Chapter 9.2

Q No: I

$$\frac{d}{dx} = \frac{x^2}{y(1+x^3)}$$

$$\frac{d}{dx} = \frac{x^2}{y(1+x^3)}$$

$$\frac{1\,d}{dy} = \frac{1}{3}\int \frac{3x^2}{} \, dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$$

$$\frac{3y^2}{2} = \ln(1+x)^3 + 3c$$

$$3y^2 = 2\ln(1+x^3)+6c$$

$$3y^2 = 2\ln(1+x^3)+C$$

QNo:2

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x \, dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$\frac{-1}{y} = \cos x + c$$

QNo:3

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$dy = \int (1+x) dx$$

Q No: 4

$\text{Cosec}y\, dx + \text{Sec}x\, dy = 0$

$\div$ by $\text{Cosec}y \;\; \text{Sec}x$

$\frac{1}{\text{Sec}x} dx + \frac{dy}{\text{cosec}y} = 0$

$\int \text{Cos}x\, dx + \int \text{Sin}y\, dy = \int 0\, dx$

$\text{Sin}x - \text{Cos}y = c$

Q No: 5

$y(1+x)\, dx + x(1+y)\, dy = 0$

$\div$ by xy

$\left(\frac{1+x}{x}\right) dx + \int \left(\frac{1+y}{y}\right) dy = 0$

$\int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy = \int 0\, dx$

$\ln x + x + \ln y + y = c$

$x + y + \ln(xy) = c$

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Q No: 6

$$\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{1+x^2} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{1-x^2} \quad |x| < 1$$

$$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = -\sin^{-1} x + C$$

$$y = \sin(C - \sin^{-1} x)$$

$$\frac{dy}{dx} + \frac{\sqrt{y^2-1}}{x^2-1} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y^2-1}}{x^2-1}$$

$$\cos\cosh^{-1} y = -\cos^{1-1} x + c$$

$$y = \cosh(c - \cosh^{-1} x)$$

Q No: 7

$(1+2y^2)\,dy = y\cos x\,dx$ $y\cos$

$\div$ by y

$\frac{(1+2y^2)}{y}\,dy = \cos x\,dx$

$\int\left(\frac{1}{y}+2y\right)dy = \int \cos x\,dx$

$\ln y + \frac{\cancel{2}y^2}{\cancel{2}} = \sin x + c$

y $\cos = 1$

$\ln 1 + 1 = 0 + c$

$1 = c$

$\ln y + y^2 = \sin x + 1$

Q No: 8

$(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$

$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$

$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$

$\div$ by $x(x+2)(y+2)$

$\frac{x+1}{x(1+2)}\,dx + \frac{1}{y+2}\,dy = 0$

$\int\frac{x+1}{x^2+2x}\,dx + \int\frac{dy}{y+2} = 0$

$\frac{1}{2}\int\frac{2x+2}{x^2+2x}\,dx + \frac{dy}{y+2} = 0$

$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$

$y+2 = \frac{C}{\sqrt{x^2+2x}}$

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Exercise No: 9.3

Q No: I

$(y^2+2xy)\,dx + x^2\,dy = 0$

$x^2\,dy = -(y^2+2xy)\,dx$

$\dfrac{dy}{dx} = -(y^2+2xy)$ $\underline{HDE}$ — i

Put $y = vx$ —— (ii)

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ —— iii

v sing (ii) of (iii) in (i)

