# GCUF

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Subject: Math

Smester: 2nd Smester (evening)

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# Excercise 9.2

Solve:-

(i) $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

$y\,dy = \frac{x^2}{(1+x^3)}dx$

$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}dx$

$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$

$\frac{3y^2}{2} = \ln(1+x^3)+c$

$3y^2 = 2\ln(1+x^3)+c$

(ii)

$\frac{dy}{dx} + y^2\sin x$

$\frac{dy}{dx} = -y^2\sin x$

$\int\frac{dy}{y^2} = -\int \sin x\,dx$

$\frac{y^{-1}}{-1} = -(-\cos x)+c$

$-\frac{1}{y} = \cos x + c$ .

(iii)

$$\frac{dy}{dx} = 1 + x + y^2 + xy.$$

$$\frac{dy}{dx} = 1(1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+y^2)(1+x)$$

$$\int \frac{dy}{(1+y^2)} = \int (1+x)\,dx$$

$$\tan^{-1} y = \int x + \frac{x^2}{2} + C$$

$$2\tan^{-1} y = 2x + x^2 + C$$

(IV) $(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$

$$[x(y+2) + 1(y+2)\,dx + x(x+2)\,dy = 0$$

$$[(y+2)(y+1)]\,dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2)}\,dx + \frac{1}{y+2}\,dy = 0$$

$$\int \frac{x+1}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln C$$

$$y + 2 = \frac{e}{\sqrt{x^2+2x}} \quad \text{Ans}$$

**(V)** $\frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$

$= 2x^2 - 2x - 2 + y - x^2y^2 + xy$

$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$

$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$

$\frac{dy}{2-y} = (x^2 - x - 1)\,dx$

$-\int \frac{-dy}{2-y} = \int (x^2 - x - 1)\,dx$

$-\ln(2-y) = \frac{x^3}{3} - \frac{x^2}{2} - x + c$

$6 - \ln(2-y) = \frac{2x^3 - 3x^2 - 6x + 6c}{\cancel{6}}$

$6 - \ln(2-y) = 2x^3 - 3x^2 - 6x + 6c$

$\ln(2-y)^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6c}$

$(2-y)^{-6} = e^{2x^3 - 3x^2 - 6x} \cdot e^{6c}$

$(2-y)^{-6} = c_1\, e^{2x^3 - 3x^2 - 6x}$

**(vi)**

$\operatorname{cosec} y\, dx + \sec x\, dy = 0$

$\div$ by $\operatorname{cosec} y \sec x$

$\frac{1}{\sec x}\, dx + \frac{dy}{\operatorname{cosec} y} = 0$

$\int \cos x\, dx + \int \sin y\, dy = \int 0\, dx$

$\sin x - \cos x = C$



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(vii)

$$y(1+x)dx + x(1+y)dy = 0$$

$\div$ by $xy$

$$\frac{1+x}{x}dx + \frac{1+y}{y}dy = 0$$

$$\int\left(\frac{1}{x}+x\right)dx + \int\left(\frac{1}{y}+y\right)dy = 0$$

$$\ln x + x + \ln y + y = C$$

$$x+y+\ln xy = C$$

(viii) $\frac{dy}{dx} + \sqrt{\frac{1+y^2}{1-x^2}} = 0$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

or $\int\frac{dy}{\sqrt{1-y^2}} = -\int\frac{dx}{\sqrt{1-x^2}}$

$$\sin^{-1}y = -\sin^{-1} + C$$

$$\frac{dy}{dx} + \sqrt{\frac{y^2-1}{x^2-1}} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{y^2-1}{x^2-1}}$$

$$\int\frac{dy}{\sqrt{y^2-1}} = -\int\frac{dx}{\sqrt{x^2-1}}$$

$$\cosh^{-1}y = -\cosh^{-1}x + C$$

$$y = \cosh(C - \cosh^{-1}x)$$



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# Excercise 9.3

**Solve 1:-**

$$(x-y)\,dx + x+y\,dy = 0$$

$$(x+y)\,dy = -(x-y)\,dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \underline{H.DE} \text{ ①}$$

