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Class " EET (2nd Smst)

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(Exercise 9.2)

Solve:-

Question No 1

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln x(1+x^3) + c$$

$$\frac{3y^2}{2} = \ln x\,(1+x^3) + 3c$$

$$\frac{3y^2}{2} = \ln x\,(1+x^3) + 3c$$

$$3y^2 = 2\ln x\,(1+x^3) + 6c$$

$$3y^2 = 2\ln x\,(1+x^3) + c$$

Question No 2

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x\,dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$\frac{-1}{y} = \cos x + C$$

Question No 3

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1} y = 2x + x^2 + C$$

Question No 4

$$(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$$

$$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$$

$$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

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$$\frac{x+1}{x(x+2x)}dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln x(y+2) = -\frac{1}{2}\ln x(x^2+2x) + \ln x$$

$$y+2 = \frac{c}{\sqrt{x^2+2x}}$$

Question No 5

$$\frac{dy}{dx} = 2x^2 x\cdot y - x^2 y + xy - 2x\cdot 2$$

$$= 2x^2 - 2x - 2 + y - x^2 y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-6\ln|2-y| = 2x^3 - 3x^2 - 6x + 6c$$

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$\ln|2-y| = (2x^3 - 3x^2 - 6x + 6c)\, lx$

$-6 \qquad 2x^3 - 3x^2 - 6x + 6c$

$\ln|2-y| = \ln e.$

$-6 \qquad 2x^3 - 3x^2 - 6x$

$|2-y| = e \qquad .e$

$|2-y|^{-6} = 2x^3 - 3x^2 - 6x$

$\longleftrightarrow$

Question No 6

$\operatorname{cosec} y \, dx + \sec x \, dy = 0$

$\div$ by $\operatorname{cosec} y \sec x$

$$\frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\Rightarrow \int \cos x \, dx + \int \sin y \, dy = \int 0 \, dx$$

$$\Rightarrow \sin x - \cos y = c$$

$\longleftrightarrow$

Question No 7

$y(1+x)dx + x(1+y)dy = 0 \quad | \div \text{by } xy$

$$\frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy = 0$$

$$\int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy = \int 0 \, dx$$

$$\Rightarrow \ln x + x \ln y + y$$

$$x + y + \ln(xy) = c$$

$\longleftrightarrow$

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Question No 8

$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$$\int \frac{\sqrt{1+x^2}}{x}\,dx + \int \frac{\sqrt{1+y^2}}{y}\,dy = \int 0\,dx$$

put $\sqrt{1+x^2} = t$ put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $y\,dy = z\,dz$

Therefore

$$\int \frac{\sqrt{1+x^2}}{x^2}\,x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}\,y\,dy = \int 0\,dx$$

$$= \int \frac{t \cdot t\,dt}{t^2-1} + \int \frac{z \cdot z\,dx}{z^2-1} = c$$

$$\int \left(\frac{t^2-1+1}{t^2-1}\right) dt + \int \frac{z^2-1+1}{z^2-1}\,dz = c$$

$$\int \left(1 + \frac{1}{t^2-1}\right) dt + \int \left(1 + \frac{1}{z^2-1}\right) dz = c$$

$$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$$

$$\sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + \sqrt{1+y^2} + \frac{1}{2}$$

$$\Rightarrow \ln\left(\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right)$$

(Exercise 9.3)

Question No 1

$(x-y)\,dx+(x+y)\,dy=0$

$(x+y)\,dy=-(x-y)\,dx$

$\frac{dy}{dx}=\frac{y-x}{x+y}$ —— (1)

Put $y=vx$ —— (2)

$\frac{dy}{dx}=v+x\frac{dv}{dx}$ —— (3)