$\dfrac{v + x\,dv}{dx} = -\dfrac{(v^2x^2 + 2x + vx)}{x^2}$

$x\dfrac{dv}{dx} = \dfrac{x^2(v^2+2v)}{x^2} - v$

$x\dfrac{dv}{dx} = (v^2+3v)$

$\int \dfrac{dv}{v^2+3v} = -\int \dfrac{dx}{x}$

$\int \dfrac{1}{v(v+3)}\,dv - \int \dfrac{dx}{x}$

$\dfrac{1}{3}\int \dfrac{1}{v} - \dfrac{1}{v+3}\,)dv = -\int \dfrac{dx}{x}$

$\dfrac{1}{3}\int \dfrac{3}{v(v+3)}\,dv - \int \dfrac{dx}{x}$

$\dfrac{1}{3}\int \dfrac{1}{v} - \dfrac{1}{v+3}\,dv\ -\int \dfrac{dx}{x}$

$\dfrac{1}{3}\ln v - \dfrac{1}{3}\ln(v+3) - \ln x + \ln c$

$\ln\left[\dfrac{v^{\frac{1}{3}}}{v+3-3}\right] \ln \dfrac{c}{x}$

$$\frac{v^{\frac{1}{3}}}{(v+3)^{\frac{1}{3}}} = \frac{c}{x}$$

$$x \cdot v^{\frac{1}{3}} = c(v+3)^{\frac{1}{3}}$$

$$x\left(\frac{y}{3}\right)^{\frac{1}{3}} = c\left(\frac{y}{x}+3\right)^{\frac{1}{3}}$$

$$\frac{x\, y^{\frac{1}{3}}}{\cancel{x^{\frac{1}{3}}}}\ \cancel{x} = c\,(y+3x)^{\frac{1}{3}}$$

$$xy^{\frac{1}{3}} = c(y+3x)^{\frac{1}{2}}$$

$$xy = c(y+3x)$$

Q. No 2

$$(x^2-3y^2)\,dx + 2xy\,dy = 0$$

$$2xy\,dy = -(x^2-3y^2)\,dx$$

$$\frac{dy}{dx} = \frac{3x^2-y^2}{2xy} \quad \text{H.D.E}$$

Put $y = vx$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2x\,vx}$$

$$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\frac{y^2-x^2}{x^2} = e^x$$

$$y^2 - x^2 = c(x)x^2$$

Q#3

$$(x^2 + xy + y^2)dx - x^2 dy = 0$$

$$(x^2 + xy + y^2)\,dx = x^2\,dy$$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Put $y = vx$

$$\frac{dy}{dx} = v + \frac{dv}{dx}$$

$$v + \frac{dv}{dx} = \frac{(1+v^2)x^2}{x^2} - v$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$\tan^{-1} v = \ln x + c$

$\tan^{-1}(y/x) = \ln x + c$

Q.4

$x \sin(y/x) dy = y(\sin y/x - x) dx$

$\frac{dy}{dx} = \frac{y \sin y/x - x}{x \sin(y/x)}$ H.D.E

put $y = vx$

using (ii) (iii) in (i)

$v + x\frac{dv}{dx} = \frac{vx \sin vx/x - x}{x \sin(vx/x)}$

$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1) - v}{\cancel{x}\sin v}$

$x\frac{dv}{dx} = \frac{\cancel{v \sin v} - 1 - \cancel{v \sin v}}{\sin v}$

$\int \sin dv = -\int \frac{dx}{x}$

$-\cos v = -\ln x + c$

$\cos v = \ln x + c$

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$\cos v/x = \ln x - c$

so we hence prove

$\cos y/x = \ln x - c$

#5

$\frac{dy}{dx} = \frac{x+y}{x}$ ——(i) $y(1) = 1$

put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

solving (ii) (iii) (iv)(i)

$v + x\frac{dv}{dx} = \frac{x + vx}{x}$

$x\frac{dv}{dx} = 1$

$\int dv = \int \frac{dx}{x}$

$v = \ln x + c$

$\frac{y}{x} = \ln x + c$

$\therefore$ $y(1) = 1$

$\frac{1}{1} = \ln 1 + c$

$1 = 0 + c$

so $y/x = \ln x + 1$

$y = x\ln x + x = x(\ln x + 1)$ Ans

Q #. 6

$(2x - 5y)dx + (4x - y)dy = y(1) = 4$

$(4x - y)\,dy = -(2x - 5y)\,dx$

$$\frac{dy}{dx} = \frac{5y - 2x}{4x - y}$$

Put $y = vx$

$$\frac{dy}{dx} = \frac{v + x\,dv}{dx}$$

solving (ii)(iii) in (i)

$$v + x\frac{dv}{dx} = \frac{5vx - 2x}{4x - x}$$

$$= \frac{x(5v - 2)}{x(4 - v)} - v$$

$$= \frac{5v - 2 - 4v + v^2}{4 - v}$$

$$x\frac{dv}{dx} = \frac{v^2 + v - 2}{4 - v}$$

$4 - v = A(v+2) + B(v-1)$

$v+2 = 0 \quad v = -2$

$B = -2$

$v-1 = 0 \quad v = 1$

$A = 1$

#7

$$\frac{dy}{dx} = \frac{3x - 4y - 2}{3y - 4y - 3}$$

Put $x - 4y = z$

$$3 - 4\frac{dy}{dx} = \frac{dz}{dx}$$

$$3 - \frac{dz}{dx} = 4\frac{dy}{dx}$$

solving (ii) (iii) in (ii)

