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Subject Applied Technology Math

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Ex 9.2

Q1, $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + c$$

3 ~~dividing~~ Multyplyg by in both side

$$\frac{3y^2}{2} = 3\frac{1}{3}\ln(1+x^2) + 3c$$

$$\frac{3y^2}{2} = \ln(1+x^2) + 3c$$

2 Multyplyg by in both side

$$\cancel{2}\times\frac{3y^2}{\cancel{2}} = 2\ln(1+x^2) + 6c$$

$$3y^2 = 2\ln(1+x^2) + 6c$$

Q2: $\frac{dy}{dx} + y^2 \sin x = 0$

$\frac{dy}{dx} = -y^2 \sin x$

$\int \frac{dy}{y^2} = -\int \sin x \, dx$

$\frac{y^{-1}}{-1} = -(-\cos x) dx + c$

$\frac{-1}{y} = \cos x + c$

Q3: $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\frac{dy}{dx} = (1+x) + y^2(1+x)$

$\frac{dy}{dx} = (1+x)(1+y^2)$

$\int \frac{dy}{1+y^2} = \int (1+x) \, dx$

$\tan^{-1} y = x + \frac{x^2}{2} + c$

$2 \tan^{-1} y = 2x + x^2 + c'$

Q4: $(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$

$$[x(y+2) + (y+2)]dx + x(x+2)dy = 0$$

$$[(y+2)(x+1)]dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2)}dx + \frac{1}{y+2}dy = 0$$

$$\int \frac{x+1}{x^2+2x}dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$$

$$y + 2 = \frac{c}{\sqrt{x^2+2x}}$$

Q5: $\frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$

$$= 2x^2 - 2x - 2 + y - x^2y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\, dx$$

$$-\int \frac{-dy}{2-y} = \int (x^2 - x - 1)\, dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-6\ln|2-y| = (2x^3 - 3x^2 - 6x + 6c)\ln e$$

$$\ln|2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln e$$

$$\ln|2-y|^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6c}$$

$$\therefore |2-y|^{-6} = e^{2x^3 - 3x^2 - 6x} \cdot e^{6c}$$

$$|2-y|^{-6} = c_1 e^{2x^3 - 3x^2 - 6x}$$

Q6: $\operatorname{cosec} y\, dx + \sec x\, dy = 0$

$$\operatorname{cosec} y\, dx + \sec x\, dy = 0$$

$\div \operatorname{cosec} y \sec x$

$$\Rightarrow \frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\Rightarrow \int \cos x\, dx + \int \sin y\, dy = \int 0\, dx$$

$$\sin x - \cos y = c$$

Q7: $y(1+x)\,dx + x(1+y)\,dy = 0$

$$\Rightarrow y(1+x)\,dx + x(1+y)\,dy = 0$$

$\div$ by xy

$$\Rightarrow \frac{1+x}{x} dx + \frac{1+y}{y} dy = 0$$

$$\Rightarrow \int \left(\frac{1}{x}+1\right) dx + \int \left(\frac{1}{y}+1\right) dy = \int 0\, dx$$

$$\Rightarrow \ln x + x + \ln y + y = c$$

$$x + y + \ln(x+y) = c$$

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Q8. $y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$\Rightarrow \int \frac{\sqrt{1+x^2}}{x}dx + \int \frac{\sqrt{1+y^2}}{y}dy = \int 0\,dx$

Put $\sqrt{1+x^2} = t$ | Put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ | $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ | $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ | $y\,dy = z\,dz$

Therefor

$\Rightarrow \int \frac{\sqrt{1+x^2}}{x^2}x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}y\,dy = \int 0\,dx$

$\Rightarrow \int \frac{t\cdot t\,dt}{t^2-1} + \int \frac{z\cdot z\,dz}{z^2-1} = c$

$\Rightarrow \int \left(\frac{t^2-1+1}{t^2-1}\right)dt + \int \frac{z^2-1+1}{z^2-1}dz = c$

$\Rightarrow \int \left(1+\frac{1}{t^2-1}\right)dt + \int \cancel{\frac{z^2-1+1}{}}\left(1+\frac{1}{z^2-1}\right)dz = c$

$\Rightarrow t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$

$\Rightarrow \sqrt{1+x^2} + \frac{1}{2}\ln\left[\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right] + \sqrt{1+y^2} + \frac{1}{2}\ln\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1} = c$