$v+x\frac{dv}{dx}=\frac{vx-x}{x+vx}$

$x\frac{dv}{dx}=\frac{\cancel{x}(v-1)}{\cancel{x}(1+v)}-v$

$=\frac{\cancel{v}-1-\cancel{v}-v^2}{1+v}$

$x\frac{dy}{dx}=-\frac{(v^2+1)}{1+v}$

$\int\frac{v+1}{v^2+1}\,dv=-\int\frac{dx}{x}$

$\frac{1}{2}\int\frac{2v\,dv}{v^2+1}+\int\frac{dv}{v^2+1}=-\int\frac{dx}{x}$

$\frac{1}{2}\ln(v^2+1)+\tan^{-1}v=-\ln x+c$

$\ln(v^2+1)^{1/2}+\tan^{-1}v+\ln x=c$

$\ln\sqrt{\frac{y^2}{x^2}+1}+\tan^{-1}\left(\frac{y}{x}\right)+\ln x=c$

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$$\ln x\sqrt{y^2+x^2} - \ln x\sqrt{x^2} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$$

$$\ln\sqrt{y^2+x^2} + \tan^{-1}\left(\frac{y}{x}\right) = c$$

$\longleftrightarrow$

Question No 2

$$(y^2+2xy)\,dx + x^2\,dy = 0$$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = -\frac{(y^2+2xy)}{x^2}$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

using (ii) & (iii) in (1)

$$v + x\frac{dv}{dx} = -\left(\frac{v^2x^2 + 2x\,vx}{x^2}\right)$$

$$x\frac{dv}{dx} = -\cancel{x^2}\frac{(v^2+2x)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = -v^2+3v$$

$$\int\frac{dv}{v^2+3v} = -\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\ln x\,v - \frac{1}{3}\ln x\,(x+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$

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$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{1/3} = c\,(v+3)^{1/3}$$

$$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x\,\frac{y^{1/3}}{\cancel{x^{1/3}}}\,\cancel{x^{1/3}} = c\,(y+3x)^{1/3}$$

$$x\,y^{1/3} = c\,(y+3x)^{1/3}$$

$$x^3 y = c\,(y+3x)$$

$\longleftrightarrow$

Question No 3

$$(x^2-3y^2)\,dx + 2xy\,dy = 0$$

$$2xy\,dy = -(x^2-3y^2)\,dx$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{—(1)}$$

Put $y = vx$ —(11)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow \text{(111)}$$

using (11) of (111) in (1)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2x\,vx}$$

$$x\frac{dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\,\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

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$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$$

$$\frac{y^2-x^2}{x^2} = cx$$

$$y^2-x^2 = (cx)x^2$$

$\longleftrightarrow$

Question No 4

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{— (i)}$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (iii)}$$

using (ii) (iii) In (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2x\,vx}$$

$$x\frac{dv}{dx} = \frac{x^2(1+3v^2)}{x^2\,2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

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$$\int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln cx$$

$$\left(1+\frac{y^2}{x^2}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$x^2+y^2 = (cx)x^2$$

Question No 5

$$(x^2+xy+y^2)\,dx - x^2\,dy = 0$$

$$(x^2+xy+y^2)\,dx = x^2\,dy$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{--- (i)}$$

Put $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (iii)}$$

using (2) (3) in (1)

$$v + x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{(1+v+v^2)x^2}{x^2} - v$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{v}$$

$$\tan^{-1}v = \ln x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$$

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Question No 6

$(x^2+3xy+y^2)dx - x^2dy = 0$

$(x^2+3xy+y^2)dx = x^2dy$

$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$ — (i)

put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ (iii)

using (ii) (iii) in (i)

$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \cancel{x^2}\left(\frac{1+3v+v^2}{\cancel{x^2}}\right) - v$

$x\frac{dv}{dx} = 1+2v+v^2$

$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$

$\left(-\frac{1}{v+1}\right) = \ln x + c$

$\left(\frac{-1}{\frac{y}{x}+1}\right) = \ln x + c$

$-\frac{1}{\frac{y+x}{x}} = \ln x + c$

$\frac{-x}{(x+y)} = \ln x + c$

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Question No (7)

