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Subject:-

MATH

Submitted to:-

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Excorsice 9.2

Question No 1

$$\frac{d}{dx} = \frac{x^2}{y(1+x^2)}$$

$$\frac{d}{dx} = \frac{x^2}{y(1+x^2)}$$

$$\frac{d}{dy} = \frac{1}{3}\int \frac{3x^2}{1+x^3}\, dx$$

$$\frac{y^2}{2} = \frac{1}{3} \ln(1+x^3)+c$$

$$\frac{3y^2}{2} = \ln(1+x^3)+3c$$

$$3y^2 = 2\ln(1+x^3)+6c$$

$$3y^2 = 2\ln(1+x^3)+c$$

Q. 2

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{dx} = -y^2 \sin x$$

$$\frac{y^{-1}}{1} = -(-\cos x)+c$$

$$\frac{-1}{y} = \cos x + c$$

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Q.3

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

Q.4

$-by \ \operatorname{cosec} y \quad \sec x$

$$\frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\int \cos x\,dx + \int \sin y\,dy = \int 0\,dx$$

$$\sin x - \cos y = c$$

Q 6

$(4x+3y^2)dx + 2xy\,dy = 0$

$M = 4x+3y^2$ $\quad N = 2xy$

$M_y = 0+6y$ $\quad N_y = 2y$

$M_y \neq N_x$ $\quad \cancel{N_y = 2y}$ Non exact

$\frac{N_x - M_y}{M} = \frac{2y-6y}{4x+3y^2}$

$\frac{M_y - N_x}{N} \cdot \frac{6y-2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$

$\int \frac{2}{x}dx \quad 2\ln x \quad \ln x^2$

$I.F = e = e = e = x^2$

Multiply both sides of (1) by I.F

$(4x^3+3y^2x^2)dx + (2x^3y)dy = 0$ —(II)

$M_y = N_x \quad \because$ Exact Eq

$\int M dx + \int (\text{terms of } N \text{ terms } x) dy = c$

$\int (4x^3+3y^2x^2)dx + \text{Nil} = c$

$4\frac{x^4}{4} + 3y^2\frac{x^3}{3} = c$

$x^4 + y^2x^3 = c$

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Q7 $\frac{dy}{dx} = e^{2}x + y - 1$

$dy = (e^{2x} + y - 1)\,dx - dy = 0$ —— (1)

$M = e^{2x} + y - 1$ $\quad N = -1$

$M_y = 1$ $\quad N_x = 0$

$M_y \neq N_x$ $\therefore$ is Non exact

$\frac{M_x - M_y}{N} = \frac{1-0}{-1} = -1 = -x^0$

$I.F = e^{\int -1\,dx} = e^{-x}$

Multiplying both sides eq (1) by e^{-x}

$e^{-x}(e^{2x} + y - 1)\,dx - e^{-x}\,dy = 0$

$(e^{x} + e^{-x}y) - e^{-x})\,dx - e^{-x}\,dy = 0$ —— (11)

$M = e^{x} + e^{-x}y - e^{-x}$ $\quad N = e^{-x}$

$M_y = e^{-x}$ $\quad N_x = e^{-x}$

$M_y = N_x$ —(11) Exact

$\int (e^{x} + e^{-x}y - e^{-x})\,dx + N_i\,1 = c$

$e^{x} - e^{-x}y + g\,e^{-x} = c$

Excercise 9.3

1 $(y+2xy)\,dx + x^3\,dy = 0$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = -\frac{(y^2+2xy)}{x^2}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

v sin y (ii) of (iii) in (i)

$$v + x\frac{dx}{dx} = \frac{-(v^2e^2+2x\,vx)}{x^2}$$

$$x\frac{dv}{dx} = e^2\frac{(v^2+2v)}{e^2} - v$$

$$x\frac{dv}{dx} = -(v^2+3v)$$

$$\int\frac{dv}{v^2+3v} = -\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{3}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(v+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{\frac{1}{3}} = c(v+3)^{1/3}$$

$$x\left(\frac{y}{3}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x\frac{y^{1/3}}{y^{1/3}}\,x^3 = c(y+3x)^{1/3}$$

