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Subject	Applied Tec Mathamatics
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Exercise 9.2

Q:1

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

Solve:

$$y\,dy = \frac{x^2\,dx}{1+x^3}$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + c$$

$$\frac{3y^2}{2} = \ln(1+x^3) + 3c$$

$$\frac{3y^2}{2} = \ln(1+x^3) + 3c$$

$$3y^2 = 2\ln(1+x^3) + 6c$$

$$3y^2 = 2\ln(1+x^3) + c$$

Q2:-

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x \, dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$\frac{-1}{y} = \cos x + c$$

Q:3

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2 (1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x) \, dx$$

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$\tan^{-1} y = x + \frac{x^2}{2} + C$

$2\tan^{-1} y = 2x + x^2 + C$

Q:4

$(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$

$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$

$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$

$\div$ by $x(x+2)(y+2)$

$\frac{x+1}{x(x+2x)}\,dx + \int \frac{dy}{y+2}$

$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$

$\ln(y+2) = \frac{1}{2}\ln(x^2+2x) + \ln c$

$y+2 = \frac{c}{\sqrt{x^2+2x}}$

Q:5

$\frac{dy}{dx} = 2x^2 x \cdot y - x^2 y + xy - 2x - 2$

$= 2x^2 - 2x - 2 + y - x^2 y + xy$

$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$

$\frac{dy}{dx} = (x^2 - x - 1)(2 - y$

$\int \frac{dy}{2-y} = \int (x^2 - x - 1) dx$

$= \int (x^2 - x - 1) dx$

$= \ell x |2 - y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$

$= \ell x |2 - y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$

$= 6 \ell x |2 - y| = 2x^3 - 3x^2 - 6x + 6c$

$\ell x |2 - y| = (2x^3 - 3x^2 - 6x + 6c) \ell x$

$= 6 \; 2x3 - 3x^2 - 6x + 6c$

$\ell x |2 - y| = \ell x_c .$

$= 6 \; 2x^3 - 3x^2 - 6x$

$|2 - y| = e \qquad . e$

$|2 - y|^6 = 2x^3 - 3x^2 - 6x$

Q:6

$\text{cosec}\,y\,dx + \sec x\,dy = 0$

$\div$ by $\text{cosec}\,y \sec x$

$\frac{1}{\sec x}dx + \frac{dy}{\text{cosec}\,y} = 0$

$\Rightarrow \int \cos x\,dx + \int \sin y\,dy = \int 0\,dx$

$\Rightarrow \sin x - \cos y = c$

Q:7

$y(1+x)dx + x(1+y)dy = 0$

$\div$ by xy

$\frac{(1+x)}{x}dx + \frac{(1+y)}{y}dy = 0$

$\int(\frac{1}{x}+1)dx + \int(\frac{1}{y}+1)dy = \int 0\,dx$

$\Rightarrow lx + xy + y$

$x + y + l x(xy) = c$

Q:8

$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$\int \frac{\sqrt{1+x^2}}{x}dx + \int \frac{\sqrt{1+y^2}}{y}dy = \int 0\,dx$

Put $\sqrt{1+x^2} = t$ Put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $y\,dy = z\,dz$

Therefore

$$= \int \frac{\sqrt{1+x^2}\, x\, dx}{x^2} + \int \frac{\sqrt{1+y^2}\, y\, dy}{y^2} = \int 0\, dx$$

$$= \int \frac{t \cdot t\, dt}{t^2 - 1} + \int \frac{z^2 \cdot 2\, dz}{z^2 - 1} = c$$

$$\int \left(\frac{t^2 - 1 + 1}{t^2 - 1} \right) dt + \int \left(\frac{z^2 - 1 + 1}{z^2 - 1} \right) dz = c$$

$$\int \left(1 + \frac{1}{t^2 - 1} \right) dt + \int \left(1 + \frac{1}{z^2 - 1} \right) dz = c$$

$$\sqrt{1+x^2} + \frac{1}{2} \ln \left(\frac{\sqrt{1+x^2} - 1}{\sqrt{1+x^2} + 1} \right) + \sqrt{1+y^2} + \frac{1}{2} \ln$$