Put $y = vx$ ——— ②

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

using ⑪ (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{vx - x}{x+vx}$$

$$x\frac{dv}{dx} = \frac{x(v-1)}{x(1+v)} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v} - 1 - \cancel{v} - v^2}{(1+v)}$$

$$= \frac{-(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + C$$

$$\ln(v^2+1)^{1/2} + \tan^{-1}v + \ln x = C$$

$$\ln\sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = C$$

$$\ln\sqrt{y^2+x} - \cancel{\ln\sqrt{x^2}} + \tan^{-1}\left(\frac{y}{x}\right) + \cancel{\ln x} = C$$

$$\ln\sqrt{y^2+x} + \tan^{-1}\left(\frac{y}{x}\right) = C$$



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Solve (ii)

$$(y^2+2xy)\,dx + x^2\,dy = 0$$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = \frac{-(y^2+2xy)}{x^2} \quad \underline{H.DE} \quad (1)$$

Put $y = vx$ ——— (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ ——— } (3)$$

using (iii) + (ii) in (i)

$$v + x\frac{dv}{dx} = \frac{-(v^2x^2+2xvx)}{x^2}$$

$$x\frac{dv}{dx} = \frac{-\cancel{x^2}(v^2+2v)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = -(v^3+3v)$$

$$\int \frac{dv}{v^2+3v} = -\int \frac{dx}{x}$$

$$\int \frac{1}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{3}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{v+3-v}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{1}{v} + \int \frac{1}{v+3} = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln v + \frac{1}{3}\ln(v+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$



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Putting

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{1/3} = c(v+3)^{1/3}$$

$$x \cdot \left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x \cdot \frac{y^{1/3}}{x^{1/3}} \cdot x^{1/3} = c(y+3x)^{1/3}$$

$$xy^{1/3} = c(y+3x)^{1/3}$$

$$x^3 y = c'(y+3x)$$

**Solve (iii)**

$$(x^2-3y)dx + 2xy\,dx = 0$$

$$2xy\,dy = -(x^2-3y^2)dx$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{H.DE} \; (i)$$

Put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

using (ii) + (i) in (i)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

**Solve (iv)**

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$x^2+3y^2\,dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \underline{\text{H.DE}} \quad (1)$$

Put $\quad y = vx \quad$ ——— (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{———} \quad (3)$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v^2)}{\cancel{x^2}2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int\frac{2v}{1+v^2}\,dv = \int\frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln cx$$

$$\left(1+\frac{y^2}{x^2}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$x^2+y^2 = (cx)x$$

## Solve (v)

$$(x^2 + xy + y^2)\,dx - x^2\,dy = 0$$

$$(x^2 + xy + y^2)\,dx = x^2\,dy$$

$$\frac{dy}{dx} = \frac{x^2 + x^2 + y^2}{x^2} \quad \text{H.PE} \text{ ——— } (1)$$

Put $y = vx$ ——— (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ ——— } (3)$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{x^2 + x(vx) + v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{(1 + v + v^2)\cancel{x^2}}{\cancel{x^2}} - v$$

$$\int \frac{dv}{(1+v^2)} = \int \frac{dx}{x}$$

$$\tan^{-1} v = \ln x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$$

## Solve (vi)

$$(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$$

$$(x^2 + 3xy + y^2)\,dx = x^2\,dy$$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} \quad \text{HDE} \text{ ——— } (1)$$

Put $y = vx$ ——— (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ ——— } (3)$$

using (ii) (iii) in (i)



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$$v + x\frac{dv}{dx} = \frac{x^2 + 3xvx + v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{x^2(1+3v+v^2)}{x^2} - v$$

$$x\frac{dv}{dx} = 1 + 2v + v^2$$

$$\int\frac{dv}{(1+v)^2} = \int\frac{dx}{x}$$

$$\frac{-1}{(v+1)} = \ln x + c$$

$$\frac{-1}{\frac{y}{x}+1} = \ln x + c$$

$$\frac{-1}{\frac{y+x}{x}} = \ln x + c$$

$$\frac{-x}{x+y} = \ln x + c$$

**Solve (vii)**

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y} \quad \text{HDE} \quad (1)$$

