π_1=0 and Π_2,Π_3,Π_4 tested.
$HEGY_2$ (3.20)	$\Delta\Delta_4 x_t = \Pi_1 * z_{1,t-1} + \Pi_2 * z_{2,t-1} + \Pi_3 * z_{3,t-2} + \Pi_4 * z_{3,t-1} + \sum_{j=1}^{p} \alpha_j * \Delta\Delta_4 x_{t-j} + u_t$	with Π_2=Π_3=Π_4=0 and Π_1 tested.
$HEGY_3$ (3.21)	$\Delta_4 x_t = \Pi_1 * z_{1,t-1} + \Pi_2 * z_{2,t-1} + \Pi_3 * z_{3,t-2} + \Pi_4 * z_{3,t-1} + \sum_{j=1}^{p} \alpha_j * \Delta_4 x_{t-j} + u_t$	with Π_1=0 and Π_2,Π_3,Π_4 tested.
$HEGY_4$ (3.22)	$\Delta_4 x_t = \Pi_1 * z_{1,t-1} + \Pi_2 * z_{2,t-1} + \Pi_3 * z_{3,t-2} + \Pi_4 * z_{3,t-1} + \sum_{j=1}^{p} \alpha_j * \Delta_4 x_{t-j} + u_t$	$\Pi_1 \neq 0$ and Π_2,Π_3,Π_4 tested.
$HEGY_5$ (3.23)	$\Delta_4 x_t = \Pi_1 * z_{1,t-1} + \Pi_2 * z_{2,t-1} + \Pi_3 * z_{3,t-2} + \Pi_4 * z_{3,t-1} + \sum_{j=1}^{p} \alpha_j * \Delta_4 x_{t-j} + u_t$	Π_2,Π_3,$\Pi_4 \neq 0$ and Π_1 tested.

Table III.1: Test statistics for a quarterly series

III.b.3. A critique of conventional unit root testing

As already indicated in section III.a., the existence of unit roots is of considerable importance from both an economic and a statistical point of view. Accordingly, unit root tests are commonly presented in applied work as justification for the imposition of particular trend removal methods and the adoption of non-standard distribution theory for OLS estimates.

Moreover, as far as the economic aspect is concerned, the statistical non-rejection of the null of unit roots can in fact be decisive for the empirical adoption or rejection of an economic theory.

Hence, considering the significance of the issue of unit roots, " ... it is important to determine whether inferences derived from conventional integration tests are fragile;" [DeJong, D. N. and Whiteman, C. H., 1991, p. 222].

In fact, it has been well documented that tests for unit roots can lack robustness against some specific alternatives, especially in sizes which usually occur in economics. This is particularly true when dealing with near-integrated or 'borderline'-processes (i. e., processes which have roots close to but not equal to unity) [cp. Cochrane, J. H., 1991, p. 276].

However, even if assuming a unit root test could provide a reliable distinction between non-stationary and near-integrated processes, this distinction might not necessarily lead to the use of the appropriate distribution theory. This is due to the fact that under certain circumstances a near-integrated process " ... behaves like an I(1) process even though it is asymptotically stationary" [Banerjee, A. et al., 1993, p. 95]. Thus, such kind of processes will be better approximated by Wiener distribution theory rather than the 'correct' standard theory. In fact, standard unit root tests have generally to be considered as being of relatively low power against some specific trend stationary alternatives.

In accordance to this, the majority of economic series has initially wrongly been taken to be difference-stationary mainly due to the DF test results published by Nelson and Plosser (1982, pp. 146-158).[27]

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[27] Maddala (1992, p. 584) mentions the additional problem of an upward-baised estimate of the autoregressive parameter in the Nelson-Plosser study.

However, as DeJong and Whiteman (1991, p. 223) point out, the above weakness is intensified by the conventional choice of non-stationarity as the null hypothesis in unit root tests which " ... involves assigning very low probability to the alternative hypothesis ... " Using a null of stationarity or a Bayesian approach often produces results contrary to those of unit root tests with the null of non-stationarity [cp. Maddala, G. S., 1992, p. 585; cp. DeJong, D. N. and Whiteman, C. H., 1991, pp. 225-251]. Therefore, it seems all the more important to use unit root tests alongside of the corresponding correlograms in order to derive a relatively reliable decision regarding the stationarity of a series.

Strictly speaking, however, to base the rejection or adoption of an economic theory solely on the empirical answer to the question whether a parameter point estimate is exactly equal to unity or not seems to be inappropriate anyway. As Maddala (1992, p. 582) points out, the " ... relevant question is not whether $\alpha=1$ or not, but how big the autoregressive parameter is or how long it takes for shocks (...) to die out" [see also Cochrane, J. H., 1991, pp. 276-277].

In addition, apart from the issue of the choice of the appropriate null hypothesis, the consideration of relatively long time series as given, for example, in the Nelson and Plosser study (1909-1970) can by itself cause the incorrect acceptance of the null of non-stationarity. This incorrect non-rejection can be due to the bias in the OLS estimate resulting from the existence of various structural breaks in the data. Hence, it seems useful to incorporate conventional parameter constancy tests and recursive least squares statistics into unit root analyses [cp. Maddala, G. S., 1992, pp. 587-588; cp. Perman, R., 1991, pp. 21-22].[28]

After all, the above discussion should have made clear that conventional unit root tests lack power and thus should be interpreted with great care. This follows not only from the problematic construction of some of the test statistics but also from the lack of precision involved when deriving the appropriate critical values on a case-by-case basis using Monte Carlo methods.

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[28] See Carruth (1987, pp. 16-17) for an example of the use of such recursive least squares statistics.

However, some of the more recently developed tests (especially as regards the issue of seasonal integration) such as the HEGY test discussed in the preceding section overcome some of the setbacks mentioned above at the price of more elaborate computations. Despite of this development, testing for unit roots at the various frequencies still represents a fragile and largely imprecise procedure.

IV. Cointegration analysis and the derivation of error-correction models

IV.a. The concept of cointegration and the Engle-Granger two step estimator

"At the least sophisticated level of economic theory lies the belief that certain pairs of economic variables should not diverge from each other by too great an extent, at least in the long-run" [Granger, C. W. J., 1986, p. 213].

However, given, for example, two series x_t and y_t integrated of order unity, any linear combination of these series will generally be I(1) so that the series diverge by ever increasing amounts [cp. Granger, C. W. J., 1981, p. 126; cp. Engle, R. F. et al., 1989, p. 46]. (First) differencing of such series will result in the loss of any potential long-run information (and, possibly, autocorrelated disturbances) [cp. Thomas, R. L., 1993, p. 152; cp. Wolters, J., 1990, p. 158]. As a consequence, no equilibrium solution between the variables can be derived. This represents also the reason for the suspicion of econometricians of the traditional Box-Jenkins approach of differencing the data in order to avoid the spurious regression problem. In time series analysis, the loss of long-run information due to differencing has readily been accepted as negligible side-effect to the modelling of dynamics. Econometricians, by contrast, emphasize the importance of the modelling of potential long-run relationships between the levels of the variables and thus the importance of any long-run information thereby ignoring the problem of potential spurious regressions [cp. Banerjee, A. et al., 1986, pp. 254-255].

Granger (1981) and Granger and Weiss (1983) provide a reconciliation of this dilemma by formalizing the observation that some systematic co-movements of economic variables have ocurred over time. According to their innovation, the series under consideration might be individually non-stationary but their co-movement might nonetheless be stationary over time giving rise to the concept of cointegration. Strictly speaking, the " ... components of the vector x_t are said to be co-integrated of order d, b, denoted x_t~CI(d,b), if (i) all components of x_t are I(d); (ii) there exists a vector $\alpha(\neq 0)$ so that $z_t = \alpha'*x_t$~I(d-b), b>0. The vector α is called the co-integrating vector" [Engle, R. F. and Granger, C. W. J., 1987, p. 253].

Clearly, d=b=1 represents the most interesting case in practice since it implies that a stochastically bounded equilibrium relationship between the variables exists. In contrast to that, if $z_t \sim I(1)$ holds, the relationship obtained by regressing z_t on x_t is entirely spurious.

Thus, the concept of cointegration derives its importance from the following facts [cp. Banerjee, A. et al., 1993, pp. 138-139; cp. Thomas, R. L., 1993, p. 164; cp. Perman, R., 1991, pp. 20-21]:

- In the case of d=b=1, testing for cointegration coincides with the idea of testing for the existence of an equilibrium relationship between the series.
- Regresssing non-stationary time series variables in their levels on each other yields meaningful results if and only if the series are cointegrated.

In addition, Granger has shown that for the case of d=b=1, the corresponding co-integrated system can be represented by a vector ARMA or a MA process and, most important in this context, by an error-correction model (ECM hereafter) [cp. Engle, R. F. and Granger, C. W. J., 1987, pp. 255-256]. "Not only must cointegrated variables obey such a model but the reverse is also true; data generated by an error-correction model (...) must be cointegrated" [Granger, C. W. J., 1986, p. 217].

The popularity of ECMs in applied work is largely due to the fact that they permit the incorporation of both the economic theory underlying the long-run relationship between the variables and the type of dynamics usually employed by time-series analysts [cp. Charemza, W. W. and Deadman, D. F., 1992, p. 155]. ECMs along with cointegration analysis therefore represent the statistical tool for the reconciliation of the time series and econometric modelling approaches [cp. Hendry, D. F., 1986, p. 204; cp. Banerjee, A. et al., 1986, p. 254; cp. Granger, C. W. J., 1981, pp. 127-129].

The following example based on Engle and Granger (1987, pp. 263-264) helps to clarify these issues:

$$x_{1t} + \beta * x_{2t} = u_{1t}, \text{ where } u_{1t} = u_{1t-1} + \varepsilon_{1t} \quad (4.1)$$

$$x_{1t} + \alpha * x_{2t} = u_{2t}, \text{ where } u_{2t} = p * u_{2t-1} + \varepsilon_{2t}, |p| < 1 \quad (4.2)$$

with $E(\varepsilon_{1t}) = E(\varepsilon_{2t}) = 0$ and $VAR(\varepsilon_{1t}) = \sigma_{11}$, $VAR(\varepsilon_{2t}) = \sigma_{22}$, $COV(\varepsilon_{1t};\varepsilon_{2t}) = \sigma_{12}$.

Since $\{u_{1t}\}$ represents a random walk and both x_{1t} and x_{2t} depend on $\{u_{1t}\}$, x_{1t} and x_{2t} are I(1). Despite of this, the linear combination $x_{1t} + \alpha^*x_{2t} = u_{2t}$ is clearly I(0) since $|p|<1$. Thus, x_{1t} and x_{2t} are CI(1,1) with cointegrating vector $(1; \alpha)'$ and the equilibrium relationship $x_{1t} + \alpha^*x_{2t}$.[29] The corresponding ECM, which can be extended by potential short-run dynamics, looks like this:

$$\Delta x_{1t} = \beta^*\delta^*z_{t-1} + \eta_{1t} \tag{4.3}$$

$$\Delta x_{2t} = (-\delta)^*z_{t-1} + \eta_{2t}, \text{ with } \delta = \frac{(1-p)}{(\alpha-\beta)} \tag{4.4}$$

where the η's are linear combinations of the ε's and $z_t = x_{1t} + \alpha^*x_{2t}$.[30]

The immediate question that arises, however, is how to test for cointegration in practice and how to estimate the parameters of the ECM (including the cointegrating vector). These issues are in fact closely related to each other.
Engle and Granger (1987, pp. 260-263) suggest a simple two-step estimator which provides also the basis for standard cointegration tests.
For the bivariate case with two series which are identically integrated of order d=1, the first step consists of estimating the cointegrating vector by a static ('cointegrating') regression in the levels of the variables using OLS.[31] This estimate is then used in the second step for the construction of an appropriate ECM. In this ECM, the estimate of the cointegrating vector determines the size of the disequilibrium error. Hence, the statistical properties of this estimate are required to be particularly favourable.

However, assuming the appropriate normalization to be known, the bivariate case is given by:

$$y_t = \alpha + \beta^* x_t + u_t, \tag{4.5}$$

[29] As shown, for example, by Banerjee et al. (1993, p. 138), the cointegrating vector and thus the equilibrium relationship are unique in the bivariate case.

[30] The derivation of ECMs from a static regression is discussed in the following section.

[31] The static regression equation is usually based on economic theory and simply represents the assumed equilibrium relationship between the variables.

where u_t usually will not be white noise because of omitted variables (omitted dynamics in particular). In spite of this, Engle and Granger (1987) justify the use of OLS, first of all, by pointing to the fact that the OLS estimation of the cointegrating vector " ... should provide a very good approximation to the true cointegrating vector because it is seeking vectors with minimal residual variance and asymptotically all linear combinations of x will have infinite variance except those which are co-integrating vectors" [Engle, R. F. and Granger, C. W. J., 1987, p. 261]. Secondly, they refer to the results presented by Stock (1987) who shows that the OLS estimate converges at a rapid rate of T rather than the conventional $\sqrt{T}$ to its probability limits. Consequently, although the OLS estimate might be biased in small samples, it is 'super-consistent' so that omitted dynamics do not matter asymptotically. Hence, any potential dynamics can be ignored until the ECM is estimated [cp. Wickens, M. R. and Breusch, T. S., 1988, p. 203]. This combination of super-consistency and two-step OLS estimation gives rise to the Engle-Granger theorem. According to this theorem, the two step estimator of a single equation ECM based on the estimated cointegrating vector will have the same limiting distribution as the maximum likelihood estimator based on the 'true' cointegrating vector [cp. Engle, R. F. and Granger, C. W. J., 1987, pp. 262-283; cp. Banerjee, A. et al., 1993, pp. 159-161].

However, in contrast to these favourable asymptotic properties, Monte Carlo evidence[32] indicates potentially substantial bias in the OLS estimates of a static cointegrating regression for sample sizes common in economics. In fact, this finite-sample bias declines at a rate faster than $\sqrt{T}$ but not as fast as T for sample sizes usually occuring in practice.[33] Moreover, dynamic regressions have been found to be more robust than static regressions under such circumstances and thus to result in good estimates in cases in which static regressions perform rather poor [cp. Banerjee, A. et al., 1986, pp. 263-265; cp. Banerjee, A. et al., 1993, pp. 214-230].
Clearly, the source of the finite-sample bias follows directly from the absence of the (asymptotic) property of super-consistency for common sample sizes [cp. Banerjee, A. et al., 1986, pp. 260-262].

32 See, for example, Banerjee et al. (1986, pp. 258-263).

Strictly speaking therefore, the omission of dynamic expressions is permissible if and only if the given sample size is relatively large so that super-consistency is likely to hold. This, however, is usually not the case in empirical economics. In other words, while " ... super-consistency theorems show that I(0) terms may be ignored asymptotically in regressions with I(1) variables, these asymptotic results have little bearing on sample sizes common in econometrics, where I(0) terms are important and need to be accomodated" [Banerjee, A. et al., 1993, p. 251; cp. Wickens, M. R. and Breusch, T. S., 1988, p. 203; cp. Wolters, J., 1990, pp. 164-165].

Similarly, the choice of the normalization is asymptotically of no further importance for the quality of the regression [cp. Engle, R. F. and Granger, C. W. J., 1987, pp. 261-262].[34] In practice, however, sample sizes are finite and the kind of normalization chosen affects the estimation results significantly.[35] If economic theory does not provide any indication of the appropriate normalization, it has to be chosen arbitrarily thereby producing potentially unreliable results. This problem can be particularly serious when dealing with multivariate series [cp. Maddala, G. S., 1992, p. 602; cp. Rüdel, T., 1989, p. 56; cp. Matthes, R. and Schulze, P. M., 1991, p. 418].[36]

Another problem related to the consideration of multivariate series is the possibility of multiple cointegrating vectors. Under these circumstances, it is no longer clear in small samples which cointegrating vector is actually estimated by OLS. In other words, the investigator faces some kind of 'identification' problem [cp. Granger, C. W. J., 1986, p. 225]. Moreover, relatively minor changes in the sample size can lead to relatively substantial changes in the coefficient estimates [cp. Rüdel, T., 1989, p. 56].

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33 Banerjee et al. (1986, pp. 261-262 and 276) show that the bias in the OLS estimate of the cointegrating vector in a static, <u>bivariate</u> regression can be expressed as a decreasing function of R^2 (this insight, however, does not hold generally for the multivariate case).

34 For two cointegrated variables, the R^2 number approaches unity asymptotically so that the obtained results do not depend on the normalization chosen (asymptotically) [cp. Rüdel, T., 1989, p. 56; cp. Perman, R., 1991, p. 23].

35 Except for the case of a high R^2 in a bivariate regression [cp. Hendry, D. F., 1986, pp. 206-207].

36 Hall (1986, p. 236) suggests to choose the normalization which has produced the highest R^2 so that the finite-sample bias can conveniently be assumed to be low. There is currently, however, no thorough approach to the problem of choosing the appropriate normalization.

These setbacks can be alleviated to some extent by using either dynamic specifications, modified static estimates or full-system estimation techniques such as the Johansen maximum likelihood procedure.[37]
In fact, provided weak exogeneity holds, estimates from a dynamic model tend to be at least as good as those from a static model no matter how close the underlying DGP is reproduced. Furthermore, the incorporation of a rich dynamic structure contributes to the detection of potential cointegrating vectors [cp. Banerjee, A. et al., 1993, pp. 240-242].

After all, given a single cointegrating vector and a relatively large sample size, the Engle-Granger two step estimator can be shown to possess some desirable properties provided a clear distinction between dependent and explanatory variables can be drawn. Under conditions of small sample sizes, however, the static regression might best be replaced by a dynamic specification in order to ensure the derivation of both a reliable test decision and an adequate ECM.

IV.b. Testing for cointegration

Test decisions on the presence of cointegration are usually based on the application of unit root tests to the OLS residuals from a static or dynamic cointegrating regression. If the series under investigation are cointegrated, the residuals resulting from a cointegrating regression must be stationary. Otherwise, the series are moving apart over time in a stochastically unbounded manner and the equation considered does not represent a valid equilibrium relationship.
However, although this procedure might appear very similar to standard unit root tests at first glance, some modifications have to be taken into account. This is because of the fact that the residuals usually have to be estimated first before any test decision can be derived whereas standard unit root tests are based on the original

[37] Gonzalo (1994, pp. 204 and 213-223) comparing the most widely used methods of estimating long-run relationships draws the conclusion that full-system maximum likelihood estimation in an ECM is the best alternative. This, however, implies the assumption that the appropriate functional form is known which will not always be the case in practice thereby leading to less robust 'long-run solutions' [cp. Carruth, A. A., 1987, p. 10].

series. As a consequence, the usual critical values have to be adjusted to permit a valid test decision on the null of no cointegration [cp. Banerjee, A. et al., 1993, p. 206].