$$\frac{1}{4}\left(3 - \frac{dz}{dx}\right) = \frac{z-2}{z-3}$$

$$3 - \frac{dz}{dx} = \frac{4z-8}{z-3}$$

$$\frac{3z - 9 - 4z + 8}{z-3} = \frac{dz}{dx}$$

$$\frac{-(1+z)}{z-3} = \frac{dz}{dx}$$

$$-dx = \frac{(z-3)}{1+z}\, dz$$

$$\frac{z-3-1+1}{1+z} = -dx$$

$$\int \frac{(z+1)-4}{1+z} = -\int dx$$

$$\int \left(1-\frac{4}{1+z}\right)dx = -\int dx$$

$$-4\ln(1+z) = -x+c$$

$$(3x-4y)-\ln(1+3x-4y) = -x+c$$

$$4x-4y-4\ln(1+3x-4y) = c$$

$$x-y-\ln(1+3x-4y) = \frac{c}{4}$$

$$x-y = \ln(1+3x-4y) = c'$$

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$$\int \frac{4-v}{v^2+v-2}\,dv = \int \frac{dx}{x}$$

$$\int \frac{(4-v)\,dv}{(v-1)(v+2)} = \int \frac{dx}{x}$$

$$\int \left[\frac{dv}{(1-v)} \quad \frac{-2}{v+2}\right] dv = \int \frac{dx}{x}$$

$$\ln(v-1) - 2\ln(v+2) = \ln x + \ln c$$

$$\ln\left[\frac{(v-1)}{(v+2)^2}\right] = \ln cx$$

$$\frac{(y/x - 1)}{y/x + 2} = cx$$

$$\frac{y-x}{x\left(\frac{y+2x^2}{x^2}\right)} \frac{1}{x} = c$$

$$y(1) = 4$$

$$\frac{4-1}{(4+2)^2} = c$$

$$c = \frac{3}{36} = \frac{1}{12}$$

$$\frac{y-x}{y+2x} = \frac{1}{12} \quad > \quad 12(y-x) = (4+2x^2)$$

$$\frac{4-v}{(v-1)(v+2)} = \frac{A}{(v-1)} + \frac{B}{v+2}$$

#8

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$$

Put $y - x = z$

$$\frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{dz}{dx}$$

solving (ii) (iii) in (i)

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

$$= \frac{z+1-z-5}{z+5}$$

$$\frac{dz}{dx} = \frac{-4}{z+5}$$

$$\int (z+5)\,dz = -4\int dx$$

$$\frac{z^2}{2} + 5z = -4x + c$$

$$z^2 + 10z = -8x + 2c$$

$$(y-x)^2 + 10(y-x) = -8x + c'$$

$$(y+x^2)+10(y-x)+8x=c'$$

$$(y-x)^2+10y-10x+8x=c'$$

$$(y-x)^2+10y-2x=c'$$

9.4

Q# 1

$$(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$$

$$M=2xy+y-\tan y,\ N=x^2-x\tan^2 y+\sec^2 y$$

$$\frac{\partial M}{\partial y}=2x+1-\sec^2 y,\ \frac{\partial N}{\partial x}=2x-\tan^2 y+0$$

$$2x-\tan^2 y$$

$$\therefore \frac{\partial M}{\partial x}=\frac{\partial N}{\partial x}$$

$$\int M dx+\int(\text{terms of } N \text{ free of } x)dy=c$$

$$\int(2xy-\tan y)dx+\int\sec^2 y\,dy=c$$

$$2x^2y+xy-x\tan y+\tan y=c$$

2

$$\frac{(x+y)}{(y-1)}\,dx = \frac{-1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$$

$$M = \frac{x+y}{y-1} \qquad N = \frac{-1}{2}\,\frac{(x+1)^2}{(y-1)^2}$$

$$N = \frac{-1}{2}\,\frac{(x^2+2x+1)}{(y-1)^2}$$

$$\frac{\delta M}{\delta y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}$$

$$\frac{\delta N}{\delta x} = \frac{-2(x+2)}{-2(y-1)^2}$$

$$-\frac{(y-1-x-1)}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$$\frac{\delta M}{\delta y} = \frac{\delta M}{\delta x} \quad \text{Prove}$$