$$\left(\frac{\sqrt{1+y^2} - 1}{\sqrt{1+y^2} + 1} \right)$$

~~Ex 9.7.~~

Q:1 Homogeneous equation

$$(x - y)\, dx - (x + y)\, dy = 0$$

$$(x+y)\, dy = -(x-y)\, dx$$

$$\frac{dy}{dx} = \frac{y - x}{x + y} \quad \text{--- (1)}$$

Put $y = vx$ --- (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (3)}$$

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{x(v-1)}{x(1+v)} - v$$

$$= \frac{v - 1 - v - v^2}{1+v}$$

$$x\frac{dy}{dx} = -\frac{(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1}dv + \int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} \int \frac{dv}{v^2+1} = \int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = \ln + c$$

$$\ln(v^2+1)^{\frac{1}{2}} + \tan^{-1}v + \ln x = c$$

$$\ln x\sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$$

$$\ln x\sqrt{y^2+x^2} - \ln x\sqrt{x^2}\ \tan^{-1}\left(\frac{y}{x}\right)$$

$$\ln x\sqrt{y^2+x^2} + \tan^{-1}\left(\frac{y}{x}\right) = c$$

Q: 2

$$(y^2+2xy)\,dx + x^2dy = 0$$

$$x^2dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = -\left(\frac{y^2+2xy}{x^2}\right)$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

using ⓐ of ⓑ in ①

$$v + x\frac{dv}{dx} = -\left(\frac{-v^2x^2 + 2xvx}{x^2}\right)$$

$$x\frac{dv}{dx} = -x^2\left(\frac{v^2 + 2x}{x^2}\right) - v$$

$$x\frac{dv}{dx} = -\cancel{x^2}\left(\frac{v^2 + 2x}{\cancel{x^2}}\right) - v$$

$$x\frac{dv}{dx} = -\left(v^2 + 3v\right)$$

$$\int\frac{dv}{v^2 + 3v} = -\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{v(v+3}{v(v+3} = -\int\frac{dx}{x}$$

$$\frac{1}{3}\, lxv - \frac{1}{3}\, lx(x+3) = l + x \cdot l_{nc}$$

$$lx\left[\frac{v^{\frac{1}{3}}}{(v+3}\,\frac{1}{3}\right] \cdot ln\frac{c}{x}$$

$$\frac{v^{\frac{1}{3}}}{(v+3)^{\frac{1}{3}}} = \frac{c}{x}$$

$$x \cdot v^{\frac{1}{3}} = c(v+3)^{\frac{1}{3}}$$

$$x\left(\frac{y}{x}\right)\frac{1}{8} = c\left(\frac{y}{x}+3\right)\frac{1}{3}$$

$$xy\frac{1}{3} = c(y+3x)\frac{1}{3}$$

$$x^3 y = c(y+3x)$$

Q: 3

$$(x^2-3y^2)dx + 2xy\,dy = 0$$

$$2xy\,dy = -(x^2-3y^2)dx$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{——— (i)}$$

Put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——— (iii)}$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2} - 1\right) = \ln cx$$

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$y^2 - x^2 = cx$

$y^2 - x^2 = cx^3$

Q:4

$(x^2+3y^2)dx - 2xy\,dy = 0$

$(x^2+3y^2)dx = 2xy\,dy$

$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx}$ —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

using (iii) in (i)

$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$

$x\frac{dv}{dx} = \frac{x^2(1+3v^2)}{x^2 2v} - v$

$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$

$x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$

$\ln(1+v^2) = \ln x + \ln c$

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$\ln(1+v^2) = \ln cx$

$\left(1+\frac{y^2}{x^2}\right) = cx$

$x^2+y^2 = (cx)x^2$

Q:5

$(x^2+xy+y^2)dx - x^2dy = 0$

$(x^2+xy+y^2)dx = x^2dy$

$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ ——— (i)

Put $y = vx$ (ii)

$\frac{dy}{dx} = v+x\frac{dv}{dx}$ (iii)

using

$v+x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \frac{(1+v+v^2)x^2}{x^2} - v$

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$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$$\tan^{-1} v = \ln x + C$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + C$$

Q: 6

$$(x^2 + 3xy + y^2)dx - x^2 dy = 0$$

$$(x^2 + 3xy + y^2)dx = x^2 dy$$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} \quad \text{——(i)}$$

Put $y = vx$ ——(ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——(iii)}$$

using (ii), (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{x^2 + 3xvx + v^2x^2}{x^2}$$