On the basis of this insight, Engle and Granger (1987, pp. 264-270) investigating seven different test statistics, recommend a Durbin-Watson type statistic for the case of first-order autoregression in the residuals rather than any conventional unit root test. This test statistic has originally been proposed by Sargan and Bhargava (1983) and is calculated identically to the standard Durbin-Watson test, i. e.:

$$\mathrm{CRDW} = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} \tag{4.6}$$

where e_t denotes the OLS residuals obtained from the cointegrating regression. In this context, the Durbin-Watson test is no longer used as a test for first-order autocorrelation as in standard regression analysis but rather as a test for the null of a random walk/no cointegration against the alternative of a stationary first-order autoregressive process [cp. Sargan, J. D. and Bhargava, A., 1983, p. 153]. This null hypothesis can conveniently be rejected if the value of the CRDW statistic is relatively large. In other words, the closer CRDW is to zero, the more likely are the series not to be cointegrated [cp. Engle, R. F. and Granger, C. W. J., 1987, pp. 266-267].[38]

However, although the CRDW test is intuitively appealing and easy to use, it suffers from a variety of drawbacks and should therefore be regarded as a very rough guide to cointegration only. First of all, as in the case of the standard Durbin-Watson test, the CRDW test depends on the number of explanatory variables in the cointegration equation (and depends weakly on the sample size). As a result, only upper and lower bounds on the critical values are available for test decisions [cp.

38 Charemza and Deadman (1992, p. 153) mention a simple rule of thumb according to which the null should not be rejected if CRDW < R^2 and vice versa.

Sargan, J. D. and Bhargava, A., 1983, pp. 158-159].[39] In addition, these " ... bounds diverge as the number of explanatory variables is increased, and eventually cease to have any practical value for the purposes of inference" [Banerjee, A. et al., 1993, p. 207]. Finally, although CRDW is of relatively high power in the case of AR(1) processes among the residuals[40], it is of extremely low power if the more common case of higher-order autoregressions is considered. This is simply due to a lack of robustness of the critical values to changes in the DGP [cp. Rüdel, T., 1989, p. 71; cp. Thomas, R. L., 1993, p. 166; cp. Molinas, C., 1986, p. 281]. In practice, " ... this implies a proliferation of tables of different critical values for different data-generation processes and simulation exercises" [Banerjee, A. et al., 1993, p. 207].

A similar line of criticism holds for the simple DF test applied to the estimated residuals. The simple DF test for cointegration proceeds from the following auxiliary regression:

$$\Delta \hat{u}_t = \varphi^* * \hat{u}_{t-1} + \varepsilon_t, \text{ where } \varphi^* = \varphi - 1 \qquad (4.7)$$

with the null of a random walk (i. e., H_0: $\varphi^* = 0$ against H_1: $\varphi^* < 0$).
Engle and Granger (1987, pp. 269-270) provide adjusted critical values since the distribution of the t-ratio depends on the number of coefficients estimated and no longer follows the conventional Dickey-Fuller distribution [cp. Banerjee, A. et al., 1993, p. 206; cp. Rüdel, T., 1989, pp. 74-75; cp. Thomas, R. L., 1993, pp. 165-166]. However, due to the restrictive assumption of a first-order process, the simple DF test for cointegration implies setbacks similar to the case of the CRDW statistic.

Not surprisingly, therefore, Engle and Granger (1987, p. 269) eventually recommend the ADF test for cointegration as the most powerful testing method by pointing to the robustness of its critical values to changes in the DGP:[41]

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39 The bounds tend to be relatively wide apart so that the CRDW might often lead to inconclusive test decisions in practice. Sargan and Bhargava (1983, pp. 160-163), however, show that the use of the Imhof routine helps to obtain the exact significance limits of the CRDW statistic.

40 See Sargan and Bhargava (1983, pp. 163-166) for some simulation evidence.

41 Molinas (1986, pp. 280-282) qualifies this result by pointing to the poor performance of both the CRDW and the simple DF as well as the augmented DF test when the variables considered are IMA(1,1) as it seems to be frequently the case in economic time series.

$$\Delta \hat{u}_t = \varphi^* * \hat{u}_{t-1} + \sum_{i=1}^{k} \varphi_i * \Delta \hat{u}_{t-i} + \varepsilon_t \qquad (4.8)$$

It should be noted that in opposite to the testing for orders of integration, " ... it seems safest to over-specify the ADF regression, and use as many lagged terms as degrees-of-freedom restrictions will allow" [Banerjee, A. et al., 1993, pp. 207-208]. This is because of the fact that the ADF test will be reasonably powerful if and only if the number of lagged $\Delta\hat{~}u_t$ terms appearing in the (unknown) DGP coincides with that corresponding to the test regression. Consequently, various specifications should be used and the corresponding test decisions should be checked for consistency in order to reduce uncertainty.

However, a similarity to integration testing follows from the fact that unit root tests (and the CRDW test) for orders of cointegration generally lack power.[42] This is again particularly true when dealing with borderline-stationary processes [cp. Engle, R. F. and Granger, C. W. J., 1987, pp. 268-269; cp. Banerjee, A. et al., 1986, p. 262; cp. Sargan, J. D. and Bhargava, A., 1983, p. 165; cp. Hendry, D. F., 1986, p. 206].

Furthermore, when dealing with (seasonal) series which turn out to have seasonal unit roots in common, cointegration analysis becomes even more problematic. Under such circumstances, as already indicated above, the standard procedure for testing for cointegration is no longer applicable due to the potential loss of super-consistency. In fact, as Engle et al. (1989, pp. 50-51) show, at the presence of zero and seasonal cointegration (with different cointegrating vectors), it is no longer clear which cointegrating vector is actually chosen by static regression.
However, if one is interested in zero frequency cointegration in seasonal series only, the standard cointegration tests (proceeding from a generalized Engle-Granger two step procedure) can still be employed provided the series are seasonally adjusted first

[42] This similarity includes the arbitrary choice of both the null hypothesis and the subsequent testing sequences, non-standard distributions, bivariate versus multivariate regressions, etc. [cp. Maddala, G. S., 1992, pp. 598-599 and 601]. It involves also the general problem of the applicability and the evaluation of pre-tests in econometric analyses [cp. Rüdel, T., 1989, pp. 111-114].

[cp. Hylleberg, S. et al., 1990, pp. 229 and 232-233].[43] Seasonal integration tests such as the HEGY test in fact indicate what kind of filter should be used.

"For zero-frequency cointegration, this procedure is probably appropriate, although the implications of the pretesting for seasonal roots has not yet been investigated" [Hylleberg, S. et al., 1990, p. 234].

Similar to this adjustment approach but less powerful, Engle et al. (1989) recommend the use of annual instead of seasonal observations in the cointegrating regression

" ... as an approach to estimating the long-run model without the need to model the seasonality" [Engle, R. F. et al., 1989, p. 52] so that standard cointegration tests could still be applied.

Furthermore, Charemza and Deadman (1992, p. 154) suggest to use seasonal dummy variables in the cointegrating regression to approximate stochastic seasonality. Standard cointegration tests can then again be used to test for cointegration at the zero frequency.

For the case of seasonal cointegration, however, a correct test procedure remains to be developed.[44] In fact, seasonal cointegration can be tested for if and only if the potential cointegrating vectors at zero and seasonal frequencies are identical and are known a priori from some economic theory. Given this information, the residuals follow directly and the conventional seasonal unit root tests can be applied in order to test for cointegration at the various frequencies [cp. Hylleberg, S. et al., 1990, pp. 234-236].[45]

Such a situation is implicitly assumed in the case of the bivariate DHSY equation relating logged consumption and income to each other on the basis of a simplified permanent income theory. The underlying theory in fact implies that there is only one, identical cointegrating vector at the zero and seasonal frequencies which, in addition, is equal to unity. Hence, the cointegrating vector has not to be estimated and, for example, the HEGY test might be applied to the residuals as a test for seasonal cointegration [see, for example, Hylleberg, S. et al., 1990, pp. 236-237].

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[43] See DHSY (1978, p. 671) for potential detrimental effects of separate seasonal adjustments of the series.

[44] See Ilmakunnas (1990, pp. 85-87) for an attempt with limited success.

Generally speaking, however, assuming favourable conditions for the validity of the (generalized) Engle-Granger two step estimator to be given, the use of an adequately augmented DF regression will yield a relatively reliable test decision on the presence of cointegration. Under such favourable conditions, the potential rejection of the null makes it possible to construct an adequate ECM using the OLS estimates. This construction is explained to some extent in the following section due to the importance of ECMs in recent empirical studies on UK consumption.

IV.c. The derivation of error-correction models

The development of 'modern' error-correction analysis was initiated by Davidson et al. (1978) who were aiming at the construction of a model which " ... (i) was consistent with the UK data, and (ii) exhibited parameter stability over time, (iii) could account for previous models and findings and (iv) conformed to steady-state postulates of economic theory" [Hadjimatheou, G., 1987, p. 162; cp. Hendry, D. F. et al., 1990, pp. 298-300; cp. Phillips, P. C. B. and Loretan, M., 1991, p. 413]. In other words, Davidson et al. introduced a completely new kind of econometric methodology mainly associated with the British econometrician David F. Hendry. The essential innovation they provided is the obligation to model estimation equations which encompass their forerunners not only in empirical power but also in theoretical reasoning. According to this, they claimed that a general-to-specific approach, as opposed to the usual specific-to-general approach, would be the more appropriate econometric research technique. In fact, the traditional method is commonly accepted as the main reason for the rather high degree of confusion caused by numerous empirical results of the past all of which showed reasonable empirical power but essentially excluded each other.
"As a result of this approach more emphasis is being given to short-time dynamics, or adjustment processes, and the long-run properties of the consumption function" [Hadjimatheou, G., 1987, p. 166].

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[45] Under these circumstances, the single cointegrating vector in fact reduces the integration of the series at all frequencies [cp. Hylleberg, S. et al., 1990, p. 230].

Econometric modelling of time-series variables before the development of cointegration analysis and ECMs suffered substantially from the lack of short-run information provided by economic theory. In fact, although economic theory provides ideas about long-run or steady-state relationships it is usually not very helpful in explaining dynamic adjustments [cp. Hadjimatheou, G., 1987, p. 162; cp. Perman, R., 1991, p. 20].

However, as the above discussion has shown, the simple omission of any dynamics in a long-run regression in the levels of the variables will result in autocorrelated residuals and spurious results (except for the case of cointegrated series which is not tested for within the 'traditional' econometric approach towards time series modelling). As a consequence, first differenced autoregressive-distributed lag (also ADL hereafter) models have frequently been used in practice to overcome these setbacks. This 'solution', however, implies the neglect of any long-run relationship between the variables and focuses exclusively on short-run parameters. It is therefore likely to be sub-optimal: If economic theory indicates a relevant long-run relationship between the variables considered any disequilibrium situation will have an implicit effect on the short-run behaviour of the dependent variable. The neglect of this effect simply implies misspecifications [cp. Matthes, R. and Schulze, P. M., 1991, pp. 415-416; cp. Hendry, D. F. and Doornik, J. A., 1992, p. 8; cp. Davidson, J. E. H. et al., 1978, p. 680].

ECMs permit the reconciliation of this dilemma by making the simultaneous consideration of dynamic and long-run aspects within the Hendry methodology of econometric modelling possible.

Such reconciling ECMs can in fact be derived from ADL models using simple reparameterisations. The validity of the resulting specifications, however, depends nonetheless on the applicability of the Granger Representation Theorem [cp. Matthes, R. and Schulze, P. M., 1991, pp. 415-416; cp. Erbsland, M. and Ulrich, V., 1992, p. 536]. Consequently, before investigating the derivation of ECMs by simple reparameterisations, the connection between cointegration and error-correction is explored more fully.

Assuming a pth order vector autoregressive process with n components to represent the 'short-run model' [cp. Rüdel, T., 1989, pp. 15-17; cp. Erbsland, M. and Ulrich, V., 1992, p. 538]:

$$x_t = A_1 * x_{t-1} + A_2 * x_{t-2} + ... + A_p * x_{t-p} + \varepsilon_t \qquad (4.9)$$

or rather $A(L)x_t = \varepsilon_t$, where x_t is a $(n\times 1)$ vector and the A_i are $(n\times n)$ matrices with $A(1)$ being of rank r,

Rüdel (1989) shows that due to $x_t \sim I(1)$, $A(1)$ has reduced rank so that the following equation holds:

$$\Delta x_t = -A(1)x_{t-1} + G_1 * \Delta x_{t-1} + G_2 * \Delta x_{t-2} + ... + G_{p-1} * \Delta x_{t-p-1} + \varepsilon_t \qquad (4.10)$$

where $G_i = -\sum_{j=i+1}^{p} A_j$, for i = 1, 2, ..., p-1.

Provided cointegration testing has confirmed the validity of the conjectured equilibrium relationship between the n components, Granger's Representation Theorem implies:

$$A_1 = \Gamma\Lambda' \qquad (4.11)$$

and thus there exists an error-correction representation of the form:

$$z_t \equiv \Lambda' * x_t,$$

with Λ consisting of the cointegrating vector(s) and Γ denoting the ECM parameters. These parameters can then be estimated, for example, by applying the second step of the (generalized) Engle-Granger two step estimator along with a Hendry type of testing down procedure to determine the adequate number of lags [cp. Engle, R. F. and Granger, C. W. J., 1987, pp. 255-256]. Hence, the above equation can be rewritten as [cp. Matthes, R. and Schulze, P. M., 1991, p. 417]:

$$\Delta x_t = -A(1) * x_{t-1} + G_1 * \Delta x_{t-1} + G_2 * \Delta x_{t-2} + ... + G_{p-1} * \Delta x_{t-p-1} + \varepsilon_t$$

$$\Leftrightarrow \quad \Delta x_t = -\Gamma * z_{t-1} + G_1 * \Delta x_{t-1} + G_2 * \Delta x_{t-2} + ... + G_{p-1} * \Delta x_{t-p-1} + \varepsilon_t \qquad (4.12)$$

which represents the corresponding error-correction model.

However, although the analysis has so far been restricted to the application of the Granger Representation Theorem to VAR models, ECMs can also be derived from the more general case of autoregressive-distributed lag models.

$$b_0(L)y_t = \sum_{i=1}^{k} b_i(L)x_{it} + \varepsilon_t \qquad (4.13)$$

The corresponding ECMs can in fact be shown to result from simple reparameterisations and thus can generally be obtained without the application of the theorem.

Restricting the analysis to first order polynomials and a single explanatory variable, Hendry et al. (1984, pp. 1041-1049) show that the majority of the empirically used models are embedded in this restricted version (including the error-correction specification):

$$y_t = \alpha_1 * y_{t-1} + \beta_0 * x_t + \beta_1 * x_{t-1} + \varepsilon_t, \qquad (4.14)$$

with $K = \frac{\beta_0+\beta_1}{1-\alpha_1}$ being the long-run response.

Using first differences in y_t and adding and subtracting $\beta_0 * x_{t-1}$ and $(\beta_0+\beta_1) * x_{t-1}$ respectively on the right-hand side, results in a balanced ECM of the form [cp. Hendry, D. F. and Doornik, J. A., 1992, p. 10; cp. Banerjee, A. et al., 1993, pp. 48-49]:

$$\Delta y_t = \beta_0 * \Delta x_t + (\alpha_1 - 1) * (y_{t-1} - K * x_{t-1}) + \varepsilon_t \qquad (4.15)$$

The DHSY ECM can in fact be obtained in a similar way [see, for example, Banerjee, A. et al., 1993, pp. 52-53; see also section V.a.].

This gives rise to the question what the distinguishing feature of an ECM is since it seems to be no more than a reparameterisation of an ADL model without any further restriction to be imposed [cp. Hylleberg, S. and Mizon, G. E., 1989, p. 120; cp. Hendry, D. F. and Doornik, J. A., 1992, p. 10]. "The answer is that in the ECM formulation, parameters describing the extent of short-run adjustment to

disequilibrium are immediately provided by the regression" [Banerjee, A. et al., 1993, p. 5].
In accordance to this, the error-correction formulation has the advantage of providing a distinct differentiation between long-run and short-run responses, each of which has an associated economic interpretation [cp. Hylleberg, S. and Mizon, G. E., 1989, p. 120; cp. Erbsland, M. and Ulrich, V., 1992, p. 536; cp. Thomas, R. L., 1993, p. 155; cp. Hendry, D. F. et al., 1990, pp. 303 and 305]. Since the estimation of ECMs represents a balanced regression in stationary variables, these adjustments can in fact be determined by OLS. In line with this, the interpretation of standard goodness-of-fit measures such as R^2 no longer suffers from spurious regression problems.

However, it has to be stressed that the regression of a reparameterised ADL-ECM model is meaningful if and only if the variables are CI(1,1) [cp. Hall, S. G., 1986, p. 230; cp. Erbsland, M. and Ulrich, V., 1992, p. 533]. Thus, although ECMs can be derived directly from ADL models, these reparameterisations will lead to useful insights only under the condition of the applicability of Granger's Representation Theorem (since the incorporated long-run relationship simply requires the variables to be CI(1,1) to be well-defined [cp. Hendry, D. F. and Doornik, J. A., 1992, p. 5]).[46] Only the connection between cointegration and error-correction therefore justifies the consideration of ECMs as a separate class of models. In line with this, the derivation of an ECM by reparameterisation does not automatically imply the reverse applicability of the Granger Representation Theorem except for the case that the ECM is incidentally stationary.
One less obvious example for this, is the original derivation of ECMs solely for variables for which economic theory suggests long-run proportionality or long-run homogeneity in the log-linear case respectively (i. e., K=1 is imposed on the equation).[47] This approach resulted from the fact that error-correction at that time was considered not to be " ... a particularly convenient form for estimation (of the

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46 In fact, if this requirement is not met, general-to-specific modelling will almost certainly lead to the elimination of the constructed long-run relationship. Thus, the error-correction mechanism is very likely not to be included in the finally preferred specifcation anyway [cp. Thomas, R. L., 1993, pp. 291-293].

47 Thus, K has not necessarily to be estimated. It can in fact even be made allowance for completely unknown long-run parameters in ECMs by simply adding lagged explanatory variables. The determination of the appropriate number of lags and the potential non-stationarity of the error-correction term, however, are likely to cause problems [cp. Banerjee, A. et al., 1993, pp. 51-52].

long-run parameter θ) - hence the attraction of setting θ equal to unity" [Wickens, M. R. and Breusch, T. S., 1988, p. 193] along with the assumption of stationarity. Provided economic theory gives support to such a proportionality hypothesis, there is no need to estimate the long-run parameter and, most important, the error-correction mechanism might well be stationary.[48]

In other words, the adoption of long-run proportionality involves the implicit assumption of the applicability of the Granger Representation Theorem. The DHSY study based on a simplified permanent income theory represents an important example for this.

The early derivation of long-run proportionality ECMs seems indeed simply to have been caused by the fact that the ECM idea historically preceded the development of cointegration analysis [cp. Rüdel, T., 1989, p. 18; cp. Phillips, P. C. B. and Loretan, M., 1991, p. 408].

Conversely, however, (so-called generalized) ECMs " ... are well-defined for $K \neq 1$, although usually K will then need to be estimated" [Hendry, D. F. and Doornik, J. A., 1992, p. 11]. The development of integration and cointegration testing along with Granger's Representation Theorem has permitted this extension of the ECM concept to applications beyond long-run proportionality theories [cp. Banerjee, A. et al., 1993, pp. 60-61; cp. Hendry, D. F. and Richard, J.-F., 1983, pp. 130-131]. This progress in empirical economics away from the need to circumvent the estimation of long-run parameters is embodied, for example, in the Engle-Granger two step estimator and the Engle-Granger theorem.