$\int M dx + \int$ (term of N term x) $dy = c$

$$\int \frac{(x+y)}{y-1}\,dx + \int \frac{-1}{2(y-1)^2}\,dy = c$$

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$$C\frac{1}{(y-1)}\int (x+y)\,dx + \left(\frac{-1}{2}\right)\int (y-1)^{-2}\,dy = C$$

$$C\frac{1}{(y-1)}\;(x^2+xy) + \left(\frac{-1}{2}\right)\left(\frac{-1}{(y-1)}\right) = C$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)}$$

$$x^2+2xy+1 = C'(y-1) \text{ Ans}$$

Q # 3

$$\frac{dy}{dx} = \frac{-(ax+hy)}{hx+by}$$

$$(hx+by)\,dx + (hx+by)\,dy = 0$$

$$M = ax+hy \qquad N = hx+by$$

$$\frac{\delta M}{\delta y} \; 0+h = \frac{\delta N}{\delta x} = h$$

$$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

$$\int M\,dx + \int (\text{term of } N \text{ term by})\,dy = C$$

$$\int (ax+hy)\,dx + \int (by\,dy) = C$$

$$\frac{ax^2}{2} + hxy + \frac{by^2}{2} = C$$

$$ax^2 + 2hxy + by^2 = C'$$

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#4

$$(1+\ln x)dx + (1+\tfrac{x}{y})dy = 0$$

$M = 1+\ln xy \quad N = 1+x/y$

$\frac{\delta M}{\delta y} = 0 + \frac{1}{xy}x \quad \frac{\delta N}{\delta x} = 0 + \frac{1}{x}$

$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$ Hence exit

$\int M dx + \int (\text{term of } N \text{ term } x) dy = c$

$\int (1+\ln xy)\, dx + \int 1\, dy = c$

$\int dx + \int 1 \ln xy\, dx + \int dy = c$

$x + \int \ln xy\, (x) - \int \frac{1}{xy} y . x dy] + y = c$

$x + x . \ln xy - \int dx + y = c$

$x + x \ln xy - x + y = c$

$x \ln xy + y = c$

Q#5

$$(y\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$$

$M = y(\cos x + 2xe^x)$, $N = \sin x + x^2e^y - 1$

$$\frac{\partial M}{\partial y} = \cos x + 2xe^x \quad \frac{\partial N}{\partial x} = \cos x + 2xe^y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Hence exit}$$

$$\int (y\cos x + 2xe^y)dx + \int -1 dy = c$$

$$y\,\frac{\sin x\,\cancel{2}x^2e^y}{\cancel{2}} - y = c$$

$$y\,\sin x^2e^y - y = c$$

Q#6

$$(y\sec^2 x + \sec x\tan x) + (\tan x + 2y)dy$$

$M = y\sec^2 + \sec x\tan x$, $N = \tan x + 2y$

$$\frac{\partial M}{\partial y} = \sec^2 x = \frac{\partial N}{\partial x}\sec^2 x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Hence we prove}$$

$\int M\,dx + \int (\text{term of } N \text{ term } x)\,dy = c$

$\int (y\sec^2 x + \sec^2 x\tan x)\,dx + \int 2y\,dy = c$

$y\tan x + \sec x + y^2 = c$

Q # 7

$(xy + 2y - 5)\,dx + (3x^2 + 4xy - 6) = 0$

$M = 6xy + 2y^2 - 5 \,,\; N = 3x^2 + 4xy - 6$

$\frac{\delta M}{\delta y} \overset{6x+4y}{=} \frac{\delta N}{\delta x} = 6x + 4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact