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y} \quad \text{—} (i)$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad (iii)$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{4vx-3x}{2x-vx}$$

$$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$$

$$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x} \quad \text{—} (iv)$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$$2-v = A(v-1) + B(v+3)$$

Put $v+3=0 \Rightarrow +5 = -4A \Rightarrow \boxed{A = \frac{-5}{4}}$

Put $v-1=0 \Rightarrow 1 = 4B = \boxed{B = \frac{1}{4}}$

$$\frac{2-v}{(v+3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v-1)}$$

$$-\frac{5}{4}\int\frac{dv}{v+3} + \frac{1}{4}\int\frac{dv}{v-1} \int\frac{dx}{x}$$

$$-\frac{5}{4}\ln(v+3)+\frac{1}{4}\ln(v-1)=\ln x+\ln c$$

$$-\ln(v+3)^5+\ln(v-1)=4\ln cx$$

$$\ln\frac{(v-1)}{(v+3)^5}=\ln c^4x^4$$

Antilog

$$\frac{(y/x-1)}{(y/x+3)^5}=c^4x^4$$

$$\frac{(y-x)\,x^5}{x(y+3x)x^4}=c'$$

$$\frac{y-x}{(y+3x)^5}=c'$$

$\longleftrightarrow$

Question No 8

$$x\sin(y/x)dy=(y\sin y/x-x)dx$$

$$\frac{dy}{dx}=\frac{y\sin y/x-x}{x\sin(y/x)} \quad \text{—(i)}$$

Put $y=vx$ (ii)

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad \text{—(iii)}$$

using (ii) (iii) in (i)

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$$v + x\frac{dv}{dx} = \frac{vx \sin\frac{vx}{x} - v}{x\sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x}\sin v} - v$$

$$x\frac{dv}{dx} = \frac{v\cancel{\sin v} - 1 - v\cancel{\sin v}}{\sin v}$$

$$\int \sin v\, dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x - c$$

$$\cos\frac{y}{x} = \ln c - c$$

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(Non Homogenious Equation)

Question No ①

$$\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$$

Put $x = x+h$

$$\frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x+h+3(y+k)-5}{x+h-(y+k)-1}$$

$$\frac{dy}{dx} = \frac{x+3y}{x-y}$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x+3vx}{x-vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(1+3v)}{\cancel{x}(1-v)}$$

$$x\frac{dv}{dx} = \frac{(1-v)^2}{(1-v)}$$

$$\int \frac{1-v}{(1-v)^2}\,dv = \int \frac{dx}{x}$$

$$\int \frac{1\,dv}{(1+v)^2} - \int \frac{v\,dv}{(1+v)^2} = \int \frac{dx}{x}$$

$\longleftrightarrow$

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Question No 2

$$\frac{dy}{dx} = -\frac{(4x+3y+15)}{2x+y+7}$$

Put $x = x+h$

$y = y+k$

$$\frac{dy}{dx} = \frac{-(4x+4h+3y+3k+15)}{2x+2h+y+k+7}$$

$$\frac{dy}{dx} = -\frac{4x-3y}{2x+y} \quad \text{--- (i)}$$

Put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

using (i) (ii) in (iii)

$$v + x\frac{dv}{dx} = \frac{-4x-3vx}{2x+vx}$$

$$x\frac{dv}{dx} = \frac{-\cancel{x}(4+3v)}{\cancel{x}(2+v)} - v$$

$$= \frac{-4-3v-2v-v^2}{2+v}$$

$$x\frac{dv}{dx} = \frac{-(v^2+5v+4)}{2+v}$$

$$\int \frac{(v+2)\,dv}{v^2+5v+4} = \int -\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{dv}{v+1} + \frac{2}{3}\int\frac{dv}{v+4} = -\int\frac{dx}{x}$$

$$\frac{1}{3}\ln(v+1) + \frac{2}{3}\ln(v+4) = -\ln x + \ln c$$

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Question No ③

$$(3y-7x-3)dx+(7y-3x-7)dy=0$$

$$(7y-3x-7)dy=-(3y-7x-3)dx$$

$$\frac{dy}{dx}=\frac{-3y+7x+3}{7y-3x-1}$$

$$\left.\begin{aligned}\text{put } x&=x+h\\ y&=y+k\end{aligned}\right]=\frac{dy}{dx}=\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx}=\frac{-3(y+k)+7(x+h)+3}{7(y+k)-3(x+h)-7}$$

$$\frac{dy}{dx}=\frac{-3y+7x}{7y-3x} \quad - (1)$$

put $y=vx$ — (ii)