Engle and Granger essentially exploit the strong prior information on the long-run parameter in form of a super-consistent estimate in order to make OLS estimation of the ECM possible. In other words, strong prior information such as long-run proportionality is still useful but generally unnecessary to obtain powerful ECMs [cp. Hylleberg, S. and Mizon, G. E., 1989, p. 120]. Apart from the characteristics already mentioned above, this power origins from the fact that the variables in an ECM tend to be less highly correlated than, for example, in the corresponding ADL version. Thus a sequential testing down procedure should provide a reliable guide to the model specification which best fits the data.

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48 In fact, all long-run proportionality theories can be reproduced in static equilibrium by an appropriate ECM [cp. Hendry, D. F. et al., 1984, p. 1048; cp. Hendry, D. F. and Doornik, J. A., 1992, p. 11].

On the other hand, error-correction modelling does not represent the ultimate tool for perfect model construction (as it was initially celebrated as) either. First of all, economic theory does usually not provide any indication of what lag length to start with (or which cointegrating vector to choose in the multivariate case). As a consequence, frequently in practice, additional explanatory variables are included into an initial ECM in an ad hoc fashion [cp. Wickens, M. R. and Breusch, T. S., 1988, p. 194; cp. Hadjimatheou, G., 1987, p. 168; cp. Thomas, R. L., 1993, p. 155]. Subsequent testing down might then turn out to yield rather poor specifications.

Moreover, the testing sequence suffers also from the fact that any finite-sample bias in the static regression is carried over to the second step within the Engle-Granger two step estimation.

Another problem is related to the use of this estimation technique alongside of the Hendry methodology of econometric modelling. Due to the potentially substantial number of non-nested models, testing down will often imply considerable judgement on non-statistical grounds. In addition, such reduction sequences may lead to the elimination of lagged error-correction terms thereby leaving a questionmark behind the initially desired separation of short-run and long-run effects [cp. Thomas, R. L., 1993, p. 170; cp. Hendry, D. F. and Doornik, J. A., 1992, p. 11].

However, ECMs have also been criticized on more fundamental grounds. As Thomas (1993, p. 170) points out, " ... instead of modelling the time series relationships in the data and then interpreting the results in terms of economics, a better approach is to derive relationships directly from economic theory, next impose the error correction mechanism as a possible short-run adjustment hypothesis, and only then specify an appropriate dynamic equation to be estimated." In line with this, it seems more realistic, for example, to try to model significant reactions to large disequilibria rather than small ones as implicitly assumed when estimating linear ECMs only.[49] It should in fact be stressed that non-linear ECMs are not excluded a priori from the Granger Representation Theorem [cp. Granger, C. W. J., 1986, pp. 225-226; cp. Thomas, R. L., 1993, p. 171].

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[49] See, for instance, Peel (1992, p. 177) for the derivation of a non-linear ECM of real consumption.

However, any further analysis of the drawbacks of ECMs is beyond the scope of this dissertation since the empirical sections focus mainly on the reconsideration of the DHSY specification in the light of the relatively recent econometric findings presented above rather than its reparameterization. Nevertheless, it should be made clear that despite of its implicit setbacks, the development of ECMs has provided the statistical instrument for the systematic investigation of the short- and long-run behaviour of economic variables.[50] If this instrument is used carefully alongside of cointegration analysis and general-to-specific modelling, it produces specifications which outperform comparable models derived from more 'traditional' econometric methodology.

50 In fact, the error-correction modelling of consumption in particular has been extensively employed and repeatedly tested in the UK, with the results giving support to the main structure [cp. Hadjimatheou, G., 1987, p. 165; cp. Davis, E. P., 1984, p. 823].

V. Reconsidering the DHSY study

V.a. The DHSY model

The rather poor state of econometric research at the time of writing provided the main incentive for DHSY to focus on the empirical and methodological reasons which might have contributed to the wide variety of 'well-determined' consumption models. DHSY illustrate this variety by examining three influential models put forward by Hendry (1974) (H), Ball et al. (1975) (B) and Wall et al. (1975) (W). Their analysis essentially proceeds from discussing seven potential explanations for the main differences between these three studies [cp. Davidson, J. E. H. et al., 1978, p. 663]:

1) Data series
2) Methods of seasonal adjustment
3) Other data transformations
4) Functional forms
5) Lag structures
6) Diagnostic tests
7) Estimation methods

However, having used both identical seasonally unadjusted data and identical data transformations as well as identical functional forms, DHSY still obtain rather contrary results. Consequently, DHSY turn to the potential effects of different lag structures, different diagnostic tests and different estimation methods. In order to consider the effects of these factors, they develop a general model in which H, B and W can be nested as special cases (based on the common seasonally unadjusted data series, data transformations and functional forms). Testing the statistical adequacy of these different special cases, DHSY eventually conclude that the Wall equation is preferable on statistical grounds:

$$\Delta c_t = \alpha + ß_1 * \Delta y_t + ß_2 * \Delta y_{t-1} \quad (5.1)$$

where lowercase letters symbolize log-form[51] and c_t denotes total consumers' expenditure.

Obviously, this statistically preferred equation implies some rather strange economic properties. First of all, although the first difference transformation might reduce or eliminate any non-stationarity of the regressors[52], the estimation equation has no static equilibrium solution.

Furthermore, it " ... implies that any adjustment to income changes is completed within six months and is independent of any disequilibrium between the levels of C_t and Y_t" [Davidson, J. E. H., 1978, p. 672]. Clearly, the statistically preferred equation accounts for short-run behaviour only and represents a 'classical' example for the neglect of the necessity to model both long-run relationships and short-run dynamics.

DHSY suggest to overcome these problems by developing an ECM of consumption and income. They proceed from the assumption of a non-stochastic steady-state theory of the form $C_t = K * Y_t$ (where K is constant) advocated by relevant economic theory. This relationship can obviously be rewritten in double log-form as follows:

$$c_t = k + y_t \tag{5.2}$$

Since economic theory has usually little to say in terms of short-run adjustments, DHSY consider it reasonable to commence with a general distributed lag model as a first general specification of the short-run adjustments, i. e.:

$$\alpha(L)c_t = k^* + \beta(L)y_t + v_t, \tag{5.3}$$

where $\alpha(L)$, $\beta(L)$ are lag polynomials of high enough order that v_t is white noise [cp. Davidson, J. E. H. et al., 1978, p. 680]. Similar to the lines on the derivation of 'early' ECMs in section IV.c., DHSY restrict their analysis to the case of first-order polynomials:

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51 This convention also holds for the remaining sections of this study.

52 Any test for the order of integration and the potential presence of deterministic or stochastic trends is of course missing.

$$c_t = k^* + \beta_1 {}^* y_t + \beta_2 {}^* y_{t-1} + \alpha_1 {}^* c_{t-1} + v_t \quad (5.4)$$

where (5.2) is included as a special case. However, in order to " ... ensure that for *all* values of the estimated parameters, the *steady-state solution of* (5.4) *reproduces* (5.2) one need ... " [Davidson, J. E. H. et al., 1978, pp. 680-681] to impose the coefficient restriction $\beta_1+\beta_2+\alpha_1 = 1$ or, equivalently, $\beta_1 = -\beta_2+\gamma$ and $\alpha_1 = 1-\gamma$ (i. e., long-run homogeneity of the parameters) resulting in the equation:

$$\Delta c_t = k^* + \beta_1 {}^* \Delta y_t + \gamma {}^* (y_{t-1} - c_{t-1}) + v_t \quad (5.5)$$

with $\gamma^*(y_{t-1} - c_{t-1})$ representing the error-correction mechanism. This error-correction mechanism can be interpreted as a kind of bi-directional 'ratchet' effect on the short-run consumption behaviour. This short-run behaviour depends exclusively on changes in income (as given in the Wall model) if and only if the previous period values of c_t and y_t were in equilibrium [cp. Davidson, J. E. H. et al., 1978, p. 683].

However, although this specification employs all long-run information available, it should be noted that the way it is derived is invalid in the light of the recent findings of cointegration analysis and error-correction modelling. Clearly, due to the lack of appropriate testing methods at the time of writing, any thorough testing for orders of integration is missing.[53] Similarly, the conjectured long-run relationship is not tested for stationary residuals leaving a questionmark behind the stationarity of the error-correction term. Nevertheless, the simulation study conducted by DHSY indicates 'estimation advantages' of (5.5) despite of its implicit systematic weaknesses [see Davidson, J. E. H. et al., 1978, p. 682]. The following empirical sections focus on the question whether this 'estimation advantage' is actually caused by the cointegrated consumption and income variables with cointegrating vector (1;-1)' and the balanced stationarity of the differenced variables in (5.5) or not.

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[53] However, DHSY are well-aware of the problem of (seasonal) non-stationarity but are obviously forced to restrict their considerations to the examination of correlograms (see, for example, Davidson, J. E. H. et al., 1978, pp. 666 and 668).

However, turning to a first empirical version of (5.5) estimated for the period 1958(1) to 1970(3), DHSY obtain:[54]

$$\Delta_4\hat{c}_t = 0.49\Delta_4 y_t - 0.17\Delta\Delta_4 y_t - 0.06(c-y)_{t-4} + 0.01\Delta_4 D_t^0 \qquad (5.6)$$

where D_t^0 takes the value 1 in 1968(2) to allow for the effect of advance warnings in 1968(1) of purchase tax increases in 1968(2). Highlighting the implicit relationship to the work by Phillips (1954, 1957), such a specification can in fact " ... be derived from a simple 'feedback theory' in which consumers plan to spend in each quarter of a year the same as they spent in that quarter of the previous year ($\ln C_t = \ln C_{t-4}$) modified by a proportion of their annual change in income ($+0.49\ \Delta_4\ln Y_t$), and by whether that change is itself increasing or decreasing ($-0.17\ \Delta_1\Delta_4\ln Y_t$) (..); these together determine a 'short-run' consumption decision which is altered by $-0.06\ \ln(C_{t-4}/Y_{t-4})$, the feedback from the previous C/Y ratio ensuring coherence with the long-run 'target' outcome $C_t = K^*Y_t$" [Davidson, J. E. H. et al., 1978, p. 684].[55]

Although this specification is found to reflect all the salient features of the data, it shows a rather poor forecast performance for the period 1970(4) to 1975(4). As a consequence, DHSY make allowance for a Deaton type of disequilibrium effect of inflation on consumption expenditure.[56] They justify this 'ad hoc'-adoption by pointing to the high rates of inflation in the forecast period. Including $\Delta_4 p_t$ and $\Delta\Delta_4 p_t$, DHSY eventually obtain (standard errors in parentheses) (5.7):

$$\Delta_4\hat{c}_t = \underset{(0.04)}{0.47\Delta_4 y_t} - \underset{(0.05)}{0.21\Delta\Delta_4 y_t} - \underset{(0.02)}{0.10(c-y)_{t-4}} + \underset{(0.003)}{0.01\Delta_4 D_t^0} - \underset{(0.07)}{0.13\Delta_4 p_t} - \underset{(0.15)}{0.28\Delta\Delta_4 p_t}$$

They interpret (5.7) as a confirmation of the significance of Deaton's 'mass illusion' effect [cp. Davidson, J. E. H. et al., 1978, p. 686]. With these alterations included,

54 See DHSY (1978, pp. 689-690) for a justification of the neglect of the intercept.

55 It has to be pointed out, however, that DHSY justify the inclusion of fourth-differenced terms on the grounds of potential explanatory power rather than any potential 'stationarity' (although this might have been accepted as a favourable side-effect).

56 Deaton's model is based on the idea that in times of accelerating inflation consumers mistake " ... unanticipated changes in inflation for relative price changes when sequentially purchasing commodities" [Davidson, J. E. H. et al., 1978, p. 686]. See also Thomas (1992, pp. 266-267) and Hadjimatheou (1987, pp.125-127 and 138-140) for more details on the Deaton model. Although initially there have been numerous studies which have given empirical support to Deaton's 'mass illusion' effect, it is no longer considered as being an

DHSY consider their forecast and stability problems as being solved even though any thorough justification for the adoption of the Deaton approach is missing. Since no signs of heteroscedasticity, autocorrelation or the importance of unemployment, interest rates and/or relative price effects are found, DHSY conclude that (5.7) " ... conforms with a range of theoretical requirements and matches all of the salient features of the data ... " [Davidson, J. E. H. et al., 1978, p. 690].

However, confronting (5.7) with an extended sample period from 1956(2) to 1989(3) and using the obtained estimates for conditional forecasts over the period 1989(4) to 1993(2), reveals certain setbacks:

Modelling $\Delta_4 c_t$ by OLS:[57]

Variable	Coefficient	Std.Error	t-value	PartR²
$\Delta_4 y_t$	0.68212	0.044838	15.213	0.6439
$\Delta\Delta_4 y_t$	-0.17723	0.070157	-2.526	0.0475
$(c-y)_{t-4}$	-0.17557	0.029140	-6.025	0.2210
$\Delta_4 p_t$	-0.12602	0.030326	-4.155	0.1189
$\Delta\Delta_4 p_t$	-0.099936	0.11517	-0.868	0.0058
DV	-0.00047912	0.0090436	-0.053	0.0000

R^2 = 0.83029 ^σ = 0.0156127 DW = 1.18
* R^2 does NOT allow for the mean *
RSS = 0.03120089348 for 6 variables and 134 observations

Analysis of 1-step forecasts:

Date	Actual	Forecast	Y - Yhat	ForecastSE	t-value
89 4	0.0164597	0.0230579	-0.00659817	0.0158737	-0.415668
90 1	0.0140887	0.0320435	-0.0179548	0.0156795	-1.14511
90 2	0.0106761	0.0171065	-0.00643036	0.0157765	-0.407592
90 3	0.00290944	0.00834085	-0.00543141	0.0157370	-0.345136
90 4	-0.00231742	0.00786827	-0.0101857	0.0157620	-0.646218
91 1	-0.0139307	0.0104626	-0.0243933	0.0158565	-1.53838
91 2	-0.0310563	0.0134611	-0.0445174	0.0158790	-2.80354
91 3	-0.0267376	0.00295024	-0.0296879	0.0157688	-1.88270
91 4	-0.0160131	0.00281977	-0.0188328	0.0156816	-1.20095
92 1	-0.0197406	0.0203751	-0.0401157	0.0158753	-2.52693
92 2	0.00153605	0.0345823	-0.0330463	0.0159102	-2.07706

appropriate mean for the incorporation of inflation effects into consumption models (see, for example, Davis, E. P., 1984, pp. 805-809 and 823).

57 Instrumental variables estimation to allow for potential simultaneity bias yields very similar results. As Davidson and Hendry (1981, p. 189) conjecture, this similarity is probably due to the fact that measurement errors in y_t lead to downward biases which compensate the upward biases caused by simultaneity.

92 3	0.00505296	0.0358959	-0.0308430	0.0156966	-1.96495
92 4	0.0111166	0.0337509	-0.0226343	0.0157062	-1.44110
93 1	0.0207734	0.0376434	-0.0168700	0.0160963	-1.04806
93 2	0.0146844	0.0228439	-0.00815947	0.0162618	-0.501758

Tests of parameter constancy over 89 (4) to 93 (2):
Forecast Chi²(15)/15 =2.4258
Chow F(15,128) = 2.1597 [0.0109] *

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 28.513 and F-Form(4, 124) = 8.3794 [0.0000] **

Normality test:
Mean 0.026751
Std.Devn. 0.980655
Skewness -0.263539
Excess Kurtosis 0.494922
Minimum -3.107612
Maximum 2.479642
Normality Chi²(2) = 2.788

(Simplified) White test for heteroscedastic errors:
CHI²(12) = 10.859 and F-Form(12, 115) = 0.84505 [0.6042]

(Original) White test of functional form:
CHI²(23) = 19.745 and F-Form(23, 104) = 0.78141 [0.7466]

RESET test for adding $\hat{Y}^2$:
RESET F(1,127) = 2.1803 [0.1423]

Similar to the original DHSY estimation, both the 'derivative' (i. e. $\Delta_4 y_t$) and the 'proportional adjustment term' (i. e. $(c-y)_{t-4}$) have the 'right' sign and are strongly significant when examining the corresponding t-values and partial R^2s. In addition, the relatively high goodness of fit and the relatively low standard error of the residuals are also in line with the original estimation results.

Turning to the issues of normal and homoscedastic residuals, both the Jarque and Bera and the original and simplified White test do not indicate any problem. Furthermore, there seem to be no residual outliers and no problems of functional form either.[58] In line with this, the graphical plot of the residuals (see figure V.1)

58 The 5% critical value for the RESET test equals 3.92 while the critical value for the original White test is equal to 1.65.

does not reveal any systematic pattern over time (although conclusions on the presence of heteroscedasticity based on graphical examinations are inevitably fragile, especially when dealing with quarterly data which allows only superficial insights [cp. Thomas, R. L., 1993, p. 95]).

It should be noted, however, that the use of a double-log specification smooths out any signs of heteroscedasticity and non-normality of the residuals so that the above results should be interpreted with care [cp. Hendry, D. F., 1983, p. 209; cp. Maddala, G. S., 1992, pp. 202 and 220].

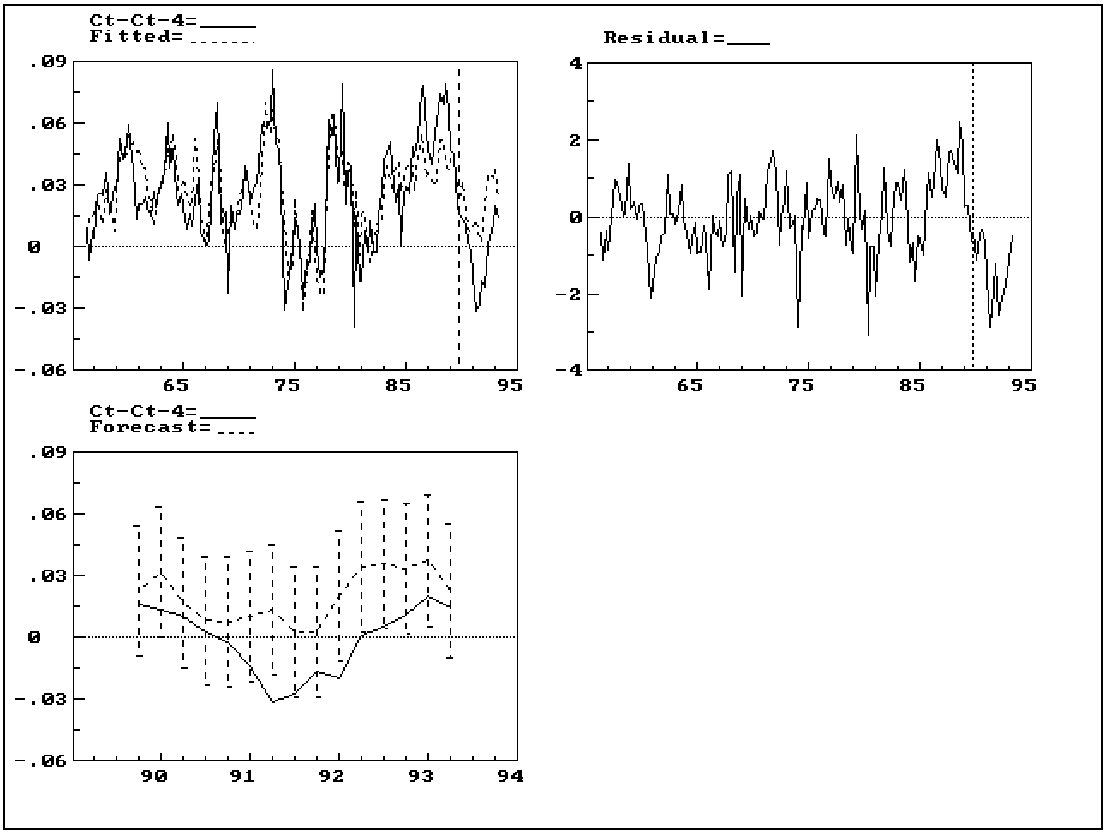

Figure V.1: Re-estimation results of the quarterly DHSY model

On the other hand, both the dummy variable (DV) and the modelled inflation effect (especially as regards $\Delta\Delta_4 p_t$) show rather poor explanatory power and low partial R^2s. Moreover, both the DW test (which is strictly speaking invalid in this context [cp. Maddala, G. S., 1992, p. 236; cp. Thomas, R. L., 1993, p. 105]) and the LM test for residual autocorrelation from lag 1 to 4 turn out to be significant at the conventional 5% level of significance.