$\int M\,dx \int (\text{term of } N \text{ term})$

$\int (6xy + 2y^2 - 5)\,dx + \int -6\,dy = c$

$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$

$3x^2y + 2xy^2 - 5x - 6y = c$

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Q # 8

$(2xy-3)\,dx + (x^2+4y)\,dy = 0$

$M = 2xy-3 \quad N = x^2+4y$

$\frac{\delta M}{\delta y} = 2x \quad \frac{\delta N}{\delta x} = 2x$

$\frac{\partial M}{\partial y} = \frac{\delta N}{\delta x}$ = Hence Exact

$\int M\,dx + \int (\text{term of } N \text{ term } x)\,dy = c$

$\int (2xy-3)\,dx + \int 4y\,dy = c$

$\int (2xy-3)\,dx + \int 4y\,dy = c$

$\frac{\cancel{2}x^2y}{\cancel{2}} - 3x + \frac{4y^2}{2} = c$

$x^2y - 3x + 2y^2 = c$

$\because y(1) = 2$

$2-3+8 = c$

$7 = c$

Hence

$x^2y - 3x + 2y^2 = 7$

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9.5 Exercise

Q#1

$$(x^2 - x - y)\,dx + x\,dy = 0$$

$M = x^2 - x - y$ $\quad N = x$

$M_y = -1$ $\quad N_x = 1$

$M_y \neq N_x$ = NON Exact

$$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$$

$$\int \frac{-2}{x}\,dx \quad -2\ln x \quad \ln x^{-2}$$

multiply by on b.s. $\frac{1}{x^2}$

$$\frac{1}{x^2}(x^2+x-y)\,dx + \frac{x}{x^2}\,dy = 0$$

$$\left(1 + \frac{1}{x} - \frac{y}{x^2}\right)dx + \frac{1}{x}\,dy = 0$$

Now $M = 1 + \frac{1}{x} - \frac{y}{x^2}$ $\quad N = \frac{1}{x}$

$M_y = N_x$ = Exact Differernce

$\int M\,dx + \int$ (term of N term x) $dy = c$

$$\int \left(1 + \frac{1}{x} - \frac{y}{y^2}\right)dx + N = C$$

Q #2

$$dy + \left(y - \frac{\sin n}{n} \right) dn = 0 \quad (1)$$

$$M = \frac{x - \sin n}{n} \quad , \quad N = 1$$

$$My = \frac{1}{n} - 0 \qquad N_n = 0$$

$My + N_n$ is Not Exact

Now

$$\frac{MY - NY}{N} = \frac{\frac{1}{n} - 0 - \frac{1}{n}}{1}$$

$$I.f = e^{\int \frac{1}{n} dn} = e^{\ln n} = n$$

multiply by on b.s.

$$z\,dy + \left(\frac{Y - \sin n}{n} \right) dn = 0$$

$$M = x - \sin y \quad , \quad N = n$$

$$My = 1 \qquad N_n = 1$$

$Mx = Nn \therefore$ is Exact

$$\int M dn + \int (\text{term of } N \text{ term } n) dy = c$$

$$\int (y - \sin n) dy = c$$

$$ny + \cos n - c$$

Q # 3 9.5

$(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$ (1)

$M = y^4 + 2y, \quad N = xy^3 + 2y^4 - 4x$

$M_y = 4y^3 + 2, \quad N_x = y^3 - 4$

$M_y \neq N_x \quad \therefore$ (1) is not exact

$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$

$\frac{-3y^3 - 6}{y(y^3+2)} = \frac{-3}{y}$

$\int -\frac{3}{y}\,dy \quad -3\ln y \quad \ln y^{-3}$

$I.f = e \quad = e \quad = e \quad y^{-3} = \frac{1}{y^3}$

$\frac{1}{y^3}(y^4 + 2y)\,dx + \frac{1}{y^3}(xy^3 + 2y^4 - 4x)\,dy = 0$

Now $M = y + \frac{2}{y^2}, \quad N = x + 2y - \frac{4x}{y^3}$

$M_y = 1 - \frac{4}{y^3} \quad N_x = 1 + 0 - \frac{4}{y^3}$

$M_y = N_x \quad \therefore$ is Not exact

No. 4

$(x^2+y^2+2x)dx+2y\,dy=0$ —(1)