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$$x\frac{dv}{dx} = x^2\frac{(1+3v+v^2)}{x^2} - v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$\int\frac{dv}{(1+v)^2} = \int\frac{dx}{x}$$

$$\frac{1}{v+1} = \ln x + C$$

$$\frac{-1}{\frac{y}{x}+1} = \ln x + C$$

$$-\frac{1}{\frac{y+x}{x}} = \ln x + C$$

$$\frac{-x}{x+y} = \ln x + C$$

Q: 7

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y} \quad \text{—— (i)}$$

Put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

using (ii) diff [illegible]

$$v + x\frac{dv}{dx} = \frac{4vx - 3x}{2x - vx}$$

$$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v - 3 - 2v + v^2}{2-v}$$

$$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x}$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$$2 - v = A(v-1) + B(v+3)$$

Put $v+3 = 0$

$v = -3$

$-1 = A(-3-1) + B(0)$

$-1 = A(-4)$

$A = \frac{-1}{-4}$

$$\boxed{A = \frac{1}{4}}$$

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Put $v-1=0$

$v=1$

$2-1=A(1-1)+B(1+3)$

$1=A(0)+B(4)$

$$\boxed{B=\frac{1}{4}}$$

$$\frac{2-4}{(2+3)(v-1)}=\frac{1}{4(v+3)}+\frac{1}{4(v-1)}$$

$$\frac{1}{4}\int\frac{dv}{v+3}+\frac{1}{4}\int\frac{dv}{v-1}=\int\frac{dx}{x}$$

$$\frac{1}{4}\ln(v+3)+\frac{1}{4}\ln(v-1)=\ln x+\ln c$$

$$\ln(v+3)^4+\ln(v-1)=4\ln cx$$

$$\ln\frac{v-1}{(v+3)^4}=\ln c^4x^4$$

Antilog

$$\left(\frac{\frac{y}{x}-1}{\frac{y}{x}+3}\right)=c^4x^4$$

$$\frac{(y-x)x^4}{x(y+3x)x^4}=c'$$

$$\frac{y-x}{(y+3x)^4}=c'$$

Q.8:-

$$x \sin\left(\frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$$

$$\frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin \frac{y}{x}} \quad \text{--- i}$$

Put $y = vx$ --- ii

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- iii}$$

$$v + x \frac{dv}{dx} = \frac{vx \sin \frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$$

$$x \frac{dv}{dx} = \frac{x(v \sin v - 1)}{x \sin v} - v$$

$$x \frac{dv}{dx} = \frac{\cancel{v \sin v} - 1 - \cancel{v \sin v}}{\sin v}$$

$$\int \sin v \, dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x - c$$

$$\cos \frac{y}{x} = \ln x - c$$

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Non Homogeneous Equation

Q.1:

$$\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$$

Put $x = x+h$

$$\frac{dy}{dx} = \frac{x+h+3(y+k)-5}{x+h-(y+k)-1}$$

$$\frac{dy}{dx} \quad y = \cancel{x}\, vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x+3vx}{x-vx}$$

$$x\frac{dv}{dx} = \frac{x(1+3v)}{\cancel{x}(1-v)}$$

$$x\frac{dv}{dx} = \frac{(1-v)^2}{1-v}$$

$$\int \frac{1-v}{(1-v)^2}dv = \int \frac{dx}{x}$$

$$\int \frac{1\,dv}{(1+v)^2} - \int \frac{v\,dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\int (1+v)^{2} dv - \int \frac{1+v-1}{(1+v)^{2}} dv = \int \frac{dx}{x}$$

$$\Rightarrow \int (1+v)^{2} dv - \int \frac{1}{1+v} dv + \int \frac{1}{(1+v)^{2}} dv = \int \frac{dx}{x}$$