Put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{4vx - 3x}{2x - vx}$$

$$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$$



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$$x\frac{dv}{dx} = \frac{4v-3 \ -2v+v^2}{2-v}$$

$$\int\frac{2-v}{v^2+2v-3}dv = \int\frac{dx}{x} \text{ ——(iv)}$$

By Partial Fraction

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{(v+3)} + \frac{B}{(v-1)}$$

$$2-v = A(v-1) + B(v+3)$$

Put $v+3=0 \Rightarrow +5 = -4A \Longrightarrow \boxed{A = \frac{-5}{4}}$

Put $v-1=0 \Rightarrow +1 = 4B \Longrightarrow \boxed{B = \frac{1}{4}}$

$$\frac{2-v}{(v+3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v-1)}$$

Form (iv)

$$-\frac{5}{4}\int\frac{dv}{v+3} + \frac{1}{4}\int\frac{dv}{v-1} = \int\frac{dx}{x}$$

$$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-1) = \ln x + \ln c$$

$$-\ln(v+3)^5 + \ln(v-1) = 4\ln xc$$

$$\ln\frac{v-1}{(v+3)^5} = \ln c^4x^4$$

Putting

$$\frac{\left(\frac{y}{x}-1\right)}{\left(\frac{y}{x}+3\right)^5} = c^4x^4$$

$$\frac{y-x}{(y+3x)^5} = c' \text{ Ans}$$



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Solve (viii)

$$x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\frac{y}{x} - x\right) dx$$

$$\frac{dy}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{H.DE} \text{ — } ①$$

Put $y = vx$ — ②

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ — } ③$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{vx \sin\frac{vx}{x} - x}{x \sin\frac{vx}{x}}$$

$$\frac{x\,dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x}\sin v} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v\sin v} - 1 - \cancel{v\sin v}}{\sin v}$$

$$\int \sin v\, dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x + c$$

$$\cos v = \ln x - c$$

$$\cos\frac{y}{x} = \ln x - c \quad \text{Ans}$$

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## Excercise 9.4

**Solve (i)**

$$(3x^2+4xy)dx+(2x^2+2y)dy=0$$

$M = 3x^2+4xy \quad , \quad N = 2x^2+2y$

$\frac{\partial M}{\partial y} = 0+4x \qquad \frac{\partial N}{\partial y} = 4x+0$

$$\frac{\partial M}{\partial o} = \frac{\partial N}{\partial o}$$

~~Now $\int M dx + \int$~~

$$\int(3x^2+4xy)dx+\int 2y\,dy = c$$

$$\cancel{3}\frac{x^3}{\cancel{3}}+\frac{\overset{2}{\cancel{4}}x^2y}{\cancel{2}}+\frac{\cancel{2}y^2}{\cancel{2}} = c$$

$$x^3+2x^2y+y^2 = c$$

**Solve (ii)**

$$(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$$

$M = 2xy+y-\tan y \quad , \quad N = x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial N} = 2x+1-\sec^2 y \quad , \quad \frac{\partial N}{\partial x} = 2x-\tan^2 y+0$

$= 2x-\tan^2 y$

$\therefore \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$

$$\int(2xy+y-\tan y)dx+\int \sec^2 y\,dy = c$$

$$\frac{\cancel{2}x^2y}{\cancel{2}}+xy-x\tan y+\tan y = c$$

$$x^2y+xy-x\tan y+\tan y = c$$

**Solve 3:** $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$$M = \frac{x+y}{y-1} \qquad N = -\frac{1}{2}\left[\frac{x+1}{y-1}\right]^2$$

$$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2} \qquad \frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$$

$$\frac{\cancel{y}-1-x-\cancel{y}}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int\left(\frac{x+y}{y-1}\right)dx + \int -\frac{1}{2(y-1)^2}dy = c$$

$$\left(\frac{1}{y-1}\right)\int(x+y)\,dx + \left(-\frac{1}{2}\right)\int(y-1)^{-2}dy = c$$

$$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2}+xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2+2xy+1 = c'(y-1) \text{ Ans}$$