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad - (iii)$$

using (ii) (iii) in (1)

$$v+x\frac{dv}{dx}=-\frac{3vx+7x}{7vx-3x}$$

$$x\frac{dv}{dx}=\frac{\cancel{x}(-3v+7)}{\cancel{x}(7v-3)}-v$$

$$x\frac{dv}{dx}=\frac{-\cancel{3v}+7-7v^2+\cancel{3v}}{7v-3}$$

$$\int\frac{7v-3}{7(1-v^2)}dv=\int\frac{dx}{x}$$

$$\frac{2}{7}\int\frac{dv}{1-v}-\frac{5}{7}\int\frac{dv}{1+v}=\int\frac{dx}{x}$$

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$-\frac{2}{7}\ln(1-v)-\frac{5}{7}\ln(1+v)=\ln x+\ln c$

$\frac{1}{7}\ln\left[\left(1-\frac{y}{x}\right)^{-2}\left(1+\frac{y}{x}\right)^{-5}\right]=7\ln cx$

$\ln\left[\left(\frac{x-y}{x}\right)^{-2}\left(\frac{x+y}{x}\right)^{-5}\right]=\ln c^7x^7$

Question No 4

$$\frac{x-2y+5}{2x+y-1}$$

put $x=x+h$

$y=Y+k$

$\therefore \frac{dy}{dx}=\frac{x+h-2y-2k+5}{2x+2h+Y+k-1}$

$\frac{dy}{dx}=\frac{x-2y}{2x+y}$ — (1)

put $y=vx$ — (11)

$\frac{dy}{dx}=v+x\frac{dv}{dx}$ — (iii)

$v+x\frac{dv}{dx}=\frac{x-2vx}{2x+vx}$

$x\frac{dv}{dx}=\frac{\cancel{x}(x-2v)}{\cancel{x}(2+v)}-v$

$x\frac{dv}{dx}=\frac{1-2v-2v-v^2}{2+v}$

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$$\int \frac{2+v}{1-4v-v^2}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\left(\ln(v^2+4v-1) = -\ln x + \ln c\right.$$

$$\ln\sqrt{v^2+4v-1} = \ln\frac{c}{x}$$

$\longleftrightarrow$

Question No 5

$$\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3} \quad \text{--- (i)}$$

put $3x-4y = z$ --- (ii)

$$3-4\frac{dy}{dx} = \frac{dz}{dx}$$

$$3-\frac{dz}{dx} = 4\frac{dy}{dx}$$

$$\frac{1}{4}\left(3-\frac{dz}{dx}\right) = \frac{dy}{dx} \quad \text{--- (iii)}$$

using (ii) (iii) in (i)

$$\frac{1}{4}\left(3-\frac{dz}{dx}\right) = \frac{z-2}{z-3}$$

$$3-\frac{dz}{dx} = \frac{4z-8}{z-3}$$

$$3-\frac{(4z-8)}{z-3} = \frac{dz}{dx}$$

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$$\frac{3z-9-4z+8}{z-3} = \frac{dz}{dx}$$

$$\frac{-(1+z)}{z-3} = \frac{dz}{dx}$$

$$-dx = \frac{z-3}{1+z}\,dz$$

$$\frac{z-3-1+1}{1+z} = -dz$$

Question No 6

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5} \quad \text{— (i)}$$

put $y-x = z$ — (ii)

$$\frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{dz}{dx} \quad \text{— (iii)}$$

using (ii) + (iii) in (i)