$$\Rightarrow \frac{-1}{1+v} = \ln(1+v) \frac{-1}{1+v} = \ln x + \ln c$$

$$\frac{-2}{1+v} = \ln(1+v) + \ln x + \ln c$$

$$\frac{-2}{1+v} = \ln c(x(1+v))$$

$$\frac{-2}{1+y} = \ln c x (1+\frac{y}{x})$$

$$\frac{-2x}{x+y} = \ln c(x+y)$$

$$\frac{-2(x-2)}{x-2+y-1} = \ln c(x-2+y-1)$$

$$\frac{-2x+4}{x+y-3} = \ln c(x+y-3)$$

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Q:2:-

$$\frac{dy}{dx} = \frac{-(4x+3y+15)}{2x+y+7}$$

Put $x = x+h$

$y = y+k$

$$\frac{dy}{dx} = \frac{-(4x+4h+3y+3k+15)}{2x+2h+y+k+7}$$

$$\frac{dy}{dx} = \frac{4x-3y}{2x+y} \quad \text{---- i}$$

Put $y = vx$ ---- ii

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{---- iii}$$

$$v + x\frac{dv}{dx} = \frac{-4x-3vx}{2x+vx}$$

$$x\frac{dv}{dx} = -\frac{x(4+3v)}{x(2+v)} - v$$

$$= \frac{-4-3v-2v-v^2}{2+v}$$

$$x\frac{dv}{dx} = \frac{-(v^2+5v+4)}{2+v}$$

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$$\int \frac{v+2}{v^2+5v+4} = \int -\frac{dx}{x}$$

$$\frac{1}{3}\int \frac{dv}{v+1} + \frac{2}{3}\int \frac{dv}{v+4} = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln(v+1) + \frac{2}{3}\ln(v+4) = \ln x + \ln c$$

$$\frac{1}{3}\ln\left[(v+1)(v+4)^2\right] = \ln \frac{c}{x}$$

$$\ln\left[\left(\frac{y}{x}+1\right)\left(\frac{y}{x}+4\right)^2\right]^{1/3} = \ln \frac{c}{x}$$

$$\frac{(y+x)}{x}\frac{(y+4x)^2}{x^2} = \frac{c^3}{x^3}$$

$$(y+x)(y+4x)^2 \frac{x^3}{x^3} = c'$$

$$(y+1+x+3)(y+1+4x+12) = c'$$

$$(x+y+4)(4x+y+13) = c'$$

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Q: 3: -

$$(3y-7x-3)dx+(7y-3x-7)dy=0$$

$$(7y-3x-7)dy=-(3y-7x-3)dx$$

$$\frac{dy}{dx}=\frac{-3y+7x+3}{7y-3x-7}$$

Put $x=x+h$

$y=y+k$

$$\frac{dy}{dx}=\frac{-3(y+k)+7(x+h)+3}{7(y+k)-3(x+h)-7}$$

$$\frac{dy}{dx}=\frac{-3y+7x}{7y-3x} \quad \text{---} \quad i$$

Put $y=vx$ ——— ii

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad \text{---} \quad iii$$

$$v+x\frac{dv}{dx}=\frac{-3vx+7x}{7vx-3x}$$

$$x\frac{dv}{dx}=\frac{x(-3v+7)}{x(7v-3)}-v$$

$$x\frac{dv}{dx}=\frac{-3v+7-7v^2+3v}{7v-3}$$

$$\int \frac{7v-3}{7(1-v^2)}\,dv = \int \frac{dx}{x}$$

$$-\frac{2}{7}\int \frac{dv}{1-v} - \frac{5}{7}\int \frac{dv}{1+v} = \int \frac{dx}{x}$$

$$\frac{-2}{7}\left[\ln(1-v) - \frac{5}{v}\ln(1+v)\right] = \ln x + \ln c$$

$$\frac{1}{7}\ln\left(1-\frac{y}{x}\right)^{2}\left(1+\frac{y}{x}\right)^{-5} = 7\ln cx$$

$$\ln\left(\frac{x-y}{x}\right)^{2}\left(\frac{x+y}{x}\right)^{-5} = \ln c^7x^7$$

$$\frac{x^2}{(x-y)^2}\,\frac{x^5}{(x+y)^5} = c^7x^7$$

$$\frac{x^7}{x^7c^7} = (x-y)^2(x+y)^5$$

$$c' = (x-y)^2(x+y)^5$$

$$c' = (x-(y-1)^2)(x+y-1)^5$$

$$c' = \cancel{x-(y-1)^2}$$

$$c' = (x-y+1)^2(x+y-1)^5$$

Q: 4:-

$$\frac{x-2y+5}{2x+y-1}$$

Put $x = x+h$
$y = y+k$

$$\frac{dy}{dx} = \frac{x+h-2y-2k+5}{2x+2h+y+k-1}$$

$$\frac{dy}{dx} = \frac{x-2y}{2x+y} \quad \text{— i}$$

Put $y = vx$ — ii

$$\frac{dy}{dx} = v+x\frac{dv}{dx} \quad \text{iii}$$

$$v+x\frac{dv}{dx} = \frac{x-2vx}{2x+vx}$$

$$x\frac{dv}{dx} = \frac{x(x-2v)}{x(2+v)} - v$$

$$x\frac{dv}{dx} = \frac{1-2v-2v-v^2}{2+v}$$

$$\int \frac{2+v}{1-4v-v^2}\,dv = \int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+4v-1) = -\ln x + \ln c$$