Ex 9.3

Q1, $(x-y)dx + (x+y)dy = 0$

$$(x+y)dy = -(x-y)dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \underline{\text{H. DE}} \quad ①$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

using ⓘ & ⓘⓘ in ①

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(1+v)} - v$$

$$= \frac{v - 1 - v - v^2}{1+v}$$

$$x\frac{dv}{dx} = \frac{-(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1} dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + C$$

Q.2 $(y^2+2xy)dx + x^2dy = 0$

$$x^2 dy = -(y^2+2xy)dx$$

$$\frac{dy}{dx} = -\frac{(y^2+2xy)}{x^2} \quad \text{H.D.E} \quad (i)$$

Put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——— (iii)}$$

using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = -\frac{(v^2x^2+2xvx)}{x^2}$$

$$x\frac{dv}{dx} = -\frac{x^2(v^2+2v)}{x^2} - v$$

$$x\frac{dv}{dx} = -(v^2+3v)$$

$$\int\frac{dv}{v^2+3v} = -\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{3}{v(v+3)}dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{v+3-v}{v(v+3)}dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\left(\frac{1}{v}-\frac{1}{v+3}\right)dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(v+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$

Antilog

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x.v^{1/3} = c(v+3)^{1/3}$$

$$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x\frac{y^{1/3}}{x^{1/3}}.x^{1/3} = c(y+3x)^{1/3}$$

$$xy^{1/3} = c(y+3x)^{1/3}$$

$$x^3y = c(y+3x)$$

Q.3 $(x^2-3y^2)dx + 2xy\,dy = 0$

$$2xy\,dx = -(x^2-3y^2)\,dx$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{H.D.E} \quad \text{—— (I)}$$

Put $y = vx$ —— (II)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (III)}$$

using (II) & (III) in (I)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$\frac{x\,dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$$

$$\frac{x\,dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{dx}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{v^2}{x^2} - 1\right) = \ln cx$$

Q.4 $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{H.D.E} \quad (I)$$

Put $y = vx$ ——— (II)

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

using (II) & (III) in (I)

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v^2)}{\cancel{x^2}\,2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln cx$$

$$\left(1+\frac{y^2}{x^2}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$x^2+y^2 = (cx)\,x^2$$

Q.5 $(x^2+xy+y^2)\,dx - x^2\,dy = 0$

$$(x^2+xy+y^2)\,dx = x^2\,dx$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{H.D.E} \quad (1)$$

Put $\quad y = vx \quad$ — (II)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (III)}$$

using (II) & (III) in (I)

$$v + x\frac{dv}{dx} = \frac{x^2 + x(vx) + v^2x^2}{x^2}$$

$$\frac{x\,dv}{dx} = \frac{(1+v+v^2)\cancel{x^2}}{\cancel{x^2}} - v$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$$\tan^{-1} v = \ln x + C$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + C$$

Q.6 $(x^2+3xy+y^2)dx - x^2dy = 0$

$$(x^2+3xy+y^2)\,dx = x^2 dx$$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \qquad \text{H.D.E} \quad (I)$$

Put $y = vx$ —— (II)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (III)}$$

using (II) & (III) in (I)

$$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$$

$$\frac{xdv}{dx} = \frac{\cancel{x^2}(1+3v+v^2)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = 1-2v+v^2$$

$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$-\frac{1}{v+1} = \ln x + C$$

$$-\frac{1}{(y/x+1)} = \ln x + C$$

$$\frac{-1}{\frac{y+x}{x}} = \ln x + C$$

$$\frac{-x}{y+x} = \ln x + C$$

Q. 7 $\quad x \sin\left(\frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$

$$\frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{H.D.E} \quad (1)$$

Put $\quad y = vx \quad$ (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{(iii)}$$

using (ii) & (iii) in (1)

$$v + x\frac{dv}{dx} = \frac{vx \sin \frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x} \sin v} - v$$