$$xy^{1/3} = c(y+3x)^{1/2}$$

$$xy = c'(y+3x)$$

2 $(x^2+y^2+2x)\,dx+2y\,dy=0$

$M=x^2+y^2+2x \qquad N=2y$

$M_y=2y \qquad N_x=0$

$M_y \neq N_x$ is not exact

$\dfrac{N_x-M_y}{M}=\dfrac{0-2y}{x^2+y^2+2x}$ Not of x dy

$IF=e^{\int 1\,dx}=e^x$

Multiply both sides of eqn (i) by IF

$e^x(x^2+y^2+2x)\,dx+e^x(2y)\,dy=0$

$M=e^x(x^2+y^2+2x) \qquad N=e^x 2y$

$M_y=e^x 2y \qquad N_x=e^x 2y$

$M_y=N_x$ is Exact

$\int M\,dx+\int$ terms of N term x dy $=c$

$\int e^x(x^2+y^2+2x)\,dx+Nil=C$

$\int e^x x^2\,dx+\int e^x y^2\,dx+\int e^x 2x\,dx=c$

$x^2e^x-\int 2xe^x\,dx+e^x y^2+\int e^x 2x\,dx=c$

$(x^2+y^2)e^x=c$

3 $(x^2+y^2)\,dx-2xy\,dy=0$

$M=x^2+y^2 \quad N=-2xy$

$M_y=2y \qquad N_x=-2y$

$M_y \neq N_x$ — is Not exact

$\dfrac{M_x-M_y}{M}=\dfrac{-2y-2y}{x^2+y^2}$

$\dfrac{M_y-N_x}{N}=\dfrac{2y+2y}{-2xy}=\dfrac{-4y}{-2x} \quad \dfrac{-2}{2}$

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$\int \frac{-2}{x} dx = -2\ln x = \ln x^{-2}$

$IF = e = e = e = x^{-2} = \frac{1}{x^2}$

Multiply both sides of eqn by $IF = \frac{1}{x^2}$

$\frac{1}{x^2}(x^2+y^2) dx - \frac{1}{x^2}(2xy) dy = 0$

$(1 + \frac{y^2}{x^2}) dx - \frac{2y}{x} dy = 0$

$M = \frac{1+y^2}{x^2}$ $N = \frac{-2y}{x}$

$M_y = \frac{2y}{x^2}$ $N_x = \frac{-2y}{x^2}$

$M_y = N_x$ is exact

$\int M dx + \int (\text{terms } N \text{ terms } x) dy = c$

$\int \frac{(1+y^2)}{x^2} dx$ Nil $= c$

$x - \frac{y^2}{x} = c$

4 $(y^4+2y) dx + (xy^3+2y^4-4x) dy = 0$

$M = y^4+2y$ $N = xy^3+2y^4-4x$

$M_y = 4y^3+2$ $N_x = y^3-4$

$M_y \neq N_x$ is Not exact

$$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3+2)} = \frac{-3}{y}$$

$\int \frac{-3}{y} \, dy = -3 \ln y \quad \ln y^{-3}$

$IF \; e = e = e = y^{-3} = \frac{1}{y^3}$

$$\frac{1}{y^3}(y^4 + 2y)\,dx + \frac{1}{y^3}(xy^2 + 2y^4 - 4x)\,dy = 0$$

Now $M = y + \frac{2}{y^2}$ $\quad N = x + 2y - \frac{4x}{y^3}$

$M_y = N_x$ is exact

$\int M\,dx + \int (\text{term of } N \text{ terms})\,dy = c$

$\int \left(y + \frac{2}{y}\right)dx + \int 2y\,dy = c$

$$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$$

$$xy + \frac{2x}{y^2} + y^2 = c$$

Excercise 9.4

Q1 $(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$

$M=2xy+y-\tan y$, $N=x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial y}=2x+1-\sec^2 y$ $\frac{\partial N}{\partial x}=2x-\tan^2 y+0$

$=2x-\tan^2 y$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\int M\,dx+\int \text{term of } N(x)\,dy=c$

$\int(2xy+y-\tan y)dx+\int \sec^2 y\,dy=c$

$\frac{2x^2y}{2}+xy-x\tan y+\tan y=c$

2 $\frac{(x+y)}{(y-1)}dx=-\frac{1}{2}\frac{(x+1)^2}{(y-1)}dy=0$

$M=\frac{x+y}{y-1}$ $N=-\frac{1}{2}\frac{(x+1)^2}{(y-1)^2}$

$N=\frac{-1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$

$\frac{\partial M}{\partial y}=\frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}$ $\frac{\partial N}{\partial x}=\frac{-2(2x+2)}{-2(y-1)^2}$