$$\ln\sqrt{v^2+4v-1} = \ln\frac{c}{x}$$

Antilog

$$\sqrt{\frac{y^2}{x^2}+\frac{4y}{x}-1} = \frac{c}{x}$$

$$\frac{y^2+4xy-x^2}{x^2} = \frac{c^2}{x^2}$$

$$y^2+4xy-x^2 = c^2$$

$$\left(y-\frac{4}{5}\right)^2 + 4\left(x+\frac{3}{5}\right)\left(y-\frac{4}{5}\right) - \left(x-\frac{3}{5}\right)^2 = c^2$$

Q: 5:-

$$\frac{dy}{dx} = \frac{3x-4x-2}{3x-4y-1} \quad \text{—i}$$

Put $3x-4y = Z$ —ii

$$3-4\frac{dy}{dx} = \frac{dZ}{dx}$$

$$3-\frac{dZ}{dx} = \frac{4dy}{dx}$$

$$\frac{1}{4}\left(3-\frac{dZ}{dx}\right) = \frac{dy}{dx} \quad \text{—iii}$$

using (i) (ii) & (iii)

$$\frac{1}{4}\left(3-\frac{dZ}{dx}\right)=\frac{Z_1-2}{Z_1-3}$$

$$3-\frac{dZ_1}{dx}=\frac{4Z_1-8}{Z_1-3}$$

$$\frac{3-(4Z_1-8)}{Z_1-3}=-\frac{dZ_1}{dx}$$

$$\frac{3Z_1-9-4Z_1+8}{Z_1-3}=-\frac{dZ_1}{dx}$$

$$-dx=\frac{Z_1-3}{1+Z_1}\,dZ_1$$

$$\frac{Z_1-3-1+1}{1+Z}=-dZ_1$$

$$\int\frac{(Z_1+1)-4}{1+Z_1}\,dZ=-\int dx$$

$$\int\left(1-\frac{4}{1+Z}\right)dZ_1=-\int dx$$

$$Z_1-4\ln(1+Z_1)=-x+C$$

$$(3x-4y)-4\ln(1+3x-4y)=-x+C$$

$$4x-4y-4\ln(1+3x-4y)=C$$

$x - y - \ln(1+3x-4y) = c$

$x - y - \ln(1+3x-4y) = c'$

Q.61 -

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5} \quad \text{--- i}$$

Put $y - x = z$ --- ii

$$\frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{dz}{dx} \quad \text{--- iii}$$

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

$$= \frac{z+1-z-5}{z+5}$$

$$\frac{dz}{dx} = \frac{-4}{z+5}$$

$$\int (Z+5)\,dZ = -4\int dx$$

$$\frac{Z^2}{2} + 5Z = -4x + c$$

$$\frac{Z^2 + 10Z}{Z} = -4x + c$$

$$Z^2 + 10Z = -8x + 2c$$

$$(y-x)^2 + 10(y-x) = -8x + c$$

$$(y-x)^2 + 10(y-x) + 8x = c'$$

$$(y-x)^2 + 10y - 10x + 8x = c'$$

$$(y-x)^2 + 10y - 2x = c'$$

Exercise 9.4
Exact Equation

Q:1-

$$(3x^2+4xy)dx+(2x^2+2y)dy=0$$

$M=3x^2+4xy$, $N=2x^2+2y$

$\frac{\partial M}{\partial y}=0+4x$ $\quad \frac{\partial N}{\partial x}=4x+0$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