**Solve 4:** $\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$

$$(hx+by)dy = -(ax+hy)dx$$

$$(ax+hy)dx + (hx+by)dy = 0$$

$$M = ax+hy \qquad N = hx+by$$

$$\frac{\partial M}{\partial y} = 0+h \qquad \frac{\partial N}{\partial x} = h$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$



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$$\int(ax+hy)dx+\int b\,dy=C$$

$$a\frac{x^2}{2}+hxy+\int b\frac{y^2}{2}=C$$

$$ax^2+2hxy+by^2=C'$$

**Solve 5** $(1+\ln xy)dx+\left(1+\frac{x}{y}\right)dy=0$

$$M=1+\ln xy \quad N=1+\frac{x}{y}$$

$$\frac{\partial M}{\partial y}=0+\frac{1}{xy}\cdot x \quad \frac{\partial N}{\partial x}=0+\frac{1}{y}.$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$$\int(1+\ln xy)dx+\int 1\cdot dy=C$$

$$\int dx+\int 1\cdot\ln xy\,dx+\int dy=C$$

$$x+\left[\ln xy\cdot(x)-\int\frac{1}{xy}\cdot y\cdot x\,dx\right]+y=C$$

$$x+x\ln xy-\int dx+y=C$$

$$\cancel{x}+x\ln xy-\cancel{x}+y$$

$$x\ln xy+y=C$$

**Solve 6:** $\frac{y\,dx+x\,dy}{1-x^2y^2}+x\,dx=0$

$$\frac{y\,dx}{1-x^2y^2}+\frac{x\,dy}{1-x^2y^2}+x\,dx=0$$

$$\left(x+\frac{y}{1-x^2y^2}\right)dx+\frac{x\,dy}{1-x^2y^2}=0$$

$$M=x+\frac{y}{1-x^2y^2}$$

$$\frac{0+(1-x^2y^2)\cdot 1-y(-2x^2y)}{(1-x^2y^2)^2}=$$

$$\frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2}=\frac{1+x^2y^2}{(1-x^2y^2)^2}$$

$N = \frac{x}{1-x^2y^2}$

$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2).1 - x(-2xy^2)}{(1-x^2y^2)^2}$

$\frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{(1-x^2y^2)^2}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int (x + \frac{y}{1-x^2y^2}) dx + Nill = C$

$\int x\,dx + \int \frac{y\,dx}{1-x^2y^2} = C$

$\frac{x^2}{2} + \int \frac{y/y^2}{1/y^2 - \frac{x^2y^2}{y^2}} dx = C$

$\frac{x^2}{2} + \frac{1}{y}\int \frac{dx}{(\frac{1}{y})^2 - x^2} = C$

$\frac{x^2}{2} + \frac{1}{y}\left(\frac{1}{2(\frac{1}{y})} \ln\left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right) = C$

$\frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| = C$

$x^2 + \ln\left|\frac{1+xy}{1-xy}\right| = C'$

**Solve 7:-** $(6xy+2y^2-5)dx + (3x^2+4xy-6) = 0$

$M = 6xy+2y^2-5$ , $N = 3x^2+4xy-6$

$\frac{\partial M}{\partial y} = 6x+4y$ $\quad \frac{\partial N}{\partial x} = 6+4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int(6xy+2y^2-5)dx + \int -6dy = C$

$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = C$

$3x^2y + 2xy^2 - 5x - 6y = C$



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**Solve 8:-**

$(y\sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$

$M: y\sec^2 x + \sec x \tan x$, $N \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x \quad \frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int (y\sec^2 x + \sec x \tan x)dx + \int 2y\, dy = c$