$$x\frac{dv}{dx} = \frac{v\cancel{\sin v} - 1 - v\cancel{\sin v}}{\sin v}$$

$$\int \sin v \, dv = \int \frac{-dx}{x}$$

$$-\cos v = -\ln + c$$

$$\cos v = \ln x - c$$

$$\cos \frac{y}{x} = \ln x - c$$

Q.8 $\frac{dy}{dx} = \frac{x+y}{x}$ — ①

Put $y = vx$ — ⑪

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — ⑪ⅰ

using ⑪ & ⑪ⅰ in ①

$v + x\frac{dv}{dx} = \frac{x+vx}{x}$

$x\frac{dv}{dx} = \frac{\cancel{x}(1+v)}{\cancel{x}} - v$

$x\frac{dv}{dx} = 1$

$\int dv = \int \frac{dx}{x}$

$v = \ln x + c$

$\frac{y}{x} = \ln x + c$

$\because y(1) = 1$

$\frac{1}{1} = \ln 1 + c$

$1 = 0 + c$

So $\frac{y}{x} = \ln x + 1$

$y = x\ln x + x$

$y = x[\ln x + 1]$ Ans

Ex 9.4

Q No 1 :- Solve $(3x^2+4xy)dx+(2x+2y)dy=0$

$M = 3x^2+4xy \quad , \quad N = 2x^2+2y$

$\frac{\partial M}{\partial y} = 0+4x \qquad \frac{\partial N}{\partial x} = 4x+0$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So given differ is Exact

Now, $\int M dx + \int ($tarmal of

Now, $\int M dx + \int ($terms of N free from $x)dy = c$

$$\int (3x^2+4xy)dx + \int 2y\,dy = c$$

$$\frac{3x^3}{3} + \frac{4x^2y}{2} + \frac{2y^2}{2} = c$$

$$x^3 + 2x^2y + y^2 = c$$

Q No 2 :-

$(2xy + y - \tan y)dx + x^2 - x\tan^2 y + \sec^2 y)dy = 0$

$M = 2xy + y - \tan y \quad , \quad N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y \quad , \quad \frac{\partial N}{\partial x} = 2x - \tan^2 y$

$= 2x - \tan^2 y$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So giving differ is Exact

(19)

$$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$$

$$\int (2xy + y - \tan y)\,dx + \int \sec^2 y\,dy = c$$

$$\frac{2x^2 y}{2} + xy - x\tan y + \tan y = c$$

$$x^2 y + xy - x\tan y + \tan y = c$$

Q No3:-

$$\frac{(x+y)}{(y-1)}\,dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$$

$$M = \frac{x+y}{y-1} \quad , \quad N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$$

$$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1) - (x+y)(1)}{(y-1)^2} \quad , \quad N = -\frac{1}{2}\left(\frac{x^2+2x+1}{(y-1)^2}\right)$$

$$= \frac{\cancel{y} - 1 - x - \cancel{y}}{(y-1)^2} \quad , \quad \frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$$

$$= \frac{-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{given differ is Exact}$$

$$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$$

$$\int \left(\frac{x+y}{y-1}\right) dx + \int -\frac{1}{2(y-1)^2}\,dy = c$$

$$\frac{1}{y-1}\int (x+y)\,dx + \left(-\frac{1}{2}\right)(y-1)^{-2}\,dy = c$$

$$\frac{1}{y-1}\left(\frac{x^2}{2}+xy\right)+\left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right)=c$$

$$\frac{x^2+2xy}{2(y-1)}+\frac{1}{2(y-1)}=c$$

$$x^2+2xy+1=c'(y-1) \quad \underline{\text{Ans}}.$$

Q NO 4:-

$$\frac{dy}{dx}=-\frac{(ax+hy)}{hx+by}$$

$$(hx+by)\,dy=-(ax+hy)\,dx$$

$$(ax+hy)\,dx+(hx+by)\,dy=0$$

$$M=ax+hy \;,\quad N=hx+by$$

$$\frac{\partial M}{\partial y}=\cancel{a+}h \;,\quad \frac{\partial N}{\partial x}=h$$

$$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \quad \text{Hence Exact differ}$$

$$\int M\,dx+\int(\text{terms of } N \text{ free from } x)\,dy=c$$

$$\int(ax+hy)\,dx+\int by\,dy=c$$

$$\frac{ax^2}{2}+hxy+\frac{by^2}{2}=c$$

$$ax^2+2hxy+by^2=c'$$

Q No5:-

$$(1+\ln xy)\,dx+\left(1+\frac{x}{y}\right)dy=0$$

$M=1+\ln xy$, $N=1+\frac{x}{y}$

$$\frac{\partial M}{\partial y}=0+\frac{1}{xy}\cdot x \quad , \quad \frac{\partial N}{\partial x}=0+\frac{1}{y}$$