However, after having investigated the issues of diagnostic tests and data coherency, the striking statistical adequacy of the DHSY specification is found to deteriorate to a certain degree (but appears still reasonable) when being applied to an extended sample period. This conclusion, which contrasts the re-estimation results reported by Davidson and Hendry (1981, pp. 181-183) (1964(1)-1979(4)), Hendry (1983, p. 201) (1954(1)-1982(2)), Davis (1984, pp. 822-823 and 825) (1966(3)-1975(4) and 1966(3)-1980(4)), Patterson (1986, p. 9) (1952(1)-1980(1)) and Hendry et al. (1990, p. 313) (1959(2)-1975(4)) but goes in line with those reported by Carruth and Henley (1990a, p. 214) (1969(2)-1984(4)), comes as no surprise since the post1982 values include the effects of financial deregulation, the phenomenon of housing equity withdrawal, the 1987/88 consumer boom and the subsequent UK recession.[59] In fact, virtually all models of consumption which showed a reasonably well performance in terms of tracing the development of consumption for the late 1970s/early 1980s break down when being confronted with the task of predicting post1982 values [cp. Muellbauer, J. N. J. and Murphy, A., 1989, pp. 26 and 29].

Despite of the reasonably well determined one-step forecasts (see also figure V.1) and as indicated by the significant Chow test, this holds also in the case of model (5.7). In addition, as illustrated by figure V.2, the remarkable parameter constancy reported by Davidson and Hendry (1981, p. 183), Hendry (1983, p. 205), Davis (1984, p. 813) and Hendry et al. (1990, pp. 319-321) gets lost when using the extended sample size for recursive least squares estimation of (5.7). It has to be taken into account, however, that the dummy variable had to be excluded from recursive least squares estimation in order to prevent a singular matrix. Since D_t^0 takes the values +1, -1 in 1973(1), 1973(2) to allow for the effect of the introduction of VAT in 1973[60], the significant recursive Chow test for 1973/74 appears less detrimental. Another problem might be the issue of measurement inaccuracies which led to particularly substantial data revisions in the late 1970s and early 1980s [cp. Hendry, D. F., 1983, p. 210].

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59 For more on this, see in particular Muellbauer and Murphy (1989), Carruth and Henley (1990a,b) and Miles (1992).

60 This innovation is due to Hendry and von Ungern-Sternberg (!981).

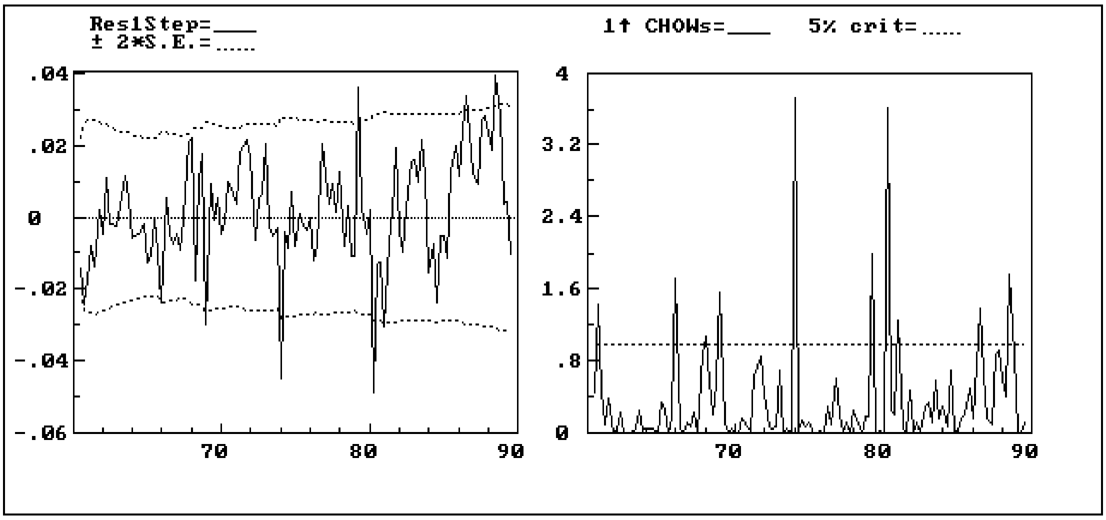

Figure V.2: One-step residuals and recursive Chow test (quarterly DHSY model)

On the other hand, both the coefficients of the level of inflation and its rate of change have already relatively shortly after the publication of the DHSY paper been shown to suffer from a high degree of instability. In fact, as early as 1980, Hendry and von Ungern-Sternberg (1981) found significant changes in parameter values for all regressors, especially in terms of $\Delta\Delta_4 p_t$. Similar results have been reported by Davis (1984, p. 812).

Hence, after all, the in- and out-sample performance of the original DHSY specification has turned out to worsen considerably when being confronted with an extended sample period. However, rather than focusing on any reparameterization of (5.7), the following empirical sections will investigate both the potential non-stationarity of the incorporated variables and the potential non-existence of any (unit-elasticity) long-run relationship between consumption and income for the period 1956(1)-1993(2) (and 1956-1992) as reasons for the poor performance of (5.7) discussed above.

V.b. Testing for orders of seasonal integration in the quarterly DHSY model

In the light of the relatively recent findings in the field of cointegration and error-correction analysis, the error-correction mechanism included in (5.7) should yield acceptable test results for the hypothesis of quarterly non-durable consumption

and personal disposable income being cointegrated of order (1;1) with cointegrating vector (1;-1). Following the discussion in chapters III.b. and IV., this hypothesis can sequentially be tested for using the concepts of seasonal integration and cointegration testing within the framework of the Engle-Granger two step estimator.
Accordingly, sections V.b.1.-V.b.3. contain testing sequences of the Ilmakunnas type on the order of seasonal integration of both logged quarterly non-durable consumption and personal disposable income as well as logged inflation (measured by annual changes in the retail price index) for the period 1955(1) to 1993(2). The results of these testing sequences are then used to check whether consumption and income (and maybe inflation) are integrated to the same order or not.[61] In addition, they indicate whether the various variables included in (5.7) (except for the error-correction mechanism) are differenced properly.
However, as outlined in the preceding sections, the determination of the appropriate number of lagged dependent variables in the test equations is based on the results of the corresponding LM autocorrelation tests. In any case, the finally obtained LM test result is reported in order to justify the assumption of white noise errors and thus the applicability of the various critical values. In addition, a variety of out- and in-sample parameter constancy tests is evaluated whereby the latter test statistics are used to control for any OLS bias resulting from potential structural breaks within the relatively large sample under investigation.

V.b.1. Quarterly consumers' expenditure

Following the testing sequence for seasonal integration described in figure III.1, non-durable consumption is first tested for H_0: c_t~I(1;1) against H_1: c_t~I(0;1) using the ADF_1 statistic. Having estimated various specifications with different numbers of lagged dependent variables, the most parsimonious regression without any sign of residual autocorrelation is given by:

Modelling $\Delta\Delta_4 c_t$ by OLS:

The present sample is: 58 (1) to 93 (2) less 10 forecasts

[61] I. e., whether condition i) of the Engle-Granger definition of cointegration for the bivariate case is fulfilled.

The forecast period is: 91 (1) to 93 (2)

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.011359	0.0037901	2.997	0.0719
$\Delta_4 c_{t-1}$	-0.37069	0.087549	**-4.234**	0.1339
$\Delta\Delta_4 c_{t-1}$	0.083499	0.10457	0.798	0.0055
$\Delta\Delta_4 c_{t-2}$	0.21056	0.10310	2.042	0.0347
$\Delta\Delta_4 c_{t-3}$	0.33529	0.10338	3.243	0.0831
$\Delta\Delta_4 c_{t-4}$	0.25314	0.089720	-2.821	0.0642
$\Delta\Delta_4 c_{t-5}$	0.18973	0.092643	2.048	0.0349
$\Delta\Delta_4 c_{t-6}$	0.15260	0.093809	1.627	0.0223
$\Delta\Delta_4 c_{t-7}$	0.26696	0.090308	2.956	0.0701
$\Delta\Delta_4 c_{t-8}$	-0.00025614	0.0016065	-0.159	0.0002
$\Delta\Delta_4 c_{t-9}$	0.00074207	0.0016065	0.462	0.0018
$\Delta\Delta_4 c_{t-10}$	0.00087634	0.0016062	0.546	0.0026
$\Delta\Delta_4 c_{t-11}$	-0.00052693	0.0015999	-0.329	0.0009
DV_1	-0.0031688	0.0041316	-0.767	0.0050
DV_2	0.00049829	0.0040982	0.122	0.0001
DV_3	-0.0029097	0.0041279	-0.705	0.0043

$R^2 = 0.3945$ $F(15, 116) = 5.0385$ [0.0000] $\hat{\sigma} = 0.0163797$ DW = 1.94
RSS = 0.03112199709 for 16 variables and 132 observations

Tests of parameter constancy over 91 (1) to 93 (2):
Forecast Chi²(10)/10 = 0.44536
Chow F(10,116) = 0.38851 [0.9495]

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 2.7345 and F-Form(4, 112) = 0.59231 [0.6689]

For p=11, the Lagrange multiplier test is clearly insignificant thereby indicating both no further problems of residual autocorrelation and the applicability of the critical values tabulated by Fuller (1976, p. 373).[62] Similarly, there are no signs of any lack of out-sample parameter constancy since both the forecast Chi^2 and the conventional Chow test are well below their 5% critical values of approximately 8.3 and 1.91 respectively. Furthermore, the more stringent examination of the issue of in-sample parameter constancy by using recursive least squares does not indicate any substantial failure to model potential 'regime shifts'. As the plot of the one-step residuals in figure V.3 illustrates, although the 1968, 1973/74 as well as the 1979 residuals are found to lie outside of the $2\hat{\sigma}_t$ error bars, they do not induce any

[62] The Durbin-Watson test (although being reported) is inappropriate when using quarterly data since (amongst other things) it tests for first order residual autocorrelation only.

'systematic' parameter non-constancy. The 1968, 1973/74, 1979 values in fact seem to be linked with the effects of both the 1968 advance warnings of purchase tax increases in 1969 and the 1970-73 'Barber boom' (along with the subsequent shift into the 1974 oil crisis) as well as the move of the UK economy into the 1980 recession. The scaled Chow tests in figure V.3 reflect the significant size of these residuals (in particular, that of the 1969 one-step residual). Although the Chow tests suggest a slightly poorer constancy performance as the one-step residuals, they give still no indication for particularly detrimental non-constancy of the ADF_1 test. In fact, following Davis (1984, p. 813) and focusing on the number of significant observations, its performance appears reasonable relative to the size of the sample.[63]

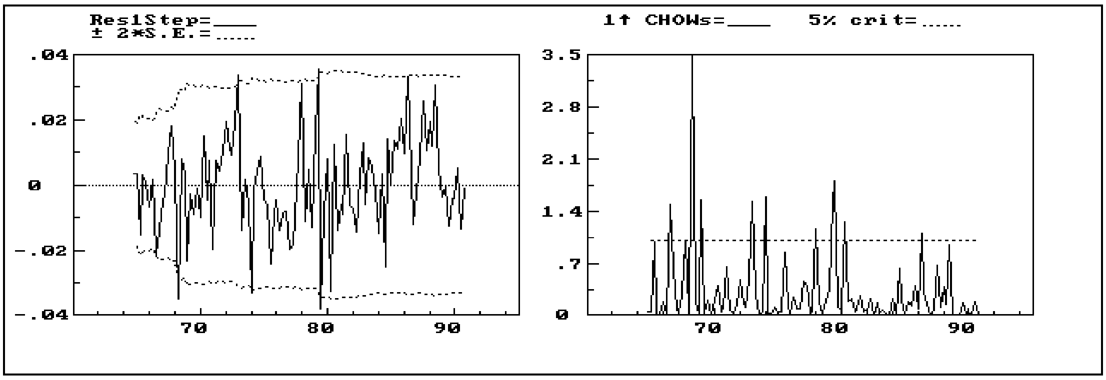

Figure V.3: One-step residuals and recursive Chow test (ADF_1 test on quarterly consumption)

Nevertheless, the various 'outliers' affect the quality of the ADF_1 statistic which therefore appears even less reliable than usual. Therefore, the strongly significant test result of -4.234 (compared with the critical value of approximately -2.89) and thus the acceptance of H_1 has to be treated with caution. Similar limitations have in fact also been encountered by the author in the case of the HEGY tests listed further below.[64]

However, these 'stability limitations' might also be the reason for the lack of clear-cut results in terms of the order of seasonally integration of consumption reported by Osborn et al. (1988, pp. 368-369). In their study, Osborn et al. fail to arrive at a clear decision on the question whether logged quarterly consumption of non-durables has to be classified as I(1;1) or I(0;1).

63 Note also the problem of cumulating type one errors when performing a large number of tests in a recursive analysis: The probability of type one errors increases with the number of tests thereby wrongly indicating parameter non-constancy [cp. Patterson, K. D., 1986, p. 7].

64 The corresponding graphics are not included to preserve space.

Ignoring these problems for a moment, computation of the more robust $HEGY_2$ test in order to investigate the ADF_1 result leads to:
Modelling $\Delta\Delta_4c_t$ by OLS (1958(1)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.0094846	0.0033537	2.828	0.0597
Z_{1t-1}	-0.26640	0.060247	**-4.422**	0.1343
$\Delta\Delta_4c_{t-1}$	-0.0041235	0.089052	-0.046	0.0000
$\Delta\Delta_4c_{t-2}$	0.18034	0.092666	1.946	0.0292
$\Delta\Delta_4c_{t-3}$	0.30456	0.093763	3.248	0.0773
$\Delta\Delta_4c_{t-4}$	-0.32003	0.079467	-4.027	0.1140
$\Delta\Delta_4c_{t-5}$	0.24620	0.094127	2.616	0.0515
$\Delta\Delta_4c_{t-6}$	0.20828	0.093532	2.227	0.0379
$\Delta\Delta_4c_{t-7}$	0.31553	0.089999	3.506	0.0889
$\Delta\Delta_4c_{t-8}$	-0.00058663	0.0015612	-0.376	0.0011
$\Delta\Delta_4c_{t-9}$	0.00068324	0.0015629	0.437	0.0015
$\Delta\Delta_4c_{t-10}$	0.00067096	0.0015620	0.430	0.0015
$\Delta\Delta_4c_{t-11}$	-0.00072697	0.0015568	-0.467	0.0017
DV_1	-0.0027340	0.0038408	-0.712	0.0040
DV_2	0.0010646	0.0038521	0.276	0.0006
DV_3	-0.0016867	0.0038714	-0.436	0.0015

$R^2 = 0.388647$ $F(15, 126) = 5.34$ [0.0000] $\hat{\sigma} = 0.0159772$ DW = 1.95
RSS = 0.03216433695 for 16 variables and 142 observations

LM test for residual autocorrelation from lags 1 to 4:
$CHI^2(4) = 3.2357$ and F-Form(4, 122) = 0.71121 [0.5858]

With the LM test for residual autocorrelation being insignificant for p=11, the t-value on the coefficient estimate of Z_{1t-1} (-4.422) is found to be significantly different from zero when comparing it with the corresponding critical value of -2.94 (for T=136 at the 5% level of significance).[65] Hence, the $HEGY_2$ test confirms the ADF_1 result and the null of c_t~I(1;1) has to be rejected in favour of c_t~I(0;1). This comes as no surprise since " ... if $\pi_2=\pi_3=\pi_4=0$ in the HEGY test, the test regression is the same as the ADF test for seasonally differenced data" [Ilmakunnas, P., 1990, p. 81]. Due to this relationship, the $HEGY_2$ test is affected by the same problem as the ADF_1 test mentioned above and might therefore also be of relatively low power in this context.