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

$$= \frac{z+1-z-5}{z+5}$$

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$$\frac{dz}{dx} = \frac{-4}{z+5}$$

$$\int (z+5)\,dz = -4\int dx$$

$$\frac{z^2}{2} + 5z = -4x + c$$

$$\frac{z^2+10z}{2} = -4x + C$$

$$z^2 + 10z = -8x + 2C$$

$$(y-x) + 10(y-x) = -8x + c$$

(Exact Equation)

$$(3x^2+4xy)\,dx + (2x^2+2y)\,dy = 0$$

$$M = 3x^2 + 4xy, \quad N = 2x^2 + 2y$$

$$\frac{\partial M}{\partial y} = 0 + 4x \qquad \frac{\partial N}{\partial x} = 4x + 0$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Now $\int M\,dx + \int (\text{terms of } N \text{ form } x)\,dy = c$

$$\int (3x^2+4xy)\,dx + \int 2y\,dy = c$$

$$3\frac{x^3}{3} + \frac{4x^2y}{2} + \frac{2y^2}{2} = c$$

$$x^3 + 2x^2y + y^2 = c$$

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Question No 2

$(2xy + y\tan y)dx + (x^2 - x\tan y + \sec^2 y)dy = 0$

$M = 2xy + y - \tan y$, $N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y$, $\frac{\partial N}{\partial x} = 2x - \tan^2 y = 0$

$= 2x - \tan^2 y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N \text{ force form}) dy = c$

$\int (2xy + y - \tan y\, dx + \int \sec^2 y\, dy = c$

$\frac{2x^2 y}{2} + xy - x\tan y + \tan y = c$

$x^2 y + xy - x\tan y + \tan y = c$

$\longleftrightarrow$

Question No 3

$\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$M = \frac{x+y}{y-1}$ $\quad N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1) - (x+y)(1)}{(y-1)^2}$ $\quad N = \frac{1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$

$\frac{\cancel{y} - 1 - x - \cancel{y}}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$

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$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \because \text{given} \ldots$$

$$\int M\,dx + \int (\text{terms of } N \ldots x)\,dy = c$$

$$\int \left(\frac{x+y}{y-1}\right) dx + \int -\frac{1}{2} \frac{dy}{(y-1)^2} = c$$

$$\left(\frac{1}{y-1}\right) \int (x+y)\,dx + \frac{-1}{2} \int (y-1)^{-2}\,dy = c$$

$$\frac{1}{(y-1)} \left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2 + 2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2 + 2xy + 1 = c\,(y-1)$$

Question No (4)

$$\frac{dy}{dx} = -\frac{(ax+hy)}{(hx+by)}$$

$$(hx+by)\,dy = -(ax+hy)\,dx$$

$$(ax+hy)\,dx + (hx + by)\,dy = 0$$

$$M = ax + hy \qquad N = hx + by$$

$$\frac{\partial M}{\partial y} = 0 + h \qquad \frac{\partial N}{\partial x} = h$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

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$\int M\,dx + \int (\text{terms of } N \ldots$

$\int (ax + hy)\,dx + \int by\,dy = c$

$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c$

Question No 5

$(1 + \ln xy)\,dx + (1 + \frac{x}{y})\,dy = 0$

$M = 1 + \ln xy \qquad N = 1 + \frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy} \cdot x \qquad \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\int M\,dx + \int (\text{terms of } N \text{ force from } x)\,dy = c$

$\int (1 + \ln xy)\,dx + \int 1 \cdot dy = c$

$\int dx + \int 1 \ln xy\,dx + \int dy = c$

$x + \left[\ln xy \cdot (x) - \int \frac{1}{xy} \cdot y \cdot x\,dx\right] + y = c$