$$\frac{\partial M}{\partial y}=\frac{1}{y} \quad , \quad \frac{\partial N}{\partial x}=\frac{1}{y}$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$ Hence Exact Differ

$$\int M\,dx+\int(\text{terms of } N \text{ free from } x)\,dy=c$$

$$\int(1+\ln xy)\,dx+\int 1.\,dy=c$$

$$\int dx+\int 1\,\ln xy\,dx+\int dy=c$$

$$x+\left[\ln xy\cdot(x)-\int\frac{1}{xy}\cdot y\cdot x\,dx\right]+y=c$$

$$x+x\ln xy-\int dx+y=c$$

$$\cancel{x}+x\ln xy-\cancel{x}+y=c$$

$$x\ln xy+y=c$$

—:————————:—

Q No7:-

$(6xy+2y^2-5)dx+(3x^2+4xy-6)dy$

$M = 6xy+2y^2-5$, $N = 3x^2+4xy-6$

$\frac{\partial M}{\partial y} = 6x+4y$, $\frac{\partial N}{\partial x} = 6x+4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact Different

$\int M dx + \int(\text{terms of } N \text{ free from } x)dy = c$

$\int(6xy+2y^2-5)dx + \int -6dy = c$

$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$

$3x^2y + 2xy^2 - 5x - 6y = c$

Q No8:-

$(y\sec^2 x + \sec x\tan x)dx + (\tan x + 2y)dy = 0$

$M = y\sec^2 + \sec x\tan x$, $N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x$, $\frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact Differ.

$\int M dx + \int(\text{terms of } N \text{ free from } x)dy = c$

$\int(y\sec^2 x + \sec x\tan x)dx + \int 2y\,dy = c$

$y\tan x + \sec x + y^2 = 0$

Ex 9.5

Solve by Finding an I.F

Q No 1:-

$$(xy^2 + y)\,dx - x\,dy = 0 \quad \text{--- (1)}$$

$M = xy^2 + y$, $N = -x$

$M_y = 2xy + 1$, $N_x = -1$

$\because M_y \neq N_x$ Non Exact

$$\frac{M_y - N_x}{N} = \frac{2xy + 1 + 1}{-x}$$ Not fnc of x alone

$$\frac{N_x - M_y}{M} = \frac{-1 - 2xy - 1}{xy^2 + y}$$

$$= \frac{-2\cancel{(1 + xy)}}{y\cancel{(xy + 1)}} = -\frac{2}{y}$$

$\therefore$ I.F $= e^{-\int \frac{2}{y} dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$

Multiplying both sides of eq (1) by I.F $= \frac{1}{y^2}$

$$\frac{1}{y^2}(x \cdot y^2 + y)\,dx - \frac{x}{y^2}\,dy = 0$$

$$\left(x + \frac{1}{y}\right)dx - \frac{x}{y^2}\,dy = 0 \quad \text{--- (ii)}$$

Now

$M = x + \frac{1}{y}$, $N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2}$, $N_x = -\frac{1}{y^2}$

$\because M_y = N_x$ $\therefore$ Exact Differ

So,

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$$

$$\int \left(x + \frac{1}{y}\right) dx + \text{Nil} = c$$

$$\frac{x^2}{2} + \frac{x}{y} = c$$

Q.2 $(x^2 + x - y)dx + xdy = 0$ —— (i)

$M = x^2 + x - y$, $N = x$

$M_y = -1$, $N_x = 1$

$M_y \neq N_x$ $\therefore$ Non Exact Diff Eq

$$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$$

$$\therefore \text{I.F} = e^{\int -\frac{2}{x} dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

Multiply both sides of eq by I.F $= \frac{1}{x^2}$

$$\frac{1}{x^2}(x^2 + x - y)dx + \frac{x}{x^2} dy = 0$$

$$\left(1 + \frac{1}{x} - \frac{y}{x^2}\right)dx + \frac{1}{x} dy = 0 \quad \text{—— (ii)}$$

Now, $M = 1 + \frac{1}{x} - \frac{y}{x^2}$ $\qquad N = \frac{1}{x}$

$M_y = N_x$ $\therefore$ (ii) is Exact Diff^l eq.