$=\frac{y-1-x-1}{(y-1)^2}$ $=\frac{-x-1}{(y-1)^2}$

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$$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial x} \quad \text{equal}$$

$$\int M\,dx + \int (\text{term of } N \text{ term } x)\,dy = c$$

$$\int \frac{(x+y)}{(y-1)}\,dx + \int \frac{-1}{2(y-1)}\,dy = c$$

$$\left(\frac{1}{y-1}\right)\int (x+y)\,dx + \left(\frac{-1}{2}\right)\int (y-1)^{-2}\,dy = c$$

$$\frac{1}{(y-1)}\left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right)\} = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)}$$

$$x^2+2xy+1 = c'(y-1)$$

Q $$\frac{dy}{dx} = \frac{-(ax+hy)}{hx+by}$$

$$(hx+by)\,dy = (-ax+hy)\,dx$$

$$(ax+hy)\,dx + (hx+by)\,dy = 0$$

$$M = ax+hy \qquad N = hx+by$$

$$\frac{\partial M}{\partial y} = 0+h \qquad \frac{\partial N}{\partial x} = h$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int \text{terms of } N \text{ terms } x)\,dy = c$$

$$\int (ax + hy)\,dx + \int by\,dy = c$$

$$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = c$$

$$ax^2 + 2hxy + by^2 = c'$$

~~4c ... dx~~

4 $(1 + \ln x)\,dx + \left(1 + \frac{x}{y}\right)dy = 0$

$M = 1 + \ln xy$ $\quad$ $N = 1 + \frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy}x$ $\quad$ $\frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\quad$ Hence exist

$\int M\,dx + \int (\text{terms of } N \text{ terms } x)\,dy = c$

$$\int (1 + \ln xy)\,dx + \int 1 \cdot dy = c$$

$$\int dx + \int 1 \ln xy\,dx + \int dy = c$$

$$x + \left[\ln xy\,(x) - \int \frac{1}{xy}\,y \cdot x\,dx\right] + y = c$$

$$x + x\ln xy - \int dx + y = c$$

$$x + x\ln xy - x + y = c$$

$$x\ln xy + y = c$$

5 $((\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$

$M = y(\cos x + 2xe^y$, $N = \sin x + x^2e^y - 1$

$\frac{dx}{dy} = \cos x + 2xe$ $\frac{\partial N}{\partial x} = \cos x + 2xe^y$

$\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial x}$ Hence exist

$\int (y\cos x + 2xe^y)dx + \int -1\,dY = c$

$Y \sin + \frac{x\ x^2e^y}{x} - y = c$

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$(Y\cdot\sec^2 x + \sec x \tan x) + (\tan x + 2y)dy$

$M = y\sec^2 + \sec x \tan x = N \tan x + 2y$

$\frac{\partial M}{\partial Y} = \sec^2 x$ $\frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial x}$ Hence exact

$\int M\,dx + \int(\text{terms } N \text{ terms } x)\,dy = c$

$\int(y\sec^2 + \sec x \tan x)dx + \int 2y\,dy = c$

$Y \tan x + \sec x + y^2 = c$

Excerse 9.5

1 $(x^2+x-y)\,dx + x\,dy = 0$

$M = x^2+x-y$ $\quad N = x$

$M_y = -1$ $\quad N_x = 1$

$M_y + N_x =$ Non exact

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$

$\int \frac{-2}{x}\,dx \quad -2\ln x \quad \ln x^{-2}$

Multiple both side $= \frac{1}{x^2}$

$\frac{1}{x^2}(x^2+x-y)\,dx + \frac{x}{x^2}\,dy = 0$

$\left(1+\frac{1}{x^2} - \frac{y}{x^2}\right)dx + \frac{1}{x}\,dy = 0$

Now $M = 1+\frac{1}{x} - \frac{y}{x^2}$ $\quad N = \frac{1}{x}$

$M_y = N_x$ Exact

$\int M\,dx + \int (\text{terms of } N \text{ terms } x)\,dy = c$

$\int \left(1+\frac{1}{x} - \frac{y}{x^2}\right)dx$ Nil $= c$

$x + \ln x + \frac{y}{x} = c$

(2) $dy + \frac{(y - \sin x)}{x} dx = 0$

$M = \frac{y - \sin x}{x}$ $N = 1$

$M_y = \frac{1}{x} = 0$ $N_x = 0$

$M_y \neq N_x$ is Not exact

Now

$\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$

$IF = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Multiplying both sides of eq (i)

$x\, dy + \frac{(y - \sin x)}{x} dx = 0$ —— (ii)