However, setting this reliability problem aside for a moment and accepting the alternative hypothesis, if a variable has to be seasonally differenced to achieve

65 See Hylleberg et al. (1990, pp. 226-227) for the list of critical values which should be applied alongside of the HEGY test regressions.

stationarity, this might in fact be caused by a single unit root at the zero frequency rather than seasonal unit roots at the half-yearly and annual frequency [cp. Osborn, D. R. et al., 1988, p. 365; cp. Ilmakunnas, P., 1990, p. 79]. Since no allowance has been made for this possibility by the ADF_1 and $HEGY_2$ specifications used above, the $HEGY_3$ statistic is computed in order to validate the provisional conclusion of c_t~I(0;1) against the alternative of c_t~I(1;0). Accordingly, if the null hypothesis is valid both the F-statistic on $\hat{\pi}_3$ and $\hat{\pi}_4$ and the t-statistic on $\hat{\pi}_2$ should turn out to be insignificant at the chosen 5% significance level given the restriction of $\pi_1=0$. The estimation of (3.21) yields:
Modelling $\Delta_4 c_t$ by OLS (1957(3)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	-0.019188	0.0080246	-2.391	0.0418
Z_{2t-1}	-0.061416	0.036444	**-1.685**	0.0212
Z_{3t-2}	-0.19737	0.060935	-3.239	0.0741
Z_{3t-1}	0.010853	0.062696	0.173	0.0002
$\Delta_4 c_{t-1}$	0.72578	0.095831	7.573	0.3045
$\Delta_4 c_{t-2}$	0.070461	0.11450	0.615	0.0029
$\Delta_4 c_{t-3}$	0.065527	0.10239	0.640	0.0031
$\Delta_4 c_{t-4}$	-0.30418	0.10220	-2.976	0.0633
$\Delta_4 c_{t-5}$	0.33795	0.10463	3.230	0.0738
$\Delta_4 c_{t-6}$	-0.14537	0.086601	-1.679	0.0211
DV_1	0.022853	0.012404	1.842	0.0253
DV_2	0.040284	0.013359	3.015	0.0649
DV_3	0.036895	0.011985	3.078	0.0675

$R^2 = 0.668578$ F(12, 131) = 22.022 [0.0000] $\hat{\sigma} = 0.0156244$ DW = 2.00
RSS = 0.03197987848 for 13 variables and 144 observations

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 4.941 and F-Form(4, 127) = 1.1281 [0.3463]

Wald test for zero restrictions on Z_{3t-2} and Z_{3t-1}:
LinRes F(2,131) = **5.255**

For T=136, the appropriate critical value for a unit root at $\hat{\pi}_2$ equals -2.9 whereas the adjusted critical value for the F-test on $\hat{\pi}_3$ and $\hat{\pi}_4$ being equal to zero is given by 6.63. Thus, the $HEGY_3$ result does not allow the rejection of the seasonal unit roots hypothesis so that first differencing alone seems not to be sufficient to achieve stationarity. In other words, the null hypothesis of c_t~I(0;1) cannot be rejected if it is tested against the alternative of a single unit root at the zero frequency. In fact, this conclusion goes in line with the HEGY test results presented by Osborn et al. (1988,

pp. 368-369).[66] However, also similar to their study, contradictory results are obtained when checking the above provisional acceptance of c_t~I(0;1) rather than c_t~I(1;1) by estimating the unrestricted and therefore more general $HEGY_4$ specification:
Modelling $\Delta_4 c_t$ by OLS (1957(3)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.017234	0.055686	0.309	0.0007
Z_{1t-1}	-0.0073217	0.011077	**-0.661**	0.0033
Z_{2t-1}	-0.061345	0.036523	**-1.680**	0.0212
Z_{3t-2}	-0.19704	0.061069	-3.227	0.0741
Z_{3t-1}	0.011441	0.062837	0.182	0.0003
$\Delta_4 c_{t-1}$	0.72915	0.096173	7.582	0.3066
$\Delta_4 c_{t-2}$	0.072990	0.11481	0.636	0.0031
$\Delta_4 c_{t-3}$	0.064736	0.10262	0.631	0.0031
$\Delta_4 c_{t-4}$	-0.30348	0.10243	-2.963	0.0633
$\Delta_4 c_{t-5}$	0.33723	0.10486	3.216	0.0737
$\Delta_4 c_{t-6}$	-0.14117	0.087019	-1.622	0.0198
DV_1	0.022765	0.012432	1.831	0.0251
DV_2	0.040136	0.013390	2.998	0.0647
DV_3	0.036868	0.012011	3.069	0.0676

R^2 = 0.669688 F(13, 130) = 20.274 [0.0000] $\hat{}\sigma$ = 0.0156581 DW = 2.00
RSS = 0.03187275531 for 14 variables and 144 observations

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 4.8626 and F-Form(4, 126) = 1.1009 [0.3592]

Wald test for zero restrictions on Z_{3t-2} and Z_{3t-1}:
LinRes F(2,130) = **5.2158**

By contrast to the ADF_1 and $HEGY_2$ findings but consistent with the $HEGY_3$ result, the insignificance of the t-value on $\hat{}\pi_1$ leaves a questionmark behind the rejection of c_t~I(1;1). This ambiguity corresponds to the classification problem encountered by Osborn et al. (1988). As in their study, the alternative classifications are obviously represented by c_t~I(0;1) on the one hand and c_t~I(1;1) on the other hand. With the comparison of the corresponding correlograms not yielding any further insight and taking the stability problems into account, it seems most reasonable to refer to both

[66] Osborn et al. (1988, p. 363), however, use a HEGY specification which is slightly different from the original one put forward by Hylleberg et al. (1990). Nevertheless, the validity of the comparison of the results obtained by Osborn et al. in their 1988 paper and those reported by the author is not affected by this difference.

the above $HEGY_4$ and the OCSB test result reported by Osborn et al. (1988, pp. 368-369) and to classify consumption as seasonally integrated of order (1;1).[67]
However, in light of this rather shaky classification, it seems more useful to control for either alternative when investigating potential cointegrating vectors rather than adopting such a restrictive approach as Osborn et al. in their 1988 paper.[68]

V.b.2. Quarterly personal disposable income

Turning to the investigation of the integration properties of the UK income series for the period 1955(1) to 1993(2), the calculation of the ADF_1 statistic for the null of y_t~I(1;1) against y_t~I(0;1) yields the following results:

Modelling $\Delta\Delta_4 y_t$ by OLS:

The present sample is: 57 (4) to 93 (2) less 10 forecasts

The forecast period is: 91 (1) to 93 (2)

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.012236	0.0042679	2.867	0.0631
$\Delta_4 y_{t-1}$	-0.40097	0.088135	**-4.550**	0.1450
$\Delta\Delta_4 y_{t-1}$	0.095239	0.10322	0.923	0.0069
$\Delta\Delta_4 y_{t-2}$	0.33575	0.10152	3.307	0.0823
$\Delta\Delta_4 y_{t-3}$	0.13112	0.088872	1.475	0.0175
$\Delta\Delta_4 y_{t-4}$	-0.23236	0.086342	-2.691	0.0560
$\Delta\Delta_4 y_{t-5}$	0.17597	0.088829	1.981	0.0312
$\Delta\Delta_4 y_{t-6}$	0.30859	0.085375	3.615	0.0967
DV_1	-0.0027958	0.0049412	-0.566	0.0026
DV_2	-0.00016592	0.0049431	-0.034	0.0000
DV_3	-0.00054821	0.0049090	-0.112	0.0001

R^2 = 0.351407 F(10, 122) = 6.6099 [0.0000] $\hat{\sigma}$ = 0.0200618 DW = 2.03
RSS = 0.04910201367 for 11 variables and 133 observations

Tests of parameter constancy over 91 (1) to 93 (2):
Forecast Chi²(10)/10 =0.61218
Chow F(10,122) = 0.58743 [0.8217]

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 2.2594 and F-Form(4, 118) = 0.5098 [0.7286]

---

67 Charemza and Deadman (1992, pp. 159-162) obtain the same conclusion. Their relatively straightforward results, however, are based on the inappropriate conventional 'testing-up' sequence and are therefore less reliable.

68 However, as will become clear after the relatively straightforward classification of income (and inflation) as being I(1;0), the classification of consumption as being I(1;1) or I(0;1) respectively does not affect the final conclusion that consumption and income cannot be found to be cointegrated when using standard seasonal integration tests within the Engle-Granger two step procedure.

Having employed various lag lengths of $\Delta\Delta_4 y_t$, p was eventually set equal to six. For this value of p, both the LM residual autocorrelation test and the out-sample parameter constancy statistics are strongly insignificant. Moreover, the usual comparison of the t-statistic on the estimated value of β (-4.55) with its corresponding critical value (-2.89) leads to the clear rejection of the null hypothesis in favour of $y_t \sim I(0;1)$. Thus, the results of the estimation of (3.15) suggest that seasonally differencing suffices to achieve stationarity.

However, taking a closer look at figure V.4, it turns out that basically the same qualifications apply in this context as in the case of the interpretation of the ADF_1 (and $HEGY_2$) test on UK consumption. In other words, although particularly detrimental parameter non-constancy is not given, the reliability of the test decision is likely to be affected by various 'outliers'. These 'outliers' are in fact again dominated by the effects of the 1970-73 'Barber boom' and the subsequent first oil price shock (along with the subsequent UK recession) on UK personal income [see also Armstrong, A. G., 1979, p. 374].

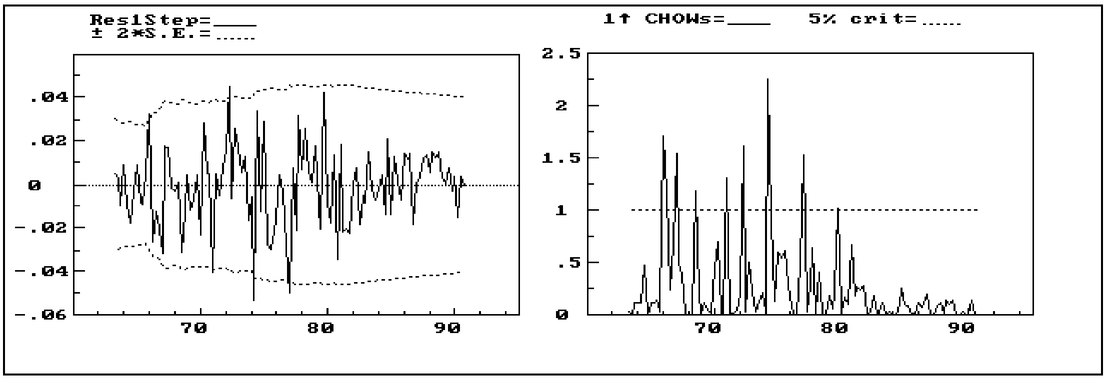

Figure V.4: One-step residuals and recursive Chow test (ADF_1 test on quarterly income)

However, in this context, it should also be noted that still considerable disagreement exists about the question which definition of 'income' to employ and how to avoid mismeasurement of income series in practice.[69] In fact, depending on the kind of definition employed, the 'parameter stability' of the regression equation considered might change considerably even though its specification has remained unchanged. Hence, parameter instability might reflect the use of a 'wrongly' defined income

69 From the wide variety of contributions to this topic, the studies by Hendry and von Ungern-Sternberg (1981), Evans and Pesaran (1984), Muellbauer and Murphy (1989, pp. 64-65) and Borooah and Sharpe (1985, pp. 244-250) are of particular interest when focusing on the impact of measurement and definition of income on consumption. Booroah and Sharpe, for example, provide evidence on the influence of income mismeasurement on consumption modelling by re-estimating prominent consumption models such as the DHSY or the Evans and Pesaran specification.

variable rather than any misspecification of the model. This insight holds not only in terms of the ADF_1 (or $HEGY_2$) test regression but also for the various other test regressions considered in this study.

However, since the number of significant Chow tests is again fairly limited in relation to the sample size, the ADF_1 result is again even less reliable than usual but there is no reason for its complete neglect either.

In an attempt to confirm the above acceptance of the alternative hypothesis of seasonal unit roots, the estimation of the $HEGY_2$ test produces the following results:

Modelling $\Delta\Delta_4 y_t$ by OLS (1957(4)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.010607	0.0040045	2.649	0.0505
Z_{1t-1}	-0.27980	0.060793	**-4.603**	0.1383
$\Delta\Delta_4 y_{t-1}$	-0.011374	0.087325	-0.130	0.0001
$\Delta\Delta_4 y_{t-2}$	0.27272	0.091271	2.988	0.0634
$\Delta\Delta_4 y_{t-3}$	0.035352	0.077887	0.454	0.0016
$\Delta\Delta_4 y_{t-4}$	-0.30464	0.078287	-3.891	0.1029
$\Delta\Delta_4 y_{t-5}$	0.23736	0.091950	2.581	0.0481
$\Delta\Delta_4 y_{t-6}$	0.35675	0.087234	4.090	0.1125
DV_1	-0.0016880	0.0046547	-0.363	0.0010
DV_2	0.00083172	0.0046897	0.177	0.0002
DV_3	0.00045961	0.0046576	0.099	0.0001

$R^2 = 0.344005$ $F(10, 132) = 6.9221$ [0.0000] $\hat{\sigma} = 0.0197458$ DW = 2.02
RSS = 0.05146626491 for 11 variables and 143 observations

LM test for residual autocorrelation from lags 1 to 4:
$CHI^2(4) = 2.683$ and F-Form(4, 128) = 0.61187 [0.6548]

In this case, p was again set equal to six after the estimation of various specifications of (3.20) with different lag orders. Setting p=6 yields the most parsimonious estimation equation which produces a Lagrange multiplier test indicating the acceptability of the null of non-autocorrelated residuals and thus the applicability of the critical values simulated by HEGY.

With both π_3 and π_4 as well as π_2 restricted to zero, the value of the t-statistic on $\hat{\pi}_1$ of -4.603 exceeds substantially its critical value of -2.94 in absolute size. Hence, seasonally differencing seems to be sufficient to obtain stationarity. Thus, the ADF_1 test decision is confirmed and the alternative hypothesis of $y_t \sim I(0;1)$ is accepted at the conventional 5% level.

However, it has again to be checked for the possibility of a single unit root at the zero frequency by estimating the $HEGY_3$ specification. OLS estimation of this restricted HEGY test under the null of $y_t \sim I(0;1)$ leads to the following results:

Modelling $\Delta_4 y_t$ by OLS (1957(3)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	-0.0083041	0.0049690	-1.671	0.0209
Z_{2t-1}	-0.075887	0.029145	**-2.604**	0.0492
Z_{3t-2}	-0.27674	0.055888	-4.952	0.1577
Z_{3t-1}	-0.087990	0.059884	-1.469	0.0162
$\Delta_4 y_{t-1}$	0.61411	0.084212	7.292	0.2887
$\Delta_4 y_{t-2}$	0.18393	0.10061	1.828	0.0249
$\Delta_4 y_{t-3}$	-0.016550	0.092287	-0.179	0.0002
$\Delta_4 y_{t-4}$	-0.29176	0.092757	-3.145	0.0702
$\Delta_4 y_{t-5}$	0.19474	0.097325	2.001	0.0297
$\Delta_4 y_{t-6}$	0.024485	0.085716	0.286	0.0006
DV_1	0.027818	0.0071244	3.905	0.1042
DV_2	0.020324	0.0067389	3.016	0.0649
DV_3	0.012793	0.0070752	1.808	0.0244

$R^2 = 0.641448$ $F(12, 131) = 19.53$ [0.0000] $\hat{\sigma} = 0.0185929$ DW = 1.94
RSS = 0.04528611194 for 13 variables and 144 observations

LM test for residual autocorrelation from lags 1 to 4:
$CHI^2(4) = 7.6818$ and F-Form(4, 127) = 1.7892 [0.1350]

Wald test for zero restrictions on Z_{3t-2} and Z_{3t-1}:
LinRes F(2,131) = **13.409 ****

Whilst the LM test does not indicate any problem of residual autocorrelation, the Wald test for the zero restrictions on $\hat{\pi}_3$ and $\hat{\pi}_4$ is highly significant (the critical value equals 6.63). Hence, although $\hat{\pi}_2$ is weakly insignificant (critical value: -2.9), the need for seasonal differencing is rejected so that y_t can safely be regarded as being seasonally integrated of order (1;0) rather than (0;1).[70] Both the examination of the correlogram of Δy_t by the author and the final calculation of the unrestricted $HEGY_5$ test have reconfirmed this result:

Modelling $\Delta_4 y_t$ by OLS (1957(3)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²

70 See again Osborn et al. (1988, pp. 367-368) for a rather similar conclusion.

Constant	0.048455	0.063414	0.764	0.0045
Z_{1t-1}	-0.011524	0.012836	**-0.898**	0.0062
Z_{2t-1}	-0.075531	0.029170	**-2.589**	0.0490
Z_{3t-2}	-0.27557	0.055944	-4.926	0.1573
Z_{3t-1}	-0.086535	0.059950	-1.443	0.0158
$\Delta_4 y_{t-1}$	0.62129	0.084653	7.339	0.2930
$\Delta_4 y_{t-2}$	0.18633	0.10072	1.850	0.0256
$\Delta_4 y_{t-3}$	-0.017429	0.092360	-0.189	0.0003
$\Delta_4 y_{t-4}$	-0.29131	0.092827	-3.138	0.0704
$\Delta_4 y_{t-5}$	0.19903	0.097514	2.041	0.0310
$\Delta_4 y_{t-6}$	0.024959	0.085781	0.291	0.0007
DV_1	0.027696	0.0071310	3.884	0.1040
DV_2	0.020203	0.0067453	2.995	0.0646
DV_3	0.012763	0.0070805	1.803	0.0244

$R^2 = 0.643657$ $F(13, 130) = 18.063$ [0.0000] $\hat{\sigma} = 0.0186067$ DW = 1.94
RSS = 0.0450070428 for 14 variables and 144 observations

LM test for residual autocorrelation from lags 1 to 4:
$CHI^2(4) = 10.216$ and F-Form(4, 126) = 2.4054 [0.0530]

Wald test for zero restrictions on Z_{3t-2} and Z_{3t-1}:
LinRes F(2,130) = **13.237 ****

As a consequence of this conclusion, the long-run relationship between consumption and income as incorporated by DHSY is rejected by the extended data series used for the above analysis. With logged quarterly consumption tentatively classified as I(1;1), logged quarterly income has been found to be I(1;0) so that the vital prerequisite for cointegration (in the bivariate case), namely identical orders of integration, is not fulfilled.[71] Hence, on the basis of this analysis, the DHSY ECM has to be judged as invalid.[72]
Furthermore, whilst $\Delta_4 c_t$ in (5.7) might well be stationary, both $\Delta_4 y_t$ and $\Delta\Delta_4 y_t$ do not represent adequately differenced regressors when focusing on the stationarity property. Although these variables might be theoretically meaningful, they are very likely to cause spurious results due to an unbalanced regression. As the following section will show, this conclusion is strongly confirmed when considering the order of seasonal integration of the annual change in the retail price index.

71 This conclusion remains of course valid when classifying consumption as being I(0;1) rather than I(1;1).
72 See, for example, Hylleberg et al. (1990, pp. 236-238) who draw the same conclusion for data on quarterly non-durable consumption and quarterly personal disposable income over the period 1955(1) to 1984(4).

V.b.3. Inflation measured by the quarterly retail price index series

Proceeding from the estimation of the ADF_1 specification, the null hypothesis of p_t~I(1;1)[73] turns out to be marginally acceptable on the basis of the following results:[74]

Modelling $\Delta\Delta_4 p_t$ by OLS:

The present sample is: 59 (2) to 93 (2) less 10 forecasts

The forecast period is: 91 (1) to 93 (2)

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.0021081	0.036611	0.058	0.0000
$\Delta_4 p_{t-1}$	-0.22328	0.087055	**-2.565**	0.0546
$\Delta\Delta_4 p_{t-1}$	0.13861	0.10418	1.331	0.0153
$\Delta\Delta_4 p_{t-2}$	0.13951	0.10397	1.342	0.0155
$\Delta\Delta_4 p_{t-3}$	0.14961	0.10384	1.441	0.0179
$\Delta\Delta_4 p_{t-4}$	-0.52228	0.10371	-5.036	0.1820
$\Delta\Delta_4 p_{t-5}$	0.064441	0.090859	0.709	0.0044
$\Delta\Delta_4 p_{t-6}$	0.073807	0.090796	0.813	0.0058
$\Delta\Delta_4 p_{t-7}$	0.068836	0.090711	0.759	0.0050
$\Delta\Delta_4 p_{t-8}$	-0.25514	0.090584	-2.817	0.0651
DV_1	0.0013425	0.051367	0.026	0.0000
DV_2	0.00027716	0.051367	0.005	0.0000
DV_3	0.00020258	0.051367	0.004	0.0000

R^2 = 0.371871 F(12, 114) = 5.6243 [0.0000] $\hat{\sigma}$ = 0.203829 DW = 1.96
RSS = 4.736262238 for 13 variables and 127 observations

Tests of parameter constancy over 91 (1) to 93 (2):
Forecast Chi²(10)/10 = 0.00060353
Chow F(10,114) = 0.00059206 [1.0000]

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 6.4116 and F-Form(4, 110) = 1.4622 [0.2186]

With the LM autocorrelation test not rejecting the null hypothesis of non-autocorrelation for p=8, the t-statistic on the estimated vale of β is slightly larger

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73 In contrast to the notation used in (5.7), the variable p_t denotes the fourth differenced quarterly retail price index series. However, the analysis has also been conducted for the original retail price index (RPI) series. According to the resulting testing sequence, RPI is seasonally integrated of order (1;1) which gives support to the conclusion of p_t~I(1;0) derived further below.