$M=x^2+y^2+2x$, $N=2y$

$M_y=2y$ $N_x=0$

$M_y \neq N_x$ ∴ is not exact

$\frac{N_x-M_y}{M}=\frac{0-2y}{x^2+y^2+2x}$ Not of x dy

if $e^{\int 1dx}=e^x$

multiply on both sides

$M=e^x(x^2+y^2+2x)$, $N=e^x 2y$

$M_y=N_x$ ∴ is exact

$\int M dx+\int$ (terms of N terms) $dy=c$

$\int e^x(x^2+y^2+2x)dx$ Ni $=c$

$x^2e^x-\int 2xe^x dx+e^x y^2+\int e^x 2x dx=c$

$(x^2+y^2)e^x=c$

Q #5

$(x^2+y^2)dx - 2xy\,dy = 0$ —(1)

$M = x^2+y^2 \quad N = 2xy$

$M_y = 2y \quad N_x = -2y$

$M_y \neq N_x \quad \therefore (1)$ is not exact

$\frac{N_x - M_y}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{2xy} = \frac{-2}{x}$

$\int \frac{-2}{x}dx = -2\ln x \quad \ln x^{-2}$

if $= e = e = e = x^{-2} = \frac{1}{x^2}$

multiply by on b.s $\frac{1}{x^2}$

$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$

$\frac{(1+y^2)}{x^2}dx - \frac{2y}{x}dy = 0$

$M = \frac{1+y^2}{x^2} \quad , N = \frac{-2y}{x}$

$\int \frac{(1+y^2)}{x^2}dx \quad Nil = c$

$x^2 - \frac{y^2}{x} = c$ ANS

Q #6

$$(4x+3y^2)dx + 2xy\,dy = 0$$

$M = 4x+3y^2 \quad N = 2xy$

$M_y = 0+6y \quad = N_x = 2y$

$M_y \neq N_x$ is Not exact

$$\frac{N_x - M_y}{N} = \frac{2y-6y}{4x+3y^2}$$

$\int \frac{2}{x}\,dx \quad 2\ln x \quad \ln x^2$

multiply by on both sides

$$(4x^3+3y^2x^2)dx\ (2x^3y)\,dy = 0$$

$M_y = N_x$ is exact

$\int M\,dx + \int(\text{term of } N \text{ term})\,dy = c$

$$\int (4x^3+3y^2x^2)\,dx + Nil = c$$

$$\frac{\cancel{4}x^4}{\cancel{4}} + \frac{\cancel{3}y^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} = c$$

$x^4 + y^2x^3 = c$ ANS

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$(y^2 + xy)dx - x^2 dy = 0$

$M = y^2 + xy$, $N = -x^2$

$M_y = 2y + x$ $N_x = -2x$

$M_y \neq N_x$ is Not exact

$\frac{M_y - N_x}{N} = \frac{2y + x + 2x}{-2x}$ Not term x

$\frac{N_x - M_y}{N} = \frac{-2x - 2y - x}{y^2 + xy}$ Not term y

$xM + yN = xy^2 + x^2 y + (y$

if $\frac{1}{xM + yN} = \frac{1}{xy^2}$

multiply by b.s.

$(\frac{1}{x} + \frac{1}{y})dx - \frac{x}{y^2} dy = 0$

$M_y = \frac{-1}{y^2}$ N_x $\frac{-1}{y^2}$

$\int \frac{1}{x} + \frac{1}{y^2})dx + Nill = c$

$\ln x + \frac{x}{y} = c$ ANS

9.6 Exercise

1 ~~#~~

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$$

$$\int P dx \quad \int \frac{2x+1}{x} dx \quad \int \left(2+\frac{1}{x}\right) dx$$

if $e - e = e$

solve is given by $\int d(y_x \, If)$

$$= \int d(ye^{2x} x) = \int e^{-2x} \cdot e^{2x} x dx + C$$

$$= xye^{x^2} = \frac{x^2}{2} + C \text{ ANS}$$

~~7~~ 2

$$\frac{dy}{dx} + \frac{3}{x} y = 6x^2$$

$$\int p dx \int \frac{3}{x} dx \quad 3 \ln x \quad \ln x^3$$

$$\int d(yx^3) = \int 6x^2 \cdot x^3 dx + C$$

$$yx^3 = \int 6x^5 dx + C$$

~~y~~

$$x^3 y = \frac{\cancel{6}x^6}{\cancel{6}} + C$$

$$x^3 y = x^6 + C \text{ ANS}$$

#3

$$\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x^2}{\ln x}$$

$$\int P\,dx \int \frac{1}{x\ln x}\,dx \int \frac{\frac{dx}{x}}{\ln x}$$

$$if = e = \ln x$$

$$\int d(y\ln x) = \int \frac{3x^2}{\ln x} \ln x\,dx + C$$

$$y\ln x = \frac{\cancel{3}x^3}{\cancel{3}} + C$$

$$y = \frac{x^3+C}{\ln x} \quad \text{Ans}$$

#4

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$$\text{I.f } e^{\int P dx} = e^{\int 3dx} = e^{3x}$$