$M = y - \sin x$ $N = x$

$M_y = 1$ $N_x = 1$

$M_y = N_x$ is exact

$\int M dx + \int (\text{terms of } N \text{ terms}) dy = c$

$\int (y - \sin x) dx = c$

$xy + \cos x = c$

③ $(y^4+2y)\,dx + (xy^3+2y^4-4x)\,dy = 0$

$M = y^4+2y$ $\qquad N = xy^3+2y^4-4x$

$M_y = 4y^3+2$ $\qquad N_x = y^3-4$

$M_y \neq N_x$ $\qquad$ ~~Nx~~ is Not exact

$$\frac{N_x - M_y}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} = \frac{-3y^3-6}{y(y^3+2)} = \frac{-3}{y}$$

$\int \frac{-3}{y}\,dy \quad -3\ln y \quad \ln y^{-3}$

IF $e = e = e \quad y^{-3} = \frac{1}{y^3}$

$\frac{1}{y^3}(y^4+2y)\,dx + \frac{1}{y^3}(x^3y+2y^4-\frac{4x}{y^3})\,dy = 0$

Now $M = y + \frac{2}{y^2}$ $\qquad N = x+2y-\frac{4x}{y^3}$

$M_y = 1-\frac{4}{y^3}$ $\qquad N_x = 1+0-4/y^3$

$M_y = N_x$ is exact

$\int M\,dx + \int (\text{tanx of } N \text{ terms } x)\,dy = c$

$\int (y+\frac{2}{y})\,dx + \int 2y\,dy = c$

$$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$$

$$xy + \frac{2x}{y^2} + y^2 = c$$

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(4) $(2xy-3)\,dx + (x^2+4y)\,dy = 0$

$M = 2xy - 3$ $\qquad$ $N = x^2 + 4y$

$\frac{\partial M}{\partial y} = 2x$ $\qquad$ $\frac{\partial N}{\partial x} = 2x$

$\frac{\partial M}{\partial N} = \frac{\partial N}{\partial x}$ Hence exact

$\int M\,dx + \int (\text{term of } N \text{ term } x)\,dy = c$

$\int (2xy-3)\,dx + \int 4y\,dy = c$

$\int (2xy-3)\,dx + \int 4y\,dy = c$

$\frac{2x^2y}{2} - 3x + \frac{4y^2}{2} = c$

$x^2y - 3x + 2y^2 = c$

$\because y(1) = 2$

$\because 2 - 3 + 8 = c$

$\because 7 = c$

Hence $x^2y - 3x + 2y^2 = 7$

Excesile 9.6

Question No 1

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-1}x$$

$$\int P dx \int \frac{2x+1}{x} dx \int \left(2+\frac{1}{x}\right) dx$$

$$I F = e = e = e$$

$\because$ Solve is given by $\int d(y \times I.F) = \int q \times I.F$

$$= \int d(ye^{2x}x) = \int e^{-2x} e^{2x} x\, dx + C$$

$$= ye^{2x}x = \int x\, dx + C$$

$$= ye^{2x} = \frac{x^2}{2} + C$$

Question No 2

$$\frac{dy}{dx} + \frac{3}{x}y = 6x^2$$

$$\int P dx \int \frac{3}{x} dx \quad 3\ln x \quad \ln x^3$$

$$I.F = e = e = e = x^3$$

Solve is given $\int d(y \times I.F) = \int q \times I.F$

$$= \int d(yx^3) = \int 6x^2 \cdot x^3 \cdot dx + C$$

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$$y x^3 = \int 6x^5 \, dx + c$$

$$y x^3 = \frac{6x^6}{6} + c$$

$$x^3 y = x^6 + c$$

question No 3

$$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$$

$$\int P dx \quad \int \frac{1}{x \ln x} dx \quad \int \frac{\frac{dx}{x}}{\ln x}$$

$$I.F = e = \ln x$$

solve is given by $\int d \int (1/x \; I.F) = \int \theta_1$.

$$\int d(y \ln x) = \int \frac{3x^2}{\ln x} \ln x \, dx + c$$

$$y \ln x = 3 \frac{x^3}{3} + c$$

$$y = \frac{x^3 + c}{\ln x}$$

Question no 4

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$$I.F \; e^{\int P dx} = e^{\int 3 dx} = e^{3x}$$

Solu given by $\int d(y \times I.F) = \int q \times I.F$

$$\int d(ye^{3x}) = \int 3x^2 e^{-3x} e^{3x} dx + C$$

$$ye^{3x} = x^3 + C$$

$$y = e^{-3x}(x + C)$$