74 It has to be emphasized that inflation is measured by annual changes in the (quarterly) retail price index rather than the implicit deflator of consumption. The use of the RPI series is rationalized by Patterson (1986, pp. 2 and 29).

than the corresponding critical value of approximately -2.89. Thus, the null is weakly acceptable.

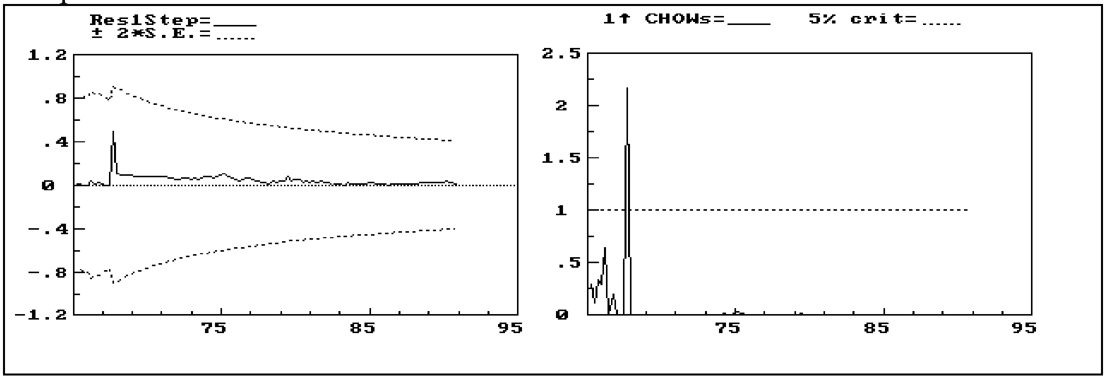

Figure V.5: One-step residuals and recursive Chow test (ADF_1 test on inflation (quarterly RPI))

This test decision still holds when taking the results of the in- and out-sample parameter constancy tests into account. First of all, as far as out-sample parameter constancy is concerned, both the forecast Chi^2 and the Chow test are clearly insignificant. In addition, as illustrated in figure V.5, apart from the 1968/69 value (which again seems to be related to the 1969 purchase tax increase), the scaled recursive Chow tests are rather satisfactory. Thus, the use of the fourth differenced quarterly retail price index series along with the ADF_1 test produces a relatively reliable test decision on the null of $p_t \sim I(1;1)$ against the alternative of $p_t \sim I(0;1)$. Due to the close similarity between the ADF_1 and the $HEGY_2$ test, this conclusion extends to the estimation of the latter test statistic.

Computing the $HEGY_2$ test (with p set equal to 8) to check the ADF_1 test decision in fact results in:

Modelling $\Delta\Delta_4 p_t$ by OLS (1959(2)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.0023250	0.033528	0.069	0.0000
Z_{1t-1}	-0.10957	0.040869	**-2.681**	0.0548
$\Delta\Delta_4 p_{t-1}$	0.024959	0.083612	0.299	0.0007
$\Delta\Delta_4 p_{t-2}$	0.027168	0.083642	0.325	0.0009
$\Delta\Delta_4 p_{t-3}$	0.038402	0.083676	0.459	0.0017
$\Delta\Delta_4 p_{t-4}$	-0.63356	0.083635	-7.575	0.3164
$\Delta\Delta_4 p_{t-5}$	0.026708	0.083623	0.319	0.0008
$\Delta\Delta_4 p_{t-6}$	0.037235	0.083667	0.445	0.0016
$\Delta\Delta_4 p_{t-7}$	0.032356	0.083628	0.387	0.0012
$\Delta\Delta_4 p_{t-8}$	-0.29109	0.083599	-3.482	0.0891
DV_1	0.00098852	0.047062	0.021	0.0000
DV_2	0.00025803	0.047402	0.005	0.0000

DV_3	0.00013682	0.047402	0.003	0.0000

$R^2 = 0.371947$ $F(12, 124) = 6.1196$ [0.0000] $\hat{\sigma} = 0.195442$ DW = 1.96
RSS = 4.736508218 for 13 variables and 137 observations

LM test for residual autocorrelation from lags 1 to 4:
$CHI^2(4) = 6.9158$ and F-Form(4, 120) = 1.5949 [0.1801]

The t-value of -2.681 implies that the null hypothesis is again marginally accepted. Thus, the ADF_1 result is confirmed and seasonal differencing alone seems not to be sufficient to obtain a stationary inflation series.
However, following the testing sequence outlined in section III.b.2., the accepted hypothesis of $p_t \sim I(1;1)$ has to be tested against $p_t \sim I(1;0)$ by computing both the $HEGY_1$ and the DHF_1 statistics.[75]
Proceeding from the estimation of equation (3.19) (with p=8), the following results are obtained:
Modelling $\Delta\Delta_4 p_t$ by OLS (1959(2)-93(2)):

Variable	**Coefficient**	**Std.Error**	**t-value**	**PartR²**
Constant	0.014492	0.032196	0.450	0.0017
Z_{2t-1}	-0.13698	0.042145	**-3.250**	0.0797
Z_{3t-2}	-0.27915	0.060114	-4.644	0.1502
Z_{3t-1}	-0.0010200	0.060177	-0.017	0.0000
$\Delta\Delta_4 p_{t-1}$	-0.059317	0.080005	-0.741	0.0045
$\Delta\Delta_4 p_{t-2}$	-0.058067	0.079974	-0.726	0.0043
$\Delta\Delta_4 p_{t-3}$	-0.045117	0.079956	-0.564	0.0026
$\Delta\Delta_4 p_{t-4}$	-0.53124	0.079994	-6.641	0.2655
$\Delta\Delta_4 p_{t-5}$	-0.058234	0.079980	-0.728	0.0043
$\Delta\Delta_4 p_{t-6}$	-0.046443	0.079928	-0.581	0.0028
$\Delta\Delta_4 p_{t-7}$	-0.051462	0.079971	-0.644	0.0034
$\Delta\Delta_4 p_{t-8}$	-0.18811	0.079998	-2.351	0.0434
DV_1	0.011462	0.045480	0.252	0.0005
DV_2	-0.0039804	0.046669	-0.085	0.0001
DV_3	-0.064104	0.045853	-1.398	0.0158

$R^2 = 0.473864$ $F(14, 122) = 7.8485$ [0.0000] $\hat{\sigma} = 0.180343$ DW = 2.01
RSS = 3.967887497 for 15 variables and 137 observations

LM test for residual autocorrelation from lags 1 to 4:
$CHI^2(4) = 2.3509$ and F-Form(4, 118) = 0.51505 [0.7248]

[75] Following DHF (1984), deterministic seasonality has been removed by replacing the p_t series by the residuals resulting from a prior regression of the series on four seasonal dummy variables rather than using any seasonal dummies and an intercept in the testing regression (as in the case of the ADF or HEGY tests respectively).

Wald test for zero restrictions on Z_{3t-2} and Z_{3t-1}:
LinRes F(2,122) = **10.782** **

Hence, both the Wald test on $\hat{}\Pi_3=\hat{}\Pi_4=0$ (critical value: 6.63) and the t-test on $\hat{}\Pi_2=0$ (crititcal value: -2.9) are significant at the chosen 5% level and therefore suggest the acceptance of the alternative hypothesis.

Similarly, after the removal of deterministic seasonality by a prior regression of the p_t series on four seasonal dummies and subsequent calculation of (3.7) and (3.8), the use of the stored residuals as the 'true' series leads to the following DHF_1 results (with p=8):

Modelling $\Delta\Delta_4 adjp_t$ by OLS (1959(2)-93(2)):

Variable	Coefficient	Std.Error	t-value	PartR²
Z_{t-4}	-0.50797	0.076944	**-6.602**	0.2540
$\Delta\Delta_4 adjp_{t-1}$	-0.012694	0.072118	-0.176	0.0002
$\Delta\Delta_4 adjp_{t-2}$	-0.0098383	0.072123	-0.136	0.0001
$\Delta\Delta_4 adjp_{t-3}$	0.0016691	0.072123	0.023	0.0000
$\Delta\Delta_4 adjp_{t-4}$	-0.49997	0.076616	-6.526	0.2496
$\Delta\Delta_4 adjp_{t-5}$	-0.011214	0.072111	-0.156	0.0002
$\Delta\Delta_4 adjp_{t-6}$	0.0022897	0.072117	0.032	0.0000
$\Delta\Delta_4 adjp_{t-7}$	-0.0052268	0.072110	-0.072	0.0000
$\Delta\Delta_4 adjp_{t-8}$	-0.15714	0.076605	-2.051	0.0318

$R^2 = 0.50431$ $\hat{}\sigma = 0.170896$ DW = 1.96
* R^2 does NOT allow for the mean *
RSS = 3.738286951 for 9 variables and 137 observations

LM test for residual autocorrelation from lags 1 to 4:
CHI²(4) = 0.46177 and F-Form(4, 124) = 0.10484 [0.9806]

The t-statistic on $\hat{}\beta$ (-6.602) exceeds substantially the corresponding critical value of approximately -4.11[76] in absolute size so that the $HEGY_1$ decision is strongly supported. Consequently, in this case, the conclusion of p_t being integrated at the zero frequency only appears reliable and straightforward.[77]

However, although (amongst other things) it has to be taken into account that the above test regressions are based on the conventional null of non-stationarity rather than any Bayesian approach, this implies again that the preferred DHSY

76 See DHF (1984, p. 362).

77 The computation of the unrestricted $HEGY_5$ test has in fact produced the same test decision. However, the corresponding regression results are not reported in order to preserve space.

specification includes non-stationary regressors. Although the Deaton approach towards the modelling of direct inflation effects on consumption might be theoretically appealing and $\Delta\Delta_4 p_t$ in (5.7) should be stationary (if measured in terms of the retail price index), $\Delta_4 p_t$ is certainly the wrongly differenced variable to be incorporated from a statistical point of view.

Consequently, after all, spurious and therefore misleading results are very likely to occur when estimating (5.7) by OLS due to the absence of any cointegrating relationship between consumption and income and the presence of generally non-stationary regressors.

Using seasonally adjusted data for the period 1966(4) to 1985(4), Drobny and Hall (1989, pp. 454-456) obtain a similar conclusion. Employing the Engle-Granger two step estimation technique, they fail to detect any cointegrating relationship between the DHSY variables. As a result, the DHSY error-correction mechanism is judged as invalid and non-stationary.

Charemza and Deadman (1992, pp. 163-164) report the same result based on the investigation of a limited sample size reaching from 1959(1) to 1975(4).

Furthermore, Hylleberg et al. (1990, p. 238) using their HEGY test in order to check the unit-elasticity assumption put forward by DHSY argue that it " ... is not valid at any frequency as long as we confine ourselves to only the consumption and income data" for the period 1955(1) to 1984(4).

In line with this, Drobny and Hall (1989, p. 456) conclude that " ... some important element of the long-run determination of consumption has been omitted." However, Drobny and Hall (1989, pp. 458-459) then succeed in deriving a set of variables which cointegrate including personal disposable income, the higher-rate tax differential and a wealth-to-income ratio. After the application of general-to-specific techniques, they eventually present a well-determined model of UK consumers' expenditure.

Favero (1993, pp. 459-463) reports another cointegrating relationship for the period 1968(1)-1989(4) consisting of personal disposable income, illiquid assets, inflation, the interest rate (measured by the quarterly rate of interest on three-month deposits with local authorities) and the percentage of the total UK population between 15 and

29 years of age. The presence of cointegration between the variables is tested for by conducting both a static regression and the Johansen maximum likelihood procedure. In contrast to the above findings and the analysis by the author, Hendry et al. (1990, pp. 315-319) find consumption, income and inflation to be cointegrated when using the Johansen procedure. The subsequent application of general-to-specific techniques enables them to derive the original DHSY specification thereby giving support to the unit-elasticity hypothesis by DHSY. It has again to be taken into account, however, that the inferences by Hendry et al. (1990) are based on a rather limited sample size (1959(2)-1976(2)) and are therefore not without qualification.

Hence, all in all, any failure to detect cointegrating relationships in consumption models seems generally to be caused by the omission of relevant variables. On the other hand, however, the inconsistent and theoretically unsatisfactory conclusion that income is no long-run determinant of consumption due to different orders of (seasonal) integration obtained above remains to be examined.

In this context, Osborn et al. (1988, p. 374) argue that the classification of consumption as being I(1;1) rather than I(1;0) " ... is due to an inappropriate treatment of seasonality. In particular, when stochastic seasonality is generalized to allow seasonally varying autoregressive coefficients, only one unit root is indicated." In other words, consumption can be shown to be I(1;0) when using the concept of periodic seasonality rather than stochastic seasonality. Thus, consumption and personal disposable income actually seem to be integrated to the same order and the existence of cointegrating relationships between these variables has no longer to be judged as impossible a priori.

However, rather than following the procedure by Osborn et al., the next section proceeds from the re-estimation of the DHSY equation for annual data. "Given the limited additional information content in quarterly data compared with annual ... " [Muellbauer, J. N. J. and Murphy, A., 1989, p. 42], this approach seems reasonable. Consequently, the treatment of seasonality can be neglected and it can be tested for any potential cointegrating vector between consumption and (amongst others) personal disposable income for the period 1955 to 1992 after the determination of the order of integration of annual consumption, income and inflation based on annual CSO data.

V.c. Testing for orders of integration in the annual DHSY model

Since the extra income and inflation dynamics present in quarterly data are meaningless for annual data, (5.7) boils down to [cp. Muellbauer, J. N. J. and Murphy, A., 1989, p. 61; cp. Hendry, D. F., 1983, p. 204; cp. Hendry, D. F. et al., 1990, p. 327; cp. Pesaran, M. H. and Evans, R. A., 1984, p. 252]:[78]

$$\Delta c_t = \alpha + \beta_1 * \Delta y_t - \beta_2 * (c-y)_{t-1} + \beta_3 * \Delta p_t \tag{5.8}$$

The OLS estimation results of (5.8) for the period 1956-1992 are included in appendix I. They show that basically the same conclusions hold for the re-estimation of the annual version of (5.7) as for the re-estimation of (5.7) itself (i. e., a considerable deterioration of performance when comparing the results for the extended sample size with those reported, for instance, by Hendry (1983) for 1952-1980).

However, the main interest in this section focuses again on the question whether the deteriorated quality of the DHSY model is partly caused by non-stationary variables and the absence of any (unit-elasticity) cointegration between consumption and income. Consequently, the Dickey-Pantula approach along with the DF/ADF test for integration are employed first in order to determine the order of integration of annual consumption, income and inflation.

V.c.1. Annual consumers' expenditure

Since the highest order of integration is assumed to be two (for all variables), the following testing sequence proceeds from the test of the null hypothesis $c_t \sim I(2)$ against $c_t \sim I(1)$ using the DF/ADF test.[79] However, due to the

[78] There is some controversy in the literature whether an intercept has to be included in (5.8) or not. However, since " ... the assumption of zero expectations for the disturbance term will be violated" [Pesaran, M. H. and Evans, R. A., 1984, p. 252] when dropping the intercept, a constant term is included.

[79] The critical values for the Dickey-Pantula-ADF t-tests on the one hand and the F-test for the presence of any deterministic trend on the other hand have been tabulated by Fuller (1976, p. 373) and Dickey and Fuller (1981, p. 1063) respectively (T=25).

insignificance of both the DW and the LM test for p=0 as well as the absence of a zero mean of the consumption series, a simple DF test with intercept seems to be the appropriate test statistic to be estimated. In order to preserve the chosen 5% level of significance, the test regression is conducted as follows:[80]

Modelling $\Delta^2 c_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.013730	0.0072330	1.898	0.0984
Δc_{t-1}	-0.50514	0.15470	**-3.265**	0.2442
Trend	-9.7612e-005	0.00027808	-0.351	0.0037

R^2 = 0.248902 F(2, 33) = 5.4678 [0.0089] ^σ = 0.0173086 DW = 1.72
RSS = 0.009886391888 for 3 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 3.8634 and F-Form(2, 31) = 1.8634 [0.1721]

Wald test for zero restrictions on Δc_{t-1} and trend:
LinRes F(2, 33) = **5.4678**

Thus, on the basis of this result, the null hypothesis cannot be rejected. However, since the F-test does not reject the null of the presence of a stochastic rather than deterministic trend, it is more appropriate to drop the trend term and to re-estimate the simple DF test:

Modelling $\Delta^2 c_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.011797	0.0046292	2.548	0.1604
Δc_{t-1}	-0.50799	0.15248	**-3.331**	0.2461

R^2 = 0.246097 F(1, 34) = 11.099 [0.0021] ^σ = 0.017084 DW = 1.71
RSS = 0.009923306676 for 2 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 4.1492 and F-Form(2, 32) = 2.0843 [0.1410]

According to this more appropriate estimation, the null hypothesis has to be rejected in favour of c_t~I(1). However, due to the fact that the sequential Dickey-Pantula procedure only stops upon acceptance of the null hypothesis, the provisional result of c_t~I(1) has to be tested against the alternative of stationarity (the F-test computed for

[80] All of the following regression results can be shown to suffer from a certain degree of parameter instability. They are therefore to be interpreted with caution. Due to this common property, the individual RLS graphics are not included.

a prior regression including the trend term indicated the absence of any deterministic trend):

Modelling $\Delta^2 c_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.055582	0.059507	0.934	0.0258
c_{t-1}	-0.0086873	0.011770	**-0.738**	0.0162
Δc_{t-1}	-0.49758	0.15416	-3.228	0.2399

R^2 = 0.25834 F(2, 33) = 5.7474 [0.0072] ^σ = 0.0171995 DW = 1.73
RSS = 0.009762159627 for 3 variables and 36 observations

Since the critical value equals -1.95, the null cannot be rejected. This result confirms the classification of consumption as being seasonally integrated of order (1;1) rather than (0;1) in section V.b.1. In addition and more important in this context, the possibility of cointegration between consumption and income as conjectured by DHSY is no longer excluded a priori.