So,

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int \left(1 + \frac{1}{x} - \frac{y}{x^2}\right) dx + \text{Nil} = C$$

$$x + \ln x + \frac{y}{x} = C$$

Q.3 $dy + \left(\frac{y - \sin x}{x}\right) dx = 0$ —— (i)

$M = \frac{y - \sin x}{x}$ $\qquad N = 1$

$M_y = \frac{1}{x} - 0$ $\qquad N_x = 0$

$M_y \neq N_x$ $\therefore$ is Non Exact Diff^l Eq

Now,

$$\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$$

$$I.F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Multiplying both sides of eq (i) by $I.F = x$

$$x dy + \cancel{x}\left(\frac{y - \sin x}{\cancel{x}}\right) dx = 0 \quad \text{—— (ii)}$$

$M = y - \sin x$ $\quad N = x$

$M_y = 1$ $\quad N_x = 1$

$M_y = N_x$ $\quad \therefore$ is Exact Diff'l Eq.

$\int M dx + \int (\text{Terms of } N \text{ free from } x)\, dy = C$

$\int (y - \sin x)\, dx = C$

$xy + \cos x = C$

Q.4 $(y^4 + 2y)\, dx + (xy^3 + 2y^4 - 4x)\, dy = 0$ — (1)

$M = y^4 + 2y$, $\quad N = xy^3 + 2y^4 - 4x$

$M_y = 4y^3 + 2$, $\quad N_x = y^3 - 4$

$M_y \neq N_x$ $\quad \therefore$ (1) is Non Exact Diff'l eq.

$$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3 + 2)} = -\frac{3}{y}$$

$$I.F = e^{\int -\frac{3}{y} dx} = e^{-3\ln y} = e^{\ln y^{-3}} = y^{-3} = \frac{1}{y^3}$$

Multiplying by $\frac{1}{y^3}$

$$\frac{1}{y^3}(y^4 + 2y)\, dx + \frac{1}{y^3}(xy^3 + 2y^4 - 4x)\, dy = 0$$

$$\left(y + \frac{2}{y^2}\right) dx + \left(x + 2y - \frac{4x}{y^3}\right) dy = 0 \text{ — (11)}$$

Now,

$M = y + \frac{2}{y^2}$, $\quad N = x + 2y - \frac{4x}{y^3}$

$M_y = 1 - \frac{4}{y^3}$, $\quad N_x = 1 + 0 - \frac{4}{y^3}$

$\because M_y = N_x$ is Exact Diff'l Eq.

$\int M dx + \int (\text{terms of } N \text{ free from } x)\, dy = C$

$\int \left(y + \frac{2}{y^2} \right) dx + \int 2y \, dy = C$

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = C$

$xy + \frac{2x}{y^2} + y^2 = C$

Q.5 $(x^2+y^2+2x)dx + 2y\,dy = 0$ —— ①

$M = x^2+y^2+2x$, $N = 2y$

$M_y = 2y$, $N_x = 0$

$M_y \neq N_x$ $\therefore$ is Non Exact Diffd Eq.

$$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2x}$$

$$\frac{M_y - N_x}{N} = \frac{2y-0}{2y} = 1 = x$$

$$I.F = e^{\int 1.dx} = e^x$$

Multiplying both sides of eq ① by $I.F = e^x$

$e^x(x^2+y^2+2x)dx + e^x(2y)dy = 0$ —— ⑪

$M = e^x(x^2+y^2+2x)$ $\quad N = e^x 2y$

$M_y = e^x 2y$ $\quad N_x = e^x 2y$

$M_y = N_x$ $\therefore$ is Exact Diffd eq.