Now $\int M dx+\int(\text{term of } N \text{ free } x)dy=c$

$$\int(3x^2+4xy)dx+\int 2y\,dy=c$$

$$3\frac{x^3}{3}+\frac{4x^2y}{2}+\frac{2y^2}{2}=c$$

$$x^3+2x^2y+y^2=c$$

Q: 2:-

$(2xy + y^2 \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

$M = 2xy + y - \tan y.$

$N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y.$

$\frac{\partial N}{\partial x} = 2x - \tan^2 y + 0$

$= 2x - \tan^2 y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$

$\int (2xy + y - \tan y) dx + \int \sec^2 y \, dy = c$

$\frac{2x^2 y}{2} + xy - x\tan y + \tan y = c$

$x^2 y + xy - x\tan y + \tan y = c$

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Q:3

$$\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$$

$$M = \frac{x+y}{y-1} \qquad N = \frac{1}{2}\left(\frac{x+1}{y-1}\right)$$

$$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{}$$

$$N = \frac{1}{2}\frac{(x^2+2x+1)}{(y-1)}$$

$$= \frac{y-1-x-y}{(y-1)^2} = \frac{x-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{givend....}$$

$\int M dx + \int$ (terms of N free from x) $dy = c$

$$\int\left(\frac{x+y}{y-1}\right)dx + \int -\frac{1}{2} \cdot \frac{dy}{(y-1)^2} = c$$

$$\left(\frac{1}{y-1}\right)\int(x+y)\,dx + -\frac{1}{2}\int(y-1)\,dy = c$$

$$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2}+xy\right) + \left(-\frac{1}{2}\left(\frac{-1}{y-1}\right)\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2+2xy+1 = c(y-1)_x =$$

Q:4

$$\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$$

$(hx+by)dy = -(ax+hy)dx$

$(ax+hy)\,dx + (hx+by)dy = 0$

$M = ax+hy \quad N = hx+by$

$\frac{\partial M}{\partial y} = 0+h \quad \frac{\partial N}{\partial x} = h$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms} \ldots N \ldots$

$\int (ax+hy)\,dx + \int by\,dy = c$

$\frac{ax^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c$

Q:5

$(1+\ln xy)\,dx + (1+\frac{x}{y})dy = 0$

$M = 1+\ln xy \quad N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = 0+\frac{1}{xy}\cdot x \quad \frac{\partial N}{\partial x} = 0+\frac{1}{y}$

$\int M dx + \int(\text{tems of } N \text{ free from } x)\,dy = c$

$\int(1+\ln xy)\,dx + \int 1.dy = c$

$\int dx + \int \ln xy\,dx + \int dy = c$

$x + [\ln xy.(x) - \int \frac{1}{xy}.y.x\,dx] + y = c$

$x + x\ln x y - \int dx + y = c$

$\phi\ x + x\ln x y - x + y = c$

$x\ln x y + y = c$

Q: 6

$$\frac{y\,dx + x\,dy}{1 - x^2y^2} + x\,dx = 0$$

$$\frac{y\,dx}{1-x^2y^2} + \frac{x\,dy}{1-x^2y^2} + x\,dy = 0$$

$$\left(\frac{x + y}{1-x^2y^2}\right)dx + \frac{x\,dy}{1-x^2y^2} = 0$$

$$M = \frac{x+y}{1-x^2y^2}$$

$$\frac{\partial M}{\partial y} = \frac{0 + (1-x^2y^2)\cdot 1 - y(-2x^2y)}{(1-x^2y^2)^2}$$

$$= \frac{1 - x^2y^2 + 2x^2y^2}{(1-x^2y^2)} = \frac{1 + x^2y^2}{1-x^2y^2}$$

$$N = \frac{x}{1-x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2)\,1 - x(-2xy^2)}{(1-x^2y^2)^2}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M\,dx + \int (\text{terms of } N \ldots\ldots x)\,dy = c$

$$\int \left(\frac{x+y}{1-x^2y^2}\right)dx + \text{Nil} = c$$

$$\int x\,dx + \int \frac{y\,dx}{1-x^2y^2} = c$$

$x^4 + y^2x^3 = c$ Ac.

Q.7:-

$$(6xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6) = 0$$

$M = 6xy + 2y^2 - 5$, $N = 3x^2 + 4xy - 6$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M dx + \int (\text{term of } N \text{ free from } x) dy = c$

$$\int (6xy + 2y^2 - 5)dx + \int -6\, dy = c$$

$$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$$

$$3x^2y + 2xy^2 - 5x - 6y = c$$

Q: 8:-

$(y\sec^2x + \sec x \tan x)dx +$
$(\tan x + 2y)dy = 0$

$M = y\sec^2x + \sec x \tan x,$
$N = \tan x\, 2y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\sec^2 x$ $\frac{\partial N}{\partial x} = \sec^2 x$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M dx + \int$ term of N ... $) dy = c$