V.c.2. Annual personal disposable income

As far as the order of integration of annual UK income is concerned, testing for H_0: y_t~I(2) against H_1: y_t~I(1) yields:
Modelling $\Delta^2 y_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.023227	0.010362	2.242	0.1321
Δy_{t-1}	-0.76005	0.16894	**-4.499**	0.3802
Trend	-0.00013060	0.00039406	-0.331	0.0033

R^2 = 0.380212 F(2, 33) = 10.122 [0.0004] ^σ = 0.0245193 DW = 1.84
RSS = 0.01983951675 for 3 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 3.7258 and F-Form(2, 31) = 1.7894 [0.1839]

Wald test for zero restrictions on Δy_{t-1} and trend:
LinRes F(2, 33) = **10.122 ****

With p=0, the DW and the LM autocorrelation test are again satisfactory. However, comparing the t-value on the estimated coefficient of Δy_{t-1} (-4.499) with its

corresponding critical value of approximately -3.6 indicates the acceptance of the alternative hypothesis. In addition, at this stage of the testing sequence, the null of a stochastic trend has to be rejected on the basis of the significant F-value of 10.122 (the critical value equals approximately 7.24).
Testing the provisional result of $y_t \sim I(1)$ against stationarity reveals:
Modelling $\Delta^2 y_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	1.8872	0.53910	3.501	0.2769
y_{t-1}	-0.39771	0.11501	**-3.458**	0.2720
Δy_{t-1}	-0.57073	0.15628	-3.652	0.2942
Trend	0.010293	0.0030335	3.393	0.2646

R^2 = 0.54882 F(3, 32) = 12.975 [0.0000] ^σ = 0.0212444 DW = 1.93
RSS = 0.01444234638 for 4 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 1.9892 and F-Form(2, 30) = 0.87729 [0.4263]

Wald test for zero restrictions on y_{t-1} and trend:
LinRes F(2, 32) = **6.0524**

Both the t-test and the F-statistic permit the acceptance of the corresponding null hypotheses. In other words, income is (marginally) found to be I(1). When dropping the trend term, however, this conclusion is much more strongly indicated:
Modelling $\Delta^2 y_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.072661	0.078156	0.930	0.0255
y_{t-1}	-0.0099582	0.014864	**-0.670**	0.0134
Δy_{t-1}	-0.75859	0.16782	-4.520	0.3824

R^2 = 0.386494 F(2, 33) = 10.395 [0.0003] ^σ = 0.0243948 DW = 1.84
RSS = 0.01963844796 for 3 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 3.2167 and F-Form(2, 31) = 1.5208 [0.2344]

Thus, income can be classified as integrated of order unity. Mindful of the apparent weakness of the DF/ADF test statistics, this result gives support to the classification of income as being integrated at the annual frequency only in section V.b.2.

V.c.3. Inflation measured by the annual retail price index series

Referring to the analysis in section V.b.3. above, inflation measured by the annual RPI series should turn out to be integrated of order zero. Checking first for the null of p_t~I(2) against p_t~I(1), it results:[81]
Modelling $\Delta^2 p_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.057495	0.30851	0.186	0.0011
Δp_{t-1}	-1.2733	0.16805	**-7.577**	0.6350
Trend	-0.0028718	0.013424	-0.214	0.0014

R^2 = 0.635111 F(2, 33) = 28.719 [0.0000] ^σ = 0.83667 DW = 2.11
RSS = 23.10057601 for 3 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 4.5399 and F-Form(2, 31) = 2.2368 [0.1238]

Wald test for zero restrictions on Δp_{t-1} and trend:
LinRes F(2, 33) = **28.719 ****

Hence, both null hypotheses are strongly rejected. Proceeding to the test of H_0: p_t~I(1) against H_1: p_t~I(0), however, the above conjecture is confirmed (the trend term was found to be neglegible in a prior regression):
Modelling $\Delta^2 p_t$ by OLS (1957-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	-1.0757	0.47285	-2.275	0.1356
p_{t-1}	-0.36861	0.15608	**-2.362**	0.1446
Δp_{t-1}	-1.0890	0.17398	-6.260	0.5428

R^2 = 0.687432 F(2, 33) = 36.289 [0.0000] ^σ = 0.774366 DW = 2.01
RSS = 19.78821904 for 3 variables and 36 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 0.82244 and F-Form(2, 31) = 0.36239 [0.6989]

Thus, the rate of inflation is narrowly indicated to be stationary for the extended sample size used in this study. Consequently, from a statistical point of view, the Δp_t term seems to be the properly differenced expression to be used in equation (5.8).

[81] In contrast to the notation used in (5.8), the variable p_t denotes the first difference of the annual RPI series.

Notice also in this context that, given these results, inflation can only be regarded as a potential long-run determinant of consumption in the multivariate case.

This conclusion does not hold for the sample sizes used in the studies referred to above. For example, the re-estimation of the DF test for the period 1957-76 covered by Hendry et al. in their 1990 paper leads to the (marginal) classification of p_t as being integrated of order unity:

Modelling $\Delta^2 p_t$ by OLS (1957-76):

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	-1.2278	0.87790	-1.399	0.1032
p_{t-1}	-0.40586	0.26031	**-1.559**	0.1251
Δp_{t-1}	-1.0827	0.26304	-4.116	0.4992

$R^2 = 0.698909$ $F(2, 17) = 19.731$ [0.0000] $\hat{\sigma} = 1.0185$ DW = 2.01
RSS = 17.63466172 for 3 variables and 20 observations

LM test for residual autocorrelation from lags 1 to 2:
$CHI^2(2) = 0.44998$ and F-Form(2, 15) = 0.17263 [0.8431]

A rather similar but even narrower result has been obtained by the author for the estimation period chosen by Hendry (1983).[82]

Ignoring the problem of the different periodicity of the underlying data, these results give some 'a priori' support to the cointegrating relationships reported by Favero (1993) and Hendry et al. (1990) all of which include inflation as a long-run determinant of consumption for a period less recent than that one used for the regressions by the author. In accordance with this, the exclusion of inflation from the cointegrating relationship by Drobny and Hall (1989) appears 'suspicious a priori' when using annual data.[83]

However, a final answer to the question of the inclusion of inflation into cointegrating relationships has to be left to future research.

Ignoring this issue, the following section mainly focuses on the possibility of cointegration between consumption and income only, especially of the type conjectured by DHSY.

[82] In fact, the null of p_t~I(1) has been found by the author to be non-rejectable for sample periods up to 1983.

V.d. Testing for cointegration between annual consumption and income and the validity of the DHSY error-correction mechanism

Proceeding from the bivariate case of consumption and income both integrated of order unity, the result of a static regression within the Engle-Granger two step framework can be used to obtain a test decision on the existence of any (unit-elasticity) equilibrium between these two variables. It has to be taken into account, however, that not all of the conditions for the meaningful application of the Engle-Granger two step estimator are fulfilled. In particular, whilst Banerjee et al. (1993, p. 220) argue that even a sample size of 200 might be too small to prevent any finite-sample bias, T is restricted to 38 in this case.
However, following the recommendation by Engle and Granger (1987, pp. 269-270), the ADF test for the stationarity of the residuals from a static regression is used alongside of the critical values (for T=50) tabulated by Engle and Yoo (1987) as a test for the null of no cointegration. Based on the analysis by Banerjee et al. (1993) mentioned in section IV.b., the ADF test is derived from the estimation of an over-specified version of (4.8). Since " ... the choice of the lag structure (...) may be *ad hoc* and different results can be obtained by changing the length of the autoregression" [Banerjee, A. et al., 1993, p. 208], various specifications of (4.8) are estimated and checked for the consistency of the corresponding test decisions.
It should be noticed, however, that due to the fact that the residuals are assumed either to contain a single unit root or to be stationary, the Dickey-Pantula approach boils down to the application of the conventional ADF test.
Finally, the CRDW test is used as a rough indicator for the presence of a first-order autoregressive relationship among the residuals. The corresponding test decision is based on the critical value (0.368) simulated by Engle and Granger (1987, p. 269) for T=100.

The static regression of consumption on income for the period 1955-92 using OLS leads to the following results (modelling c_t):

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83 Thorough cointegration testing within a common methodological framework only could yield conclusive evidence on this issue.

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	0.42633	0.058198	7.325	0.5985
y_t	0.88662	0.011098	79.892	0.9944

R^2 = 0.994391 F(1, 36) = 6382.7 [0.0000] ^σ = 0.0198374 CRDW = **0.340**
RSS = 0.01416684421 for 2 variables and 38 observations

Diagnostic tests are not reported since the performance of this model is anyway relatively certain to be poor. More interesting for the 'quality' of a static regression is the value of R^2 as an indicator for the extend of finite-sample bias in the estimated parameters. However, although the R^2 is rather large, the consideration of the relatively small sample size leads to the conclusion that the response surface provided by Banerjee et al. (1986, p. 261) for the approximation of the bias in a simple static regression (for T=90) is not applicable without qualifications. Thus, the finite-sample bias is likely to be substantial despite of $(1-R^2)\approx 0.006$.[84]
However, testing the estimated deviations of c_t from its long-run path for stationarity produces the following results:
Modelling Δe_t by OLS (1960-92):

Variable	Coefficient	Std.Error	t-value	PartR²
e_{t-1}	-0.33661	0.14011	**-2.403**	0.1709
Δe_{t-1}	0.45958	0.18958	2.424	0.1735
Δe_{t-2}	0.016707	0.20613	0.081	0.0002
Δe_{t-3}	0.15913	0.19480	0.817	0.0233
Δe_{t-4}	0.10651	0.19871	0.536	0.0102

R^2 = 0.260141 ^σ = 0.011067 DW = 2.01
* R^2 does NOT allow for the mean *
RSS = 0.003429372063 for 5 variables and 33 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 1.4367 and F-Form(2, 26) = 0.59172 [0.5607]

In this case, the approximate critical value for the 5% significance of the t-value on the coefficient of e_{t-1} equals -3.29. Therefore, the t-statistic on e_{t-1} is obviously not sufficiently negative to allow the rejection of the null hypothesis of no cointegration. The use of various other (presumably) over-specified ADF tests by the author has

84 Furthermore, the CRDW statistic is both below the (questionable) R^2 value and the critical value of 0.368. Thus, when assuming a first-order scheme among the residuals, the null of no cointegration cannot be rejected. However, in the light of this shaky assumption and the unknown distribution of the CRDW statistic for the given sample size, it seems safest to rely on the following ADF/DF test decisions only.

yielded fairly similar t-values and thus identical test decisions. However, the computation of the corresponding LM tests for residual autocorrelation has indicated that the simple DF test represents the most 'parsimonious' specification without any sign of autocorrelation:

Modelling Δe_t by OLS (1956-92):

Variable	Coefficient	Std.Error	t-value	PartR²
Δe_{t-1}	-0.19230	0.091718	**-2.097**	0.1088

R² = 0.108819 ^σ = 0.0109158 DW = 1.38
* R² does NOT allow for the mean *
RSS = 0.004289595795 for 1 variables and 37 observations

Hence, on the basis of the DF test, the null of $e_t \sim I(1)$ again cannot be rejected at conventional levels of significance. Consequently, the variables c_t and y_t although being integrated to the same order seem not to be cointegrated.

The ADF/DF tests for the stationarity of the residuals from static regressions including various additional regressors (integrated of order unity) calculated by the author also led to the (weak) acceptance of the null hypothesis.[85] Therefore, the DHSY error-correction mechanism has to be judged as invalid and non-stationary. This conclusion is confirmed when testing more directly for its validity by applying the ADF/DF tests to the residuals resulting from the expression c_t-y_t:

Modelling Δe_t by OLS (1960-92):

Variable	Coefficient	Std.Error	t-value	PartR²
e_{t-1}	0.016931	0.014988	**1.130**	0.0436
Δe_{t-1}	0.23127	0.19418	1.191	0.0482
Δe_{t-2}	-0.25469	0.19862	-1.282	0.0555
Δe_{t-3}	-0.0080947	0.19778	-0.041	0.0001
Δe_{t-4}	-0.11829	0.19519	-0.606	0.0129

R² = 0.13494 ^σ = 0.0139532 DW = 1.95
* R² does NOT allow for the mean *
RSS = 0.005451393207 for 5 variables and 33 observations

LM test for residual autocorrelation from lags 1 to 2:
CHI²(2) = 1.6863 and F-Form(2, 26) = 0.70006 [0.5057]

85 The additional regressors consisted of net personal wealth, the mortgage interest rate and/or the UK base rate.

Notice that this test represents the equivalent to a conventional integration test since the residuals are no longer derived from a static regression but on the basis of the original c_t and y_t series. Hence, the critical values tabulated by Fuller (1976, p. 373) have to be used.

However, the t-value on the coefficient of e_{t-1} is obviously not sufficiently negative to be smaller than its corresponding critical value of -1.95. As a result, the null of no cointegration cannot be rejected. Very similar (positive) t-statistics are in fact obtained for various other specifications, the most 'parsimonious' of which is given by:

Modelling Δe_t by OLS (1959-92):

Variable	Coefficient	Std.Error	t-value	PartR²
e_{t-1}	0.016180	0.014483	**1.117**	0.0399
Δe_{t-1}	0.23435	0.19170	1.222	0.0475
Δe_{t-2}	-0.23250	0.18932	-1.228	0.0479
Δe_{t-3}	-0.0046714	0.19091	-0.024	0.0000

R^2 = 0.12281 $\hat{\sigma}$ = 0.0137998 DW = 1.90
* R^2 does NOT allow for the mean *
RSS = 0.00571306092 for 4 variables and 34 observations

LM test for residual autocorrelation from lags 1 to 2:
$CHI^2(2)$ = 1.1043 and F-Form(2, 28) = 0.46998 [0.6299]

Hence, the null is accepted consistently by all test regressions.

However, it has to be pointed out that the " ... failure to reject non-stationarity of the residuals (...) is by no means conclusive" [Thomas, R. L., 1993, p. 289]. The ADF/DF tests on the estimated and the directly obtained residuals lack power. In addition, the limited sample sizes available for both the simple (T=38) and the extended static regression (T=11) implies the absence of any 'super-consistency'. Under these circumstances, the estimated residuals themselves, for example, are likely to contribute to misleading inferences.

Nevertheless, the DHSY hypothesis of a unit-elasticity error-correction mechanism can safely be rejected in the light of the test results based on the directly obtained residuals, the seasonal integration analysis in section V.b. as well as the dynamic regression results reported in appendix II and discussed below.

The dynamic regression in second order lags of both variables was performed in order to circumvent the problem of potentially large finite-sample biases in the static regression. As illustrated by (A1) in appendix II, all parameters of the dynamic equation are well-determined with largely satisfactory t-values and relatively high partial R^2s. Furthermore, both the conventional R^2 and the F-test are of considerable size. In addition, there are no signs of autocorrelation, heteroscedasticity or non-normality among the residuals. Moreover, out-sample parameter instability as measured by the conventional Chow test seems to be absent. Finally, neither the original White test nor the RESET test indicate any problem of functional form misspecification.

The long-run solution of (A1) is given by (A2). Taking the poor determination of the long-run equation into account, the finding of non-stationarity among its residuals comes as no surprise:

Modelling Δe_t by OLS (1962-87):

Variable	Coefficient	Std.Error	t-value	PartR²
e_{t-1}	-0.16697	0.23977	**-0.696**	0.0226
Δe_{t-1}	-0.54063	0.27043	-1.999	0.1599
Δe_{t-2}	-0.59881	0.27309	-2.193	0.1863
Δe_{t-3}	-0.28198	0.24573	-1.148	0.0590
Δe_{t-4}	-0.34933	0.20139	-1.735	0.1253

$R^2 = 0.452022$ $\hat{\sigma} = 0.892972$ DW = 2.27
* R^2 does NOT allow for the mean *
RSS = 16.7453667 for 5 variables and 26 observations

LM test for residual autocorrelation from lags 1 to 2:
$CHI^2(2) = 4.1898$ and F-Form(2, 19) = 1.825 [0.1884]

The simple DF test was again found to yield the most 'parsimonious' test specification:

Modelling Δe_t by OLS (1958-87):

Variable	Coefficient	Std.Error	t-value	PartR²
e_{t-1}	-0.50062	0.16370	**-3.058**	0.2438

$R^2 = 0.243848$ $\hat{\sigma} = 0.917465$ DW = 2.15
* R^2 does NOT allow for the mean *
RSS = 24.41050279 for 1 variables and 30 observations

Comparing the two t-statistics with the critical value of -3.75 results again in the non-rejection of the null of no cointegration. Consequently, the hypothesis of the existence of any cointegrating relationship between consumption and income alone cannot be accepted no matter whether a static regression or its usually more powerful dynamic counterpart is considered.

This conclusion implicitly goes in line with those put forward by, for example, Drobny and Hall (1989), Hylleberg et al. (1990) and Thomas (1993, p. 289) who point to the omission of relevant long-run determinants of consumption in cointegrating regressions as reason for the failure to detect any cointegration between consumption and income.

Thus, after all, the above application of the integration and cointegration techniques discussed in chapter III.b. and IV. to the issue of testing for both the potential non-stationarity of the DHSY variables and the non-existence of the hypothesized unit-elasticity error-correction mechanism has led to a variety of insights.

First of all, when using seasonal integration techniques, the original DHSY specification has to be judged as unbalanced and non-stationary in the regressors. The conjectured error-correction mechanism is not valid due to different orders of seasonal integration of quarterly consumption and income.

Secondly, when considering the annual version of (5.7), the problem of non-stationarity in the regressors (apart from the error-correction mechanism) seems to be reduced although inflation might be wrongly differenced when working with less recent samples. As far as the unit-elasticity hypothesis is concerned, however, the seasonal integration result is confirmed. In other words, although annual consumption and income have been found to be integrated to the same order, the null of no cointegration cannot be rejected so that the error-correction mechanism has again to be regarded as non-stationary. This conclusion has been shown to hold consistently for tests on estimated residuals from static and dynamic regressions as well as for tests on directly obtained residuals. In fact, consumption and income generally cannot be found to be cointegrated within the Engle-Granger two step framework as long as no additional (potential) long-run determinants of consumption such as wealth or rates of interest are considered.

VI. Conclusion

Despite of the clear rejection of the validity of the DHSY model for a more recent sample period in the previous chapter, it should have become clear that its publication meant a major step forward in terms of the statistical and theoretical analysis of consumption data in the UK.

As far as consumption theory is concerned, it undoubtedly paved the way for the seminal contributions by Hendry and von Ungern-Sternberg (1981), Pesaran and Evans (1984), Patterson (1986), Muellbauer and Murphy (1989), etc.

Similarly, on the statistical front, it stimulated the systematic development of integration and cointegration analysis which nowadays allows econometricians to test for the existence of long-run relationships rather than assuming them.

In fact, the methodological developments based on the DHSY study[86] have constituted a kind of " ... 'progressive research programme' that suggests an efficient and systematic way of making progress towards modelling consumers expenditure" [Hadjimatheou, G., 1987, p. 166].

On the other hand, these apparently favourable developments have not come along without qualifications.