$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x)\, dy = 0$

$$\int e^x(x^2+y^2+2x)dx + \text{Nil} = C$$

$$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x\,dx = C$$

$$x^2 e^x - \cancel{\int 2x e^x dx} + e^x y^2 + \cancel{\int e^x 2x\,dx} = 0$$

$$(x^2+y^2)e^x = C$$ Ans

Q.6 $(x^2+y^2)dx - 2xy\,dy = 0$ ——— ①

$M = x^2+y^2$, $N = -2xy$

$M_y = 2y$, $N_y = -2y$

$M_y \neq N_x$ $\therefore$ is Non Exact Diffl eq.

$$\frac{N_x - M_y}{M} = \frac{-2y-2y}{x^2+y^2}$$

$$\frac{M_y - N_x}{N} = \frac{-2y+2y}{-2xy} = \frac{4y}{-2xy} = \frac{-2}{x}$$

$$I.F = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

Multiplying both sides of eq ① by $I.F = \frac{1}{x^2}$

$$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$$

$$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0 \quad \text{——— (II)}$$

$M = 1+\frac{y^2}{x^2}$ $\qquad N = \frac{-2y}{x}$

$M_y = \frac{2y}{x^2}$ $\qquad N_x = \frac{2y}{x^2}$

$M_y = N_x$ $\therefore$ (II) is Exact Diffl Eq.

$\therefore \int M\,dx + \int(\text{terms of } N \text{ free from } x)\,dy = 0$

$\int\left(1+\frac{y^2}{x^2}\right)dx + \text{Nil} = C$

$$x - \frac{y^2}{x} = C \qquad \underline{\text{Ans}}$$

Q.7 $(y^2+xy)dx - x^2dy = 0$

$$\frac{dy}{dx} = \frac{y^2+xy}{x^2} \quad \text{H.D.E} \quad \text{— (I)}$$

Put $y = vx$ — (II)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (III)}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 + xvx}{x^2}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x^2}(v^2+v)}{\cancel{x^2}}$$

$$x\frac{dv}{dx} = v^2 + v - v$$

$$\frac{dv}{dx} = \frac{dx}{x}$$

$$\int v^2 dv = \int \frac{dx}{x}$$

$$\frac{-1}{v} = \ln x + C$$

$$0 = \ln x + \frac{1}{v} + C$$

$$= \ln x + \frac{x}{y} + C \qquad = y = vs$$

$$\frac{y}{x} = v$$

Q.8 $y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$

$$y - x = \frac{dy}{dx}(x+y)$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{H.D.E} \text{ —— (1)}$$

Put, $y = vx$ —— (11)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ —— (111)}$$

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{x(v-1)}{x(1+v)} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v} - 1 - \cancel{v} - v^2}{1+v}$$

$$\int \frac{(1+v)}{1+v^2} dv = -\int \frac{dx}{x}$$

$$\int \frac{dv}{1+v^2} + \int \frac{v dx}{1+v^2} = -\int \frac{dx}{x}$$

$$\tan^{-1} v + \frac{1}{2}\int \frac{2v dv}{1+v^2} = -\ln x$$

$$\tan^{-1} v + \frac{1}{2}\ln(1+v^2) \ln x = C$$

$$\tan^{-1} v + \ln(1+v^2)^{1/2} + \ln x = C$$

$$\tan^{-1}\frac{y}{x} + \ln\left(1 + \frac{y^2}{x^2}\right)^{1/2} + \ln x = C$$

Ex. 9.6 :-

Q No 1 :-

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x} \quad \text{LDE is } Y$$

$$I.F = e^{\int P dx} = e^{\int \frac{2x+1}{x} dx} = e^{\int \left(\frac{2+1}{x}\right) dx}$$

$$I.F = e^{2x + \ln x} = e^{2x} e^{\ln x} = e^{2x} x$$

$\because$ Sol is given by $\int d\,(Y \times I.F) = \int Q \times I.F\, dx + C$

$$\Rightarrow Ye^{2x}x = \int x\, dx + c$$

$$x\,Ye^{2x} = \frac{x^2}{2} + C$$

Q No 2 :-

$$\frac{dy}{dx} + \frac{3}{x}Y = 6x^2$$

$$I.F = e^{\int P dx} = e^{\int \frac{3}{x} dx} = e^{3\ln x} = e^{\ln x^3} = x^3$$