$x + \ln xy - \int dx + y = c$

$\cancel{x} + x\ln xy - \cancel{x} + y = c$

$x\ln xy + y = c$

Question No ⑥

$$\frac{y\,dx + x\,dy}{1-x^2y^2} + x\,dx = 0$$

$$\frac{y\,dx}{1-x^2y^2} + \frac{x\,dy}{1-x^2y^2} + x\,dy = 0$$

$$\left(\frac{x+y}{1-x^2y^2}\right)dx + x\frac{dy}{1-x^2y^2} = 0$$

$$M = \frac{x+y}{1-x^2y^2}$$

$$\frac{\partial M}{\partial y} = \frac{0+(1-x^2y^2).1-y(-2x^2y)}{(1-x^2y^2)^2}$$

$$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{1-x^2y^2}$$

$$N = \frac{x}{1-x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2)-x(-2xy^2)}{(1-x^2y^2)^2}$$

$$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int(\text{terms of } N \ldots x)\,dy = c$$

$$\int\left(x + \frac{y}{1-x^2y^2}\right)dx + \text{Nil} = c$$

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$$\int x\,dx + \int \frac{y\,dx}{1-x^2y^2} = c$$

$$\frac{x^2}{2} + \int \frac{y/y^2}{\frac{1}{y^2} - \frac{x^2y^2}{y^2}}\,dx = c$$

$$\frac{x^2}{2} + \frac{1}{y}\int \frac{dx}{\left(\frac{1}{y}\right)^2 - x^2} = c$$

$$\frac{x^2}{2} + \frac{1}{y}\left[\frac{1}{2}\left(\frac{1}{y}\right)\ln\left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right] = c$$

$$\frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| = c$$

$$x^2 + \ln\left|\frac{1+xy}{1-xy}\right| = c$$

Question No 7

$$(6xy+2y^2-5)dx + (3x^2+4xy-6) = 0$$

$M = 6xy + 2y^2 - 5$ $\quad N = 3x^2 + 4xy - 6$

$\frac{\partial M}{\partial y} = 6x + 4y$, $\frac{\partial N}{\partial x} = 6x + 4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact Difficiency

$\int M\,dx + \int(\text{terms of } N \text{ force from } x)\,dy = c$

$$\int(6xy + 2y^2 - 5)\,dx + \int -6\,dy = c$$

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$$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$$

$$3x^2y + 2xy^2 - 5x - 6y = c$$

$\longleftrightarrow$

Question No 8

$$(y\sec^2x + \sec x\tan x)dx + (\tan x + 2y)dy = 0$$

$M = y\sec^2x + \sec x\tan x$, $N = \tan x + 2y$

$$\frac{\partial M}{\partial y} = \sec^2x \qquad \frac{\partial N}{\partial x} = \sec^2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int(\text{terms of } N \ldots)dy = c$$

$$\int(y\sec^2x + \sec x\tan x)\,dx + \int 2y\,dy = c$$

$$y\tan x + \sec x + y^2 = c$$

$\longleftrightarrow$

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(Exercise 9.5)

Q Non Exact Equation

Question No 1

$$(xy^2+y)\,dx - x\,dy = 0 \quad \text{---(i)}$$

$M = xy^2 + y \qquad N = -x$

$M_y = 2xy + 1 \qquad N_x = -1$

$\because M_y \neq N_x \qquad \therefore$ Non Exact

$$\frac{M_y - N_x}{N} = \frac{2xy+1+1}{-x}$$

$$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$$

$$= \frac{-2(1+xy)}{y(xy+1)} = \frac{-2}{y}$$

$-\int \frac{2}{y}\,dy - 2\ln y$

$$\frac{1}{y}(xy^2+y)\,dx - \frac{x}{y^2}\,dy = 0$$

$$\left(x + \frac{1}{y}\right)dx - \frac{x}{y^2}\,dy = 0 \quad \text{---(ii)}$$

Now $M = x + \frac{1}{y} \qquad N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2} \qquad N_x = -\frac{1}{y^2}$