As the empirical study in chapter V has revealed, integration and cointegration testing still suffers from a substantial lack of precision and reliability. In line with this, appropriate tests for seasonal or periodic cointegration are not yet available [see Hylleberg, S. et al., 1990, p. 234, Osborn, D. R. et al., 1988, pp. 370-374 and Birchenhall, C. R. et al., 1989]. Moreover and particularly detrimental in this context, a powerful estimation technique at the presence of a large set of variables and/or stringent degrees-of-freedom restrictions remains to be developed [cp. Gonzalo, J., 1994, pp. 224-225; cp. Thomas, R. L., 1993, p. 170]. Finally, although the understanding of time series problems such as multicollinearity, measurement errors, omitted variables, etc. has benefitted from the findings of cointegration and integration analysis, there is still considerable disagreement about the effect of pre-tests such as integration and cointegration tests on subsequent hypothesis testing [cp. Banerjee, A. et al., 1993, p. 305].

[86] See, for example, Spanos (1988, pp. 342-360) for a well-written exposition.

Extending this discussion to error-correction modelling, there is still a lack of consensus as to what exactly defines an ECM. The variety of interpretations runs from Phillips (1957) via Sargan (1964), Hendry et al. (1984), Hendry and von Ungern-Sternberg (1981), Nickell (1985) to Engle and Granger (1987).

In addition, econometric researchers seem not to be concerned about the atheoretical nature of their ECMs. Accordingly, the modelling of expectations and the incorporation of formal economic theory into ECMs has largely been neglected in practice. This includes the wide-spread ignorance of the theoretically appealing modelling of non-linear ECMs [cp. Alogoskoufis, G. and Smith, R., 1991, pp. 97-109; cp. Peel, D. A., 1992, p. 173].

This lack of concern has also led to the tendency to include additional regressors in a rather ad hoc-fashion into non-cointegrating relationships. The subsequent investigation of the implications of such a procedure for the steady-state solution of the model might yield statistically satisfactory but in any case theoretically unfounded results [see also Hadjimatheou, G., 1987, pp. 167-168 and Carruth, A. A., 1987, p. 28].[87] Hence, as indicated in section IV.c., it appears generally " ... preferable to derive relationships from theory at the outset rather than to rely on a data based approach and then simply interpret results in terms of theory" [Thomas, R. L., 1993, p. 171].

Thus, all in all, many difficult empirical and theoretical obstacles in the field of integration/cointegration and error-correction analysis remain to be overcome. Similar conclusions in fact hold for the state of consumption modelling in the UK.[88] The econometric instruments and methodology which originated from the DHSY study, however, still provide a powerful basis for the circumvention of these problems by future research.

87 The extended static regression mentioned in section V.d. illustrates this conclusion.

88 This includes in particular the issues of the adequate definition and measurement of income, wealth and interest rate variables as well as the consideration of liquidity constraints, non-human wealth, etc. [see, for

instance, Hadjimatheou, G., 1987, pp. 50-51 and 117-118, Muellbauer, J. N. J. and Murphy, A., 1989, p. 27, Thomas, R. L., 1993, pp. 283-284].

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APPENDIX I: Re-estimation results of the annual DHSY model

Modelling Δc_t by OLS:

The present sample is: 56 to 92 less 5 forecasts

The forecast period is: 88 to 92

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	-0.026343	0.012391	-2.126	0.1390
Δy_t	0.58494	0.066766	8.761	0.7327
$(c-y)_{t-1}$	-0.12327	0.048294	-2.553	0.1888
Δp_t	-0.0049341	0.0020294	-2.431	0.1743

$R^2 = 0.786811$ $F(3, 28) = 34.446$ [0.0000] $\hat{\sigma} = 0.00846329$ DW = 1.29

RSS = 0.002005564757 for 4 variables and 32 observations

Analysis of 1-step forecasts:

Date	**Actual**	**Forecast**	**Y - Yhat**	**Forecast SE**	**t-value**
88 1	0.0648812	0.0441392	0.0207420	0.00887079	2.33824
89 1	0.0306200	0.0327666	-0.00214656	0.00873171	-0.245834
90 1	0.0130920	0.0219519	-0.00885990	0.00864390	-1.02499
91 1	-0.0122112	0.00849223	-0.0207034	0.00890599	-2.32466
92 1	0.00155502	0.0275915	-0.0260365	0.00888201	-2.93137

Tests of parameter constancy over 88 to 92:

Forecast $Chi^2(5)/5 = 4.523$

Chow $F(5, 28) = 3.9986$ [0.0073] **

LM test for residual autocorrelation from lags 1 to 2:

$CHI^2(2) = 5.2307$ and F-Form(2, 26) = 2.5402 [0.0982]

Normality test:

Mean	0.000000
Std.Devn.	0.950382
Skewness	0.460412
Excess Kurtosis	-0.187822
Minimum	-1.513986

Maximum 2.322058
Normality $Chi^2(2)$= 1.0304

(Simplified) White test for heteroscedastic errors:
$CHI^2(6)$ = 8.3835 and F-Form(6, 21) = 1.2424 [0.3251]

(Original) White test of functional form:
$CHI^2(9)$ = 16.452 and F-Form(9, 18) = 2.1162 [0.0841]

RESET test for adding $\hat{}Y^2$:
RESET F(1, 27) = 0.23749 [0.6300]

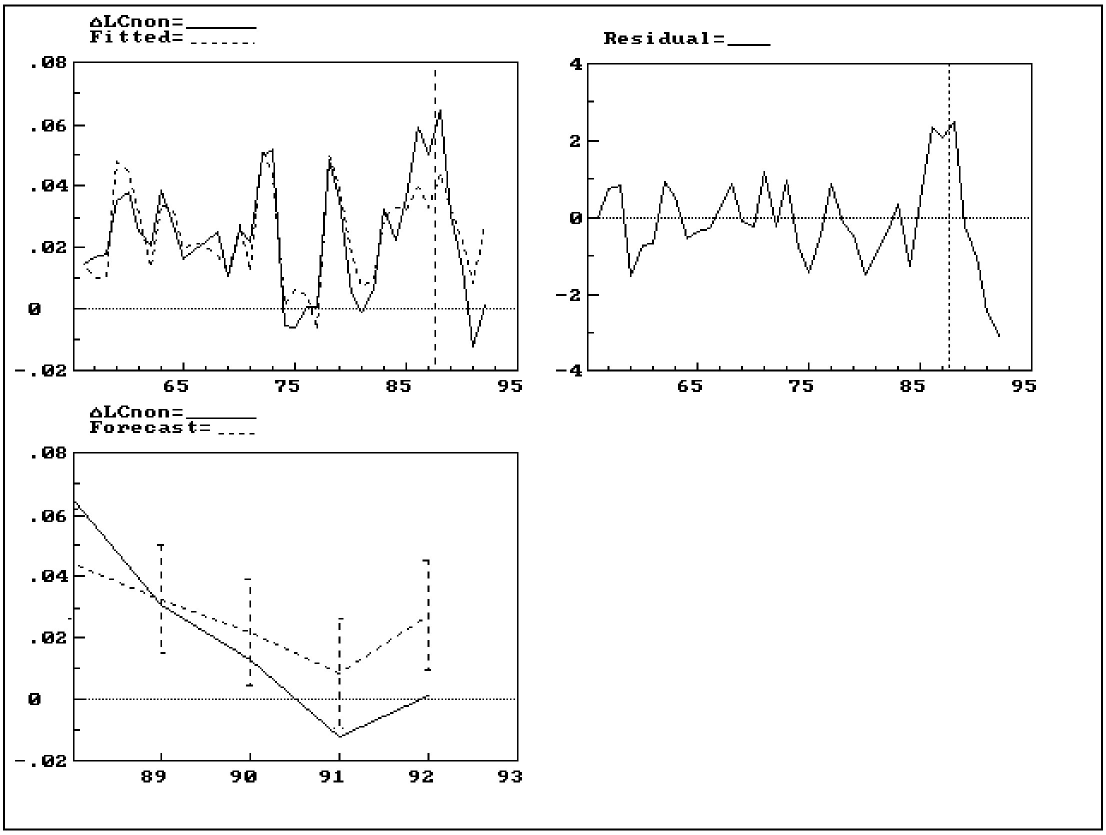

Figure A1.1: Re-estimation results of the annual DHSY model

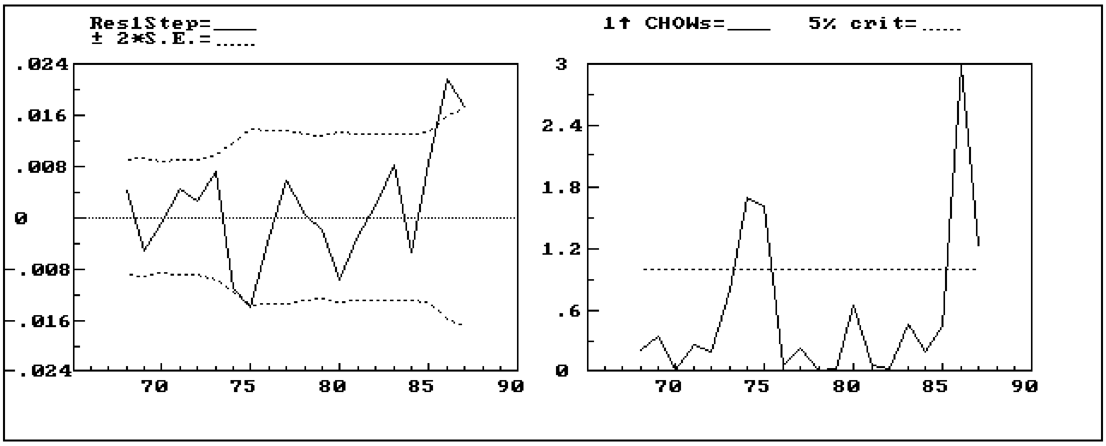

Figure A1.2: One-step residuals and recursive Chow test (annual DHSY model)

APPENDIX II: Dynamic regression results

Modelling c_t by OLS: (A1)

The present sample is: 57 to 92 less 5 forecasts

The forecast period is: 88 to 92

Variable	Coefficient	Std.Error	t-value	PartR²
Constant	-0.038612	0.14349	-0.269	0.0029
c_{t-1}	1.4594	0.21884	6.669	0.6402
c_{t-2}	-0.43972	0.27893	-1.576	0.0904
y_t	0.55301	0.080110	6.903	0.6559
y_{t-1}	-0.80992	0.14433	-5.611	0.5574
y_{t-2}	0.24642	0.18336	1.344	0.0674

R^2 = 0.998369 F(5, 25) = 3059.9 [0.0000] $\hat{\sigma}$ = 0.00886972 DW = 1.79

RSS = 0.001966800013 for 6 variables and 31 observations

Tests of parameter constancy over 1988-92:

Forecast Chi²(5)/5 = 3.1807

Chow F(5, 25) = 1.0333 [0.4199]

LM test for residual autocorrelation from lags 1 to 2:

CHI²(2) = 2.7259 and F-Form(2, 23) = 1.1087 [0.3470]

Normality test:

Mean	0.000000
Std.Devn.	0.912871
Skewness	0.111031
Excess Kurtosis	-0.022547
Minimum	-1.669175
Maximum	2.373364

Normality Chi²(2) = 0.051896

(Simplified) White test for heteroscedastic errors:

CHI²(10) = 9.8189 and F-Form(10, 14) = 0.649 [0.7514]

(Original) White test of functional form:

$CHI^2(20) = 21.946$ and F-Form(20, 4) = 0.48479 [0.8760]

RESET test for adding $\hat{}Y^2$:

RESET F(1, 24) = 4.1442 [0.0530]

Solved static long run equation (standard errors in parentheses): (A2)

$$c_t = \underset{(14.55)}{1.959} - \underset{(308.9)}{28.06}*y_t + \underset{(448.5)}{41.09}*y_{t-1} - \underset{(143.1)}{12.5}*y_{t-2}$$

WALD test $Chi^2(3) = 5.8556$

APPENDIX III

Quarterly data

C_t - Non-durable consumption (pounds in million)

Y_t - Personal disposable income (pounds in million)

RPI_t - Retail price index

	C_t	Y_t	RPI_t
55 - 1	32058	34000	11.3
55 - 2	34034	36473	11.5
55 - 3	34878	36185	11.6
55 - 4	36166	36006	11.8
56 - 1	32614	35270	11.9
56 - 2	34360	36881	12.2
56 - 3	34666	36693	12.1
56 - 4	36465	37299	12.3
57 - 1	32767	35506	12.4
57 - 2	35241	37881	12.5
57 - 3	35598	37099	12.7
57 - 4	37398	38024	12.8
58 - 1	33964	36621	12.9
58 - 2	35816	38106	13
58 - 3	36324	37840	12.9
58 - 4	38510	38406	13.1
59 - 1	34901	37348	13.1
59 - 2	37756	40488	13
59 - 3	37927	40023	13
59 - 4	40329	40879	13.1
60 - 1	37043	39746	13.1
60 - 2	39429	42895	13.1
60 - 3	39365	42706	13.1
60 - 4	40898	43852	13.3
61 - 1	37850	41997	13.4
61 - 2	40292	44424	13.5
61 - 3	40352	44941	13.7
61 - 4	41705	44894	13.9

62 - 1	38438	42411	14
62 - 2	41410	44414	14.3
62 - 3	41143	45132	14.2
62 - 4	42934	46329	14.2
63 - 1	39559	43537	14.4
63 - 2	42967	46009	14.5
63 - 3	43707	47608	14.4
63 - 4	44641	48272	14.5
64 - 1	41500	46380	14.6
64 - 2	44122	48131	14.9
64 - 3	44597	48859	15.1
64 - 4	45825	49877	15.2
65 - 1	42306	47735	15.3
65 - 2	44524	49105	15.7
65 - 3	45346	49440	15.8
65 - 4	46317	50718	15.9
66 - 1	43296	51094	16
66 - 2	46064	49924	16.3
66 - 3	45710	49462	16.4
66 - 4	46480	50727	16.5
67 - 1	43327	48900	16.6
67 - 2	46217	51025	16.7
67 - 3	47378	51899	16.6
67 - 4	49063	52347	16.8
68 - 1	46489	51895	17.1
68 - 2	46729	52332	17.4
68 - 3	48066	51369	17.6
68 - 4	49925	52176	17.8
69 - 1	45467	51750	18.1
69 - 2	47682	52411	18.4
69 - 3	48494	52139	18.5
69 - 4	50723	53384	18.7
70 - 1	46228	52180	19
70 - 2	48922	54774	19.5
70 - 3	50435	54814	19.7
70 - 4	52288	55907	20.1
71 - 1	47254	52680	20.6

71 - 2	50353	55011	21.4
71 - 3	52035	55422	21.7
71 - 4	54497	57231	22
72 - 1	50173	55665	22.3
72 - 2	53405	60776	22.7
72 - 3	55138	59748	23.1
72 - 4	58036	62555	23.7
73 - 1	54685	60741	24.1
73 - 2	56166	64388	24.9
73 - 3	57861	64355	25.2
73 - 4	59903	64845	26.1
74 - 1	53065	61683	27.2
74 - 2	55198	61929	28.8
74 - 3	57026	64359	29.5
74 - 4	60028	64389	30.8
75 - 1	53859	64634	32.7
75 - 2	55667	62971	35.8
75 - 3	56462	63599	37.3
75 - 4	58592	62610	38.6
76 - 1	53509	62457	40
76 - 2	55260	62657	41.5
76 - 3	57050	65244	42.5
76 - 4	59847	62654	44.4
77 - 1	53426	59836	46.7
77 - 2	54469	60748	48.7
77 - 3	56523	62293	49.5
77 - 4	60474	64818	50.2
78 - 1	56839	63696	51.1
78 - 2	57523	66206	52.4
78 - 3	60163	67571	53.4
78 - 4	62384	68452	54.3
79 - 1	58697	67670	56
79 - 2	62282	69552	8
79 - 3	61533	69617	61.9
79 - 4	64700	74245	63.6
80 - 1	61087	70661	66.7
80 - 2	59936	70901	70.5

80 - 3	62511	72051	72
80 - 4	63651	71798	73.4
81 - 1	60104	71891	75.1
81 - 2	60409	70604	78.8
81 - 3	62403	70129	80.2
81 - 4	64486	70552	82.1
82 - 1	59964	70098	83.5
82 - 2	60271	70096	86.2
82 - 3	63352	70640	86.5
82 - 4	66265	70888	87.2
83 - 1	62549	70836	87.6
83 - 2	63135	72193	89.4
83 - 3	66658	72648	90.6
83 - 4	68858	73527	91.6
84 - 1	63958	73532	92.2
84 - 2	65077	74662	94
84 - 3	66738	74369	94.9
84 - 4	70713	77193	96
85 - 1	65886	75454	97.2
85 - 2	66822	78173	100.6
85 - 3	70092	77299	100.8
85 - 4	73942	78895	101.3
86 - 1	69440	78866	102
86 - 2	72178	81538	103.3
86 - 3	75841	81517	103.5
86 - 4	78163	81821	104.8
87 - 1	72812	80932	106
87 - 2	75138	83413	107.7
87 - 3	80160	84473	107.9
87 - 4	83124	86063	109.1
88 - 1	78435	85583	109.6
88 - 2	80564	88498	112.3
88 - 3	86775	89421	113.8
88 - 4	88817	91427	116.2
89 - 1	82250	90555	118.1
89 - 2	84317	92989	121.5
89 - 3	88548	94053	122.6

89 - 4	90291	94759	125
90 - 1	83417	93895	127.2
90 - 2	85222	94260	133.2
90 - 3	88806	95668	135.4
90 - 4	90082	96269	137.5
91 - 1	82263	93203	138.3
91 - 2	82616	94614	141.2
91 - 3	86463	94505	141.8
91 - 4	88651	95867	143.2
92 - 1	80655	93993	144
92 - 2	82743	97369	147.1
92 - 3	86901	98176	147
92 - 4	89642	99051	147.6
93 - 1	82348	95117	146.6
93 - 2	83967	96051	148.9

APPENDIX IV

Annual data

	Y_t	P_t%	C_t
55 - 1	112140	4.5	103150
56 - 1	115090	4.3	104670
57 - 1	116930	4.1	106430
58 - 1	118690	3.2	108360
59 - 1	124750	0	112250
60 - 1	132960	1.5	116630
61 - 1	138520	3	119620
62 - 1	140020	4.4	122150
63 - 1	146060	2.1	126960
64 - 1	152270	3.4	130460
65 - 1	155440	4.7	132630
66 - 1	158870	3.8	13530
67 - 1	161240	2.5	138310
68 - 1	164060	4.8	141810
69 - 1	165580	5.1	143330
70 - 1	172090	6.5	147040
71 - 1	174320	9.2	150330
72 - 1	188990	7.5	158030
73 - 1	201020	9.1	166420
74 - 1	199430	15.9	165580
75 - 1	200420	24.2	164650
76 - 1	19970	16.5	16480
77 - 1	195330	15.8	164950
78 - 1	210310	8.3	173170
79 - 1	22230	13.4	17950
80 - 1	225880	18	180410
81 - 1	224150	11.9	18030
82 - 1	223040	8.6	181480
83 - 1	228950	4.6	187480
84 - 1	236930	5	19170
85 - 1	244820	6.1	198780
86 - 1	254850	3.4	21090

87 - 1	263810	4.2	221740
88 - 1	279690	4.9	236610
89 - 1	292350	7.8	243970
90 - 1	29960	9.4	247180
91 - 1	298150	5.9	244180
92 - 1	304940	3.8	244560