$$\int y \sec^2 x + \sec x \tan x \, dx + \int 2y \, dy = c$$

$$y \tan x + \sec x + y^2 = c$$

Exercise # 4.5

Q#1

Solve

$$(xy^2+y)dx - xdy = 0 \text{ — (i)}$$

$M = xy^2+y \quad N = -x$

$My = 2xy+1 \quad Nx = -1$

$My \neq Mx \therefore$ Non Exact

$\frac{My \cdot Nx}{N} = \frac{2xy+1+1}{-x}$

$\frac{Nx - My}{M} = \frac{-1-2xy-1}{xy^2+y}$

$= \frac{-2}{y}\frac{(1+xy)}{(xy+1)} = \frac{2}{y}$

$\int \frac{2}{y} dy - 2 \log y$

$\frac{1}{y}(xy^2+y)dx - \frac{x}{y} = dy = 0$

$(x+\frac{1}{y})dx - \frac{x}{y}dy = 0$ — (ii)

Now $M = x+\frac{1}{y} \quad N = \frac{x}{y^2}$

$My = -\frac{1}{y^2} \quad Nx = \frac{-1}{y^2}$

$My = Nx$

$\int M dx + \int (\text{terms of } N$

$\int (x+\frac{1}{y}) dx + \text{Nil} = c$

$\frac{x^2}{2} + \frac{x}{y} = c$

Q: 2

$(x^2+x-y)\,dx + x\,dy = 0$ — ①

$M = x^2+x-y \quad N = x$

$M_y = -1 \quad N_x = 1$

$M_y \neq N_x$ ∴ Non Exact

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{2}{x}$

$\int -\frac{2}{x}\,dx - 2\ln x \; \ln x^{-2}$

Multiply both sides of eq

$\frac{1}{x^2}(x^2+x-y)\,dx + \frac{x}{x^2}\,dy$

$(1+\frac{1}{x} - \frac{y}{x^2})\,dx + \frac{1}{x}\,dy = 0$ — ②

Now $M = 1+\frac{1}{x} - \frac{y}{x^2}$ $\quad N = \frac{1}{x}$

$M_y = \frac{-1}{x^2} \quad N_x = \frac{-1}{x^2}$

$M_y = N_x$

so $\int M\,dx + \int$ term of N free of x dy = c

$\int (1+\frac{1}{x} - \frac{y}{x^2})\,dx + \text{Nil} = c$

$x + \ln x + \frac{y}{x} = c$

Q: 3

$y\,dx + (2xy - e - 2y)\,dy = 0$

$M = y \quad N = 2xy - e - 2y$

$M_y = 1 \quad N_x = 2y$

$M_y \neq N_x$ is Non exact diff eq

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$$\frac{M_y - N_x}{N} = \frac{1-2y}{2xy - e^{-2y}}$$

$$\frac{N_x - M_y}{M} = \frac{2y-1}{y} = 2 - \frac{1}{y}$$

$$I.F = e^{\int (2 - \frac{1}{y}) dy} = e^{2y - \ln y}$$

$$= e^{2y + \ln y^{-1}} = e^{2y} e^{\ln (\frac{1}{y})}$$

$$= e^{2y} \frac{1}{y}$$

Multiply (1) by $IF = e^{2y} \frac{1}{y}$

$$e^{2y} \frac{1}{y} \times dx + e^{2y} \frac{1}{y} (2xy - e^{2y})$$

$$e^{2y} dx = (e^{2y} \frac{1}{y} (2xy - e^{2y})) dy = 0$$

$M = e^{2y}$ $N = e^{2y} 2x - \frac{1}{y}$

$M_y = 2e^{2y}$ $N_x = 2e^{2y} - 0$

$\therefore \int M dx + \int$ term of N free $dy = c$

$\Rightarrow \int e^{2y} dx + \int -\frac{1}{y} dy = c$

$\Rightarrow xe^{2y} - \ln y = c$

Q#4

$$dy + \left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{—} (1)$$

$M = \frac{y - \sin x}{x}$ $N = 1$

$M_y = \frac{1}{x} - 0$ $N_x = 0$

$M_y \neq N_x \therefore$ (1) is not.....

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$

Mulibly in both sides.