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$\therefore M_y = N_x$

$\int M dx + \int (\text{term of } N \ldots) dy = c$

$\int (x + \frac{1}{y}) dx + Nil = c$

$\frac{x^2}{2} + \frac{x}{y} = c$

Question No 2

$(x^2 + x - y) dx + x\, dy = 0$ — ①

$M = x^2 + x - y$ $\quad N = x$

$M_y = -1$ $\quad N_x = 1$

$M_y \neq N_x$ $\quad \therefore$ Non Exact

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$

$\int \frac{-2}{x} dx \quad -2 \ln x \quad \ln x^{-2}$

Multiply both sides of Equation ①

$\frac{1}{x^2}(x^2 + x - y) dx + \frac{x}{x^2} dy = 0$

$(1 + \frac{1}{x} - \frac{y}{x^2}) dx + \frac{1}{x} dy = 0$ — ⑪

Now $M = 1 + \frac{1}{x} - \frac{y}{x^2}$. $N = \frac{1}{x}$

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$M_y = \frac{-1}{x^2}$ $N_x = \frac{-1}{x^2}$

$M_y = N_x$

So

$\int M\,dx + \int(\text{terms of } N \text{ from } x)\,dy = c$

$\int(1 + \frac{1}{x} - \frac{y}{x^2})\,dx + Nil = c$

$x + \ln x + \frac{y}{x} = c$

$\longleftrightarrow$

Question No 3

$dy + \left(\frac{y - \sin x}{x}\right)dx = 0$ — (1)

$M = \frac{y - \sin x}{x}$ $N = 1$

$M_y = \frac{1}{x} - 0$ $N_x = 0$

$M_y \neq N_x$ ∴ (1) is Non Exact ---

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$

Multiply in Both sides

$x\,dy + x\left(\frac{y - \sin x}{x}\right)dx = 0$

$M = y - \sin x$ $N = x$

$M_y = 1$ $N_x = 1$

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$M_y = N_x \quad \therefore$ (11) is Exact

$\int (y - \sin x)\, dx = c$

$xy + \cos x = c$

$\because M_y = N_x \quad \therefore$ (11) is Exact

$\int M\, dx + \int (\text{terms of } N \ldots x)\, dy = c$

$\int \left(y + \frac{2}{y^2}\right) dx + \int 2y\, dy = c$

$xy + \frac{2x}{y^2} + \frac{2y^2}{\cancel{2}} = c$

$xy + \frac{2x}{y^2} + y^2 = c$

$\longleftrightarrow$

Question No (4)

$y(2xy + e^x)\, dx - e^x dy = 0$

$(2xy^2 + e^x y)\, dx - e^x dy = 0$

$M = 2xy^2 + e^x y \qquad N = -e^x$

$M_y = 4xy + e^x \qquad N_x = -e^x$

$\frac{M_y - N_x}{N} = \frac{4xy + e^x + e^x}{-e^x}$

$\frac{N_x - M_y}{M} = \frac{-e^x - 4xy - e^x}{2xy^2 + ye^x} = \frac{-2e^x - 4xy}{y(2xy + e^x)}$

$= \frac{-2\cancel{(e^x + 2xy)}}{y\cancel{(2xy + e^x)}} = \frac{-2}{y}$

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Multiply Both sides

$\frac{1}{y^2}(2xy^2+e^x y)dx - \frac{1}{y^2}e^x dy = 0$

$(2x + \frac{e^x}{x})dx - \frac{e^x}{y^2}dy = 0$ — (11)

$M = 2x + \frac{e^x}{y}$ $\quad N = \frac{-e^x}{y^2}$

$M_y = 0 + \left(\frac{-e^x}{y^2}\right)$ $\quad N_x = \frac{-e^x}{y^2}$

$M_y = N_x$

$\int M dx + \int(\tan \text{ of } N \text{ from } x)dy = c$

$\int(2x + \frac{e^x}{y})dx + Nil = c$

$x + \frac{e^x}{y} = c$

Question No (5)