$y\tan x + \sec x + y^2 = c$

## Excercise 9.5

**Solve 1:-**

$(xy^2 + y)dx - x\,dy = 0$ — (1)

$M = xy^2 + y \qquad N = -x$

$M_y = 2xy + y \qquad N_x = -1$

$M_y \neq N_x$

$\frac{M_y - N_x}{N} = \frac{2xy + 1 + 1}{-x}$

$\frac{N_x - M_y}{M} = \frac{-1 - 2xy - 1}{xy^2 + y}$

$= \frac{-2(1 + xy)}{y(xy + 1)} = \frac{-2}{y}$

$\therefore$ If $= e^{-\int \frac{2}{y} dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$

Multiply both side of equ by If $= \frac{1}{y^2}$

$$\frac{1}{y^2}(x.y^2+y)dx - \frac{x}{y^2}dy = 0$$

$$\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0 \quad \text{--- (ii)}$$

Now $M = x+\frac{1}{y}$ $\quad N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2}$ $\quad N_x = -\frac{1}{y^2}$

$M_y = N_x$

$$\int\left(x+\frac{1}{y}\right)dx + Nill = C$$

$$\frac{x^2}{2}+\frac{x}{y} = C$$

**Solve 3** $(x^2+x-y)dx + x\,dy = 0$ --- (i)

$M = x^2+x-y$ $\quad N = x$

$M_y = -1$ $\quad N_x = 1$

$M_y \neq N_x$

$$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$$

If $e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$

$$\frac{1}{x^2}(x^2+x-y)dx + \frac{x}{x^2}dy = 0$$

$$\left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + \frac{1}{x}dy = 0 \quad \text{--- (ii)}$$

Now $M = 1+\frac{1}{x}-\frac{y}{x^2}$ $\quad N = \frac{1}{x}$

$M_y = -\frac{1}{x^2}$ $= N_x = -\frac{1}{x^2}$

$M_y = N_x$

$$\int\left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + Nil = C$$

$$x+\ln x-\frac{y}{x} = C$$

## Solve 4:-

$$dy + \left(\frac{y - \sin x}{x}\right)dx = 0 \quad \text{—— (1)}$$

$M = \frac{y - \sin x}{x}$ $\quad N = 1$

$M_y = \frac{1}{x} - 0$ $\quad N_x = 0$

$M_y \neq N_x$

$$\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$$

$I.f = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$x\,dy + \cancel{x}\left(\frac{y - \sin x}{\cancel{x}}\right)dx = 0$ —— (ii)

$M = y - \sin x$ $\quad N = x$

$M_y = 1$ $\quad N_x = 1$

$M_y = N_x$

$\int (y - \sin x) = C$

$xy + \cos x = C$

## Solve 5:-

$$y(2xy + e^x)dx - e^x dy = 0$$

$$(2xy^2 + e^x y)\,dx - e^x dy = 0 \quad \text{—— (i)}$$

$M = 2xy^2 + e^x y$ $\quad N = -e^x$

$M_y = 4xy + e^x$ $\quad N_x = -e^x$

$M_y \neq N_x$

$$\frac{M_y - N_x}{N} = \frac{4xy + e^x + e^x}{-e^x}$$

$$\frac{N_x - M_y}{M} = \frac{-e^x - 4xy - e^x}{2xy^2 + ye^x} = \frac{-2e^x - 4xy}{y(2xy + e^x)}$$

$$= \frac{-2\cancel{(e^x + 2xy)}}{y\cancel{(2xy + e^x)}} = \frac{-2}{y}$$

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$If = e^{\int -\frac{2}{y} dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \boxed{\frac{1}{y^2}}$

$\frac{1}{y^2}(2xy^2 + e^x y)dx - \frac{1}{y^2} e^x dy = 0$

$(2x + \frac{e^x}{y})dx - \frac{e^x}{y^2} dy = 0$ (ii)