$x dy + x\left(\frac{y - \sin x}{x}\right) dx = 0$

$M = y - \sin x \quad N x$

$My = 1 \quad Nx = 1$

$My = Nx$ ∴ ① is Bact

$\int (y - \sin x) dx = c$

$xy + \cos x = c$

∴ $My = Nx$ ∴ ① is Fast

$\int M dx + \int$ (terms of ...... of x) $dy = c$

$\int (cy + \frac{2}{y^2}) dx + \int 2y dy = c$

$xy + \frac{2x}{2y^2} + \frac{2y^2}{2} = C$

$xy + \frac{2x}{y^2} + y^2 = c$

Q.5

$(x^2 + y^2 + 2x) dx + 2y dy = 0$ — ①

$M = x^2 + y^2 + 2x \quad N = 2y$

$My = 2y \quad Nx = 0$

$My \neq Nx$ ∴ is Non.....

$\frac{Nx - My}{M} = \frac{0 - 2y}{x^2 + y^2 + 2y}$

$\frac{My - Nx}{N} = \frac{2y - 0}{2y}$

Multibly both side

$e^x (x^2 + y^2 + 2x) dx + e^x (2y) dy = 0$ — ②

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$M = e^x(x^2+y^2+2x) \quad N = e^x 2y$

$My = e^x 2y \quad Nx = e^x 2y$

$My = Nx \therefore$ ① is

$\int e^x(x^2+y^2+2x)\,dx + Nil = c$

$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x\,dx = c$

$x^2 e^x - \int 2x e^x dx + e^x y^2 + \int e^x 2x + d = c$

$(x^2+y^2)e^x = c$

Q:6

$(4x+3y^2)dx + 2xy\,dy = 0$ — ①

$M = 4x+3y^2 \quad N = 2xy$

$My = 0+6y \quad Nx = 2y$

$My \neq Nx$

$$\frac{Nx - My}{M} = \frac{2y-6y}{4x+3y^2}$$

$$\frac{My - Nx}{N} = \frac{6y-2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$$

Multibly both sides

$(4x^3+3y^2x^2)dx + (2x^3y)dy = 0$ — ②

$My = 6yx^2 \quad Nx = 6x^2y$

$My = Nx$

$\therefore \int M dx + \int (\text{term} \ldots\ldots mx)dy = c$

$\int (4x^3+3y^2x^2)\,dy + Nil = c$

$\frac{4x^4}{4} + \frac{3y^2x^3}{3} = c \quad x^4 + y^2x^3 = c$ Az

Q: 7

$(x^2+y^2)dx - 2xy\,dy = 0$ — ①

$M = x^2+y^2 \quad N = -2xy$

$My = 2y \quad Nx = -2y$

$My \neq Nx$

$\frac{Nx - My}{M} = \frac{-2y - 2y}{x^2+y^2}$

$\frac{My - Nx}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{2xy} = \frac{2}{x}$

Multiply bothsides

$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$

$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$

$M = 1+\frac{y^2}{x^2} \quad N = -\frac{2y}{x}$

$My = \frac{2y}{x^2} \quad Nx = +\frac{2y}{x^2}$

$My = Nx$

$\therefore \int M dx + \int (\text{term free } x) dy = c$

$\int \left(1+\frac{y^2}{x^2}\right)dx + \text{Nil} = c$

$x - \frac{y^2}{x} = c$ (Ans)

Q: 8

$\frac{dy}{dx} = e^{x+y} - 1$

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$dy = (e^{2x} + y - 1)dx$

$(e^{2x} + y - 1)dx - dy = 0$ — ①

$M = e^{2x} + y - 1 \quad N = -1$

$M_y = 1 \quad N_x = 0$

$M_y \neq N_x \therefore$ ① is nonexact

$\frac{N_x - M_y}{M} = \frac{0 - 1}{e^{2x} + y - 1}$

$\frac{M_y - N_x}{N} = \frac{1 - 0}{-1} = -1 = e^{-x}$

Multiply both side

$(e^{-x} + y - 1)dx - e^{-x}dy = 0$

$M = e^{x} + e^{-x}y - e^{-x} \quad N = -e$

$M_y = e \quad , N_x = +e$

$M_y = N_x$ so ① is exact so

$\int M dx + \int (\text{term fo} \ldots x) dy = c$

$\int (e^{x} + e^{-x}y - e^{-x}) dx + Nil = c$

$e^{x} - e^{-x}y + e^{-x} = c$