$(y^4+2y)dx + (xy^3+2y^4-4x)dx = 0$

$M = y^4 + 2y$ $\quad N = xy^3 + 2y^4 - 4x$

$M_y = 4y^3 + 2$ $\quad N_x = y^3 - 4$

$M_y \neq N_x$

$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3+2)} = \frac{-3}{y}$

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$\int \frac{-3}{y} dy - 3 \ln y \qquad \ln y^{-3} - 3$

I.F $= e \quad = e \quad = e \quad = y$

$\frac{1}{y^3}(y^4 + 2y) dx + \frac{1}{y^3}(xy^3 + 2y^4 - 4x) dy = 0$

$(y + \frac{2}{y^2}) dx + (x + 2y - \frac{4x}{y^3}) dy = 0$

Now $M = y + \frac{2}{y^2} \qquad N = x + 2y - \frac{4x}{y^2}$

$M_y = 1 - \frac{4}{y^3} \qquad N_x = 1 + 0 - \frac{4}{y^3}$

$\int M dx + \int$ tan of $N \ldots) dy = c$

$\int (y + \frac{2}{y^2}) dx \int 2y dy = c$

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$

$xy + \frac{2x}{y^2} + y^2 = c$

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Question No (6)

$(x^2+y^2+2x)dx+2y\,dy=0 \quad -(1)$

$M=x^2+y^2+2x \qquad N=2y$

$M_y=2y \qquad N_x=0$

$M_y \neq N_x \quad \therefore (1)$ is Non Ex....

$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2y}$

$\frac{M_y - N_x}{N} = \frac{2y-0}{2y}$

Multiply Both Sides

$e^x(x^2+y^2+2x)dx + e^x(2y)dy=0 \quad -(1)$

$M=e^x(x^2+y^2+2x) \qquad N=e^x 2y$

$M_y=e^x 2y \qquad N_x=e^x 2y$

$M_y=N_x \quad \therefore (ii)$ is

$\int e^x(x^2+y^2+2x)dx + Nil = c$

$\int e^x x^2\,dx + \int e^x y^2\,dx + \int e^x 2x\,dx = c$

$x^2e^x - \int 2xe^x\,dx + e^x y^2 + \int e^x 2x\,d = c$

$(x^2+y^2)e^x = c$

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Question No 7

$(x^2+y^2)dx - 2xy\,dy = 0$ — (1)

$M = x^2+y^2$ $\quad N = -2xy$

$M_y = 2y$ $\quad N_x = -2y$

$M_y \neq N_x$

$\frac{N_x - M_y}{M} = \frac{-2y-2y}{x^2+y^2}$

$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = \frac{-2}{x}$

Multiply Both sides

$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$

$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$

$M = 1+\frac{y^2}{x^2}$ $\quad N = \frac{-2y}{x}$

$M_y = \frac{2y}{x^2}$ $\quad N_x = +\frac{2y}{x^2}$

$M_y = N_x$

$\therefore \int M\,dx + \int(\text{term of} \dots x)\,dy = c$

$\int\left(1+\frac{y^2}{x^2}\right)dx + \text{Nil} = c$

$x - \frac{y^2}{x} = C$

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Question No (8)

$(4x + 3y^2)dx + 2xy\,dy = 0$ — (1)

$M = 4x + 3y^2$ $\quad N = 2xy$

$M_y = 0 + 6y$ $\quad N_x = 2y$

$M_y \neq N_x$

$\frac{N_x - M_y}{M} = \frac{2y - 6y}{4x + 3y^2}$

$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$

Multiply Both sides

$(4x^3 + 3y^2x^2)dx + (2x^3y)dy = 0$ — (11)

$M_y = 6yx^2$ $\quad N_x = 6x^2y$

$M_y = N_x$

$\therefore \int M dx + \int (\text{term} \dots x)dy = c$

$\int (4x^3 + 3y^2x^2)\,dy + \text{Nil} = c$

$\cancel{4}\frac{x^4}{\cancel{4}} + \cancel{3}y^2\frac{x^3}{\cancel{3}} = c$

$x^4 + y^2x^3 = c$