$M = 2x + \frac{e^x}{y}$ $\quad N = \frac{-e^x}{y^2}$

$M_y = 0 + (\frac{-e^x}{y^2})$ $\quad N_x = \frac{-e^x}{y^2}$

$M_y = N_x$

$\int (2x + \frac{e^x}{y}) dx + N_1 d = c$

$x^2 + \frac{e^x}{y} = c$

**Q. Solve 6:-**

$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

$M = y^4 + 2y$ $\quad N = xy^3 + 2y^4 - 4x$

$M_y = 4y^3 + 2$ $\quad N_x = y^3 - 4$

$M_y \neq N_x$ ∴ (1) is Non Exact Diff eqn

$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3 + 2)} = -\frac{3}{y}$

$I.f = e^{\int -\frac{3}{y} dy} = e^{-3\ln y} = e^{\ln y^{-3}}$

$= e^{\ln y^{-3}} = y^{-3} = \boxed{\frac{1}{y^3}}$

$\frac{1}{y^3}(y^4 + 2y)dx + (x + 2y^4 - 4x)dy = 0$

Now $M = y + \frac{2}{y^2}$ $\quad N = x + 2y - \frac{4x}{y^3}$

$M_y = 1 - \frac{4}{y^3}$ $\quad N_x = 1 + 0 - \frac{4}{y^3}$

$M_y = N_x$

$\int (y + \frac{2}{y^2}) dx + \int 2y\, dy = C$

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = C$

$xy + \frac{2x}{y^2} + \frac{2}{y^2} = C$

**Solve 7:-**

$(x^2 + y^2 + 2x)\, dx + 2y\, dy = 0$ — ①

$M = x^2 + y^2 + 2x$ $\quad N = 2y$

$M_y = 2y$ $\quad N_x = 0$

$M_y \neq N_x$ $\therefore$ ① is No Exact Diff

$\frac{M_y - N_x}{N} = \frac{2y - 0}{2y} = 1 = x^0$

$If = e^{\int 1 \cdot dx} = e^x$

Multiply both side of eqn ① by

$I.f = \boxed{e^x}$

$e^x(x^2 + y^2 + 2x)\, dx + e^x(2y)\, dy = 0$ — (ii)

$M = e^x(x^2 + y^2 + 2x)\, dx + Nill = C$

$\int x^2 e^x - \int \cancel{2x e^x dx} + e^x y^2 \int e^x \cancel{2x}\, dx = C$

$(x^2 + y^2) e^x = C$

## Excercise 9.6

**Solve 1**

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y = e^{-2x}$$

$$I.f = e^{\int P\,dx} = e^{\int \frac{2x+1}{x}dx} = e^{\int\left(2+\frac{1}{x}\right)dx}$$

$$= e^{2x+\ln x} = e^{2x}e^{\ln x} = e^{2x}x$$

$$\Rightarrow \int d(ye^{2x}x) = \int x\,dx + C$$

$$xye^{2x} = \frac{x^2}{2} + C$$

**Solve 2**

$$\frac{dy}{dx}+\frac{3}{x}y = 6x^2$$

$$I.f = e^{\int P\,dx} = e^{\int \frac{3}{x}dx} = e^{3\ln x} = e^{\ln x^3}$$

$$x^3$$

Solve is given by $\int d(Y \times I.f) = \int Q \times I.f\,dx + C$

$$\Rightarrow \int d(yx^3) = \int 6x^2 \cdot x^3\,dx + C$$

$$\Rightarrow Yx^3 = \int 6x^5 + C$$

$$Yx^3 = \frac{6x^6}{6} + C$$

$$x^3y = x^6 + C$$



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**Solve 3** $\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x^2}{\ln x}$

$$If = e^{\int P dx} = e^{\int \frac{1}{x\ln x} dx} = e^{\int \frac{\frac{dx}{x}}{\ln x}}$$