solve given by $\int d(y \times \text{I.f})$

$$\int d(ye^{3x}) = \int 3x^2 e^{-3x} e^{3x}\,dx + C$$

$$ye^{3x} = x^3 + C$$

$$y = e^{-3x}(x + C) \quad \text{Ans}$$

#5

$$\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$$

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\int p\,dx \quad \int \sec^2 x\,dx$$

$$i.f = e = e = e^{\tan x}$$

solving by $\int d \int y \times i.f$

$$ye^{\tan x} = \int e^t t\,dt + C$$

$$-te^t - \int 1 e^t\,t\,dt + C$$

$$= te - t + C$$

$$ye^{\tan} = e^t(t-1) + C$$

$$ye^{\tan x} = e^{\tan x}(\tan x - 1) + C$$

$$y = (\tan x - 1) + Ce^{\tan x}$$

Q#6

$$(x^2+1)\frac{dy}{dx}+2xy=4x^2$$

$$\frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{4x^2}{x^2+1}$$

$$\int\left(\frac{2x}{x^2+1}\right)dx$$

if $= e = e = x^2+1$

solve is given by

$$\int d(y \cdot if) = \int Q \cdot i.f \, dx + c$$

$$\int d\left(y(x^2+1)\right) = \int \frac{4x^2}{(x^2+1)}(x^2+1)dx + c$$

$$y(x^2+1) = \frac{4x^3}{3} + c$$

$$3y(x^2+1) = 4x^3 + C$$ Ans

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$$x\frac{dy}{dx} + 2y = \sin x$$

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{\sin x}{x}$$

$$\text{if} = e = e = e = e^{\ln x^2} = x^2$$

solve is given by

$$\int d(yx^2) = \int \frac{\sin x}{x} x^2 dx + C$$

$$yx^2 = \int x \sin x \, dx + C$$

$$yx^2 = x(-\cos x) - (1 - \cos x) + C$$

$$y = \frac{1}{x}(-\cos x + \sin x + C)$$

ANS

#8

$$(x+2y^3)\frac{dy}{dx} = y$$

$$\left(\frac{1}{x+2y^3}\right)\frac{dx}{dy} = \frac{1}{y}$$

$$\frac{dx}{dy} = \frac{x+2y^3}{y}$$

$$\frac{dx}{dy} = \frac{x}{y} + 2y^3$$

Solve is given by

$$\int d\left(x\frac{1}{y}\right) = \int 2y^2 \frac{1}{y}\,dy + c$$

$$\frac{x}{y} = \int 2y\,dy + C$$

$$x = 1\left(\frac{2y^2}{2}\right. + C$$

$$x = y^3 + cy + C \quad A$$

Q 9

$(x^2+1)\dfrac{dy}{dx} + 4xy = x, \; y(2)=1$

$\dfrac{dy}{dx} + \left(\dfrac{4x}{x^2+1}\right)y = \dfrac{x}{(x^2+1)}$

$e^{\int \frac{4x}{(x^2+1)}dx} \quad e^{2\ln(x^2+1)} = (x^2+1)^2$

solve given by

$\int d\left(y(x^2+1)^2\right) = \int \dfrac{x}{x^2+1}(x^2+1)^2 dx + c$

$y(x^2+1)^2 = \int x(x^2+1)\,dx + c$

$y(x^2+1)^2 = \int (x^3+x)\,dx + c$

$y(x^2+1)^2 = \dfrac{x^4}{4} + \dfrac{x^2}{2} + c$

$y(2)=1$

$1(25) = 6 + c$

$19 = c$

$y(x^2+1)^2 = \dfrac{x^4}{4} + \dfrac{x^2}{2} + 19$