$$If = e^{\ln(\ln x)} = \ln x$$

Solve is given $\int d(y \times I.f) = \int Q \times If\, dx + C$

$$\int d(y \ln x) = \int \frac{3x^2}{\cancel{\ln x}} \cancel{\ln x}\, dx + C$$

$$y \ln x = \frac{\cancel{3}x^3}{\cancel{3}} + C$$

$$y = \frac{x^3}{\ln x} + C$$

**Solve 4** $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$

$$I.f = e^{\int P dx} = e^{\int 3 dx} = e^{3x}$$

Solve is given $\int d(y \times I.f) = \int Q \times I.f\, dx + c$

$$\Rightarrow \int d(ye^{3x}) = \int 3x^2 \cancel{e^{-3x}}\, \cancel{e^{3x}}\, dx + c$$

$$ye^{3x} = x^3 + C$$

$$y = e^{-3x}(x^3 + C$$

**Solve 5:-** $\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$



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$$\frac{dy}{dx} + \sec^2 x\, Y = \sec^2 x \tan$$

$I.f = e^{\int P dx} = e^{\int \sec^2 x\, dx} = \boxed{e^{\tan x}}$

Solve is given by $\int d(Y \times I.f) = \int Q \times I.f\, dx + C$

$\Rightarrow \int d(Y \times e^{\tan x}) = \int \sec^2 x \tan x\, e^{\tan x}\, dx + c$

$\Rightarrow Y e^{\tan x} = \int \underset{II}{e^t}\, \underset{I}{t}\, dt + c$ $\quad \tan x = t$, $\sec^2 x = dt$

$= t e^t - \int 1 \cdot e^t\, dt + c$

$t e^t - e^t + c$

$Y e^{\tan x} = e^t (t-1) + c$

$Y e^{\tan x} = e^{\tan x}(\tan x - 1) + C$

$Y = (\tan x - 1) + Ce^{-}$

**Solve 6 :-** $x \frac{dy}{dx} + (1 + x \cot x) Y = x$

$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right) Y = 1$

$I.f = e^{\int P dx} = e^{\int (\frac{1}{x} + \cot x) dx} = e^{\ln x + \ln \sin}$

$I.f = e^{\ln(x \sin x)} = \boxed{x \sin x}$

Sol is given by $\int d(Y \times I.f) = \int Q \times I.f\, dx + C$

$\Rightarrow \int d(Y x \sin x) = \int \underset{I}{x} \underset{II}{\sin x}\, dx + c$

$Y x \sin x = x(-\cos x) - \int 1 \cdot (-\cos x)\, dx$

$= x(-\cos x) + \int \cos x\, dx$

$Y x \sin x = -x \cos x + \sin x + C$

$$Y = -\cot x + \frac{1}{x} + \frac{e}{x} \operatorname{cosec} x$$

**Solve 7:-** $(x+1)\frac{dy}{dx} - ny = e^x (x+1)^{n+1}$

$$\frac{dy}{dx} - \frac{n}{(x+1)} Y = e^x (x+1)^n$$

$$I.f = e^{\int P dx} = e^{\int \frac{n}{x+1} dx} = e^{-n \ln(x+1)} = e^{\ln(x+1)^{-n}}$$

$$If = (x+1)^{-n} = \boxed{\frac{1}{(x+1)^n}}$$

Solve is given by $\int d(Y \times I.f) = \int Q \times If\, dx + C$

$$\Rightarrow \int d\left(Y \cdot \frac{1}{(x+1)^n}\right) = \int e^x \cancel{(x+1)^n} \cdot \frac{1}{\cancel{(x+1)^n}} dx + C$$

$$\frac{Y}{(x+1)^n} = e^x + C$$

$$Y = (e^x + C)(x+1)^n \text{ Ans}$$

**Solve 8** $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

$$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right) Y = \frac{4x^2}{x^2+1}$$

$$If \quad e^{\int \left(\frac{2x}{x^2+1}\right) dx} = e^{\ln(x^2+1)} = (x^2+1)$$

$$\Rightarrow \int d(Y(x^2+1)) = \int \frac{4x^2}{\cancel{(x^2+1)}} \cancel{(x^2+1)}\, dx + C$$

$$Y(x^2+1) = \frac{4x^3}{3} + C$$

$$3y(x^2+1) = 4x^3 + C'$$



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