$\because$ Sol is giving by $\int d(Y \times I.F) = \int Q \times I.F\, dx + C$

$$\Rightarrow \int d(Yx^3) = \int 6x^2 \cdot x^3\, dx + C$$

$$\Rightarrow Yx^3 = \int 6x^5\, dx + C$$

$$Yx^3 = \frac{6x^6}{6} + C$$

$$x^3 y = x^6 + C$$

Q No. 3:-

$$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$$

$$I.F = e^{\int P dx} = e^{\int \frac{1}{x \ln x} dx} = e^{\int \frac{dx/x}{\ln x}} = e^{\ln(\ln x)}$$

$$I.F = \ln x$$

Solve is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + c$

$$\Rightarrow \int d(y \ln x) = \int \frac{3x^2}{\cancel{\ln x}} \cancel{\ln x}\, dx + c$$

$$y \ln x = \frac{\cancel{3}x^3}{\cancel{3}} + c$$

$$y = \frac{x^3 + c}{\ln x}$$

Q No 4:-

$$\cos^3 x \frac{dy}{dx} + y \cos x = \sin x$$

$$\frac{dy}{dx} + \frac{y \cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \sec^3 x\, y = \sec^2 x \tan x$$

$$I.F = e^{\int P dx} = e^{\int \sec^2 x\, dx} = e^{\tan x}$$

Solve is given by $\int d(y \times I.F) - \int Q \times I.F\, dx + c$

$$\Rightarrow \int d(y \times e^{\tan x}) = \int \sec^2 x \tan x\, e^{\tan x}\, dx + c$$

$$\Rightarrow \; ye^{\tan x} = \int \underset{II}{e^t}\,\underset{I}{t}\,dt + c \qquad \tan x = t,\; \sec^2 x\,dx = dt$$

$$= te^t - \int 1 \cdot e^t\,dt + c$$

$$= te^t - e^t + c$$

$$ye^{\tan x} = e^t(t-1) + c$$

$$ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$$

$$y = \tan x - 1 + ce^{-\tan x}$$

Q No 5:-

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$$I.F = e^{\int P\,dx} = e^{\int 3\,dx} = e^{3x}$$

Solve given by $\int d(y \times I.F) = \int Q \times I.F\,dx + c$

$$\Rightarrow \int d(ye^{3x}) = \int 3x^2 \cancel{e^{-3x}}\,\cancel{e^{3x}}\,dx + c$$

$$ye^{3x} = x^3 + c$$

$$y = e^{-3x}(x^3 + c)$$

Q No 6:-

$$\frac{x\,dy}{dx} + (1 + x\cot x)\,y = x$$

$$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

$$I.F = e^{\int P\,dx} = e^{\int \left(\frac{1}{x} + \cot x\right)dx} = e^{\ln x + \ln \sin x}$$

$I.F = e^{\ln(x\sin x)} = x\sin x$

Sol is given by $\int d(Y \times I.F) = \int Q \times I.F\,dx + c$

$\Rightarrow \int d(Yx\sin x) = \int \underset{I}{x}\,\underset{II}{\sin x}\,dx + c$

$Yx\sin x = x(-\cos x) - \int 1\cdot(-\cos x)\,dx$

$= x(-\cos x) + \int \cos dx$

$Yx\sin x = -x\cos x + \sin x + c$

$Y = -\cot x + \frac{1}{x} + \frac{c}{x}\operatorname{cosec} x$

Q No 7:-

$$(x+1)\frac{dy}{dx} - xy = e^x(x+1)^{x+1}$$

$$\frac{dy}{dx} - \frac{x}{(x+1)}Y = e^x(x+1)^x$$

$I.F = e^{\int P dx} = e^{-\int \frac{x}{x+1}dx} = e^{-x\ln(x+1)} = e^{\ln(x+1)^{-x}}$

$I.F = (x+1)^{-x} = \frac{1}{(x+1)^x}$

Solve is given by $\int d(Y \times I.F) = Q \times I.F\,dx + c$

$\Rightarrow \int d\left(y\cdot\frac{1}{(x+1)^x}\right) = \int e^x\cancel{(x+1)^x}\frac{1}{\cancel{(x+1)^x}}dx + c$

$$\frac{Y}{(x+1)^x} = e^x + c$$

$$Y = (e^x + c)(x+1)^x$$

QNo:8.

$$(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{4x^2}{x^2+1} \quad (LDE\ in\ y)$$

$$I.F = e^{\int\left(\frac{2x}{x^2+1}\right)dx} = e^{\ln(x^2+1)} = \underline{\overline{|x^2+1|}}$$

sol is given by

$$\int d(y\,I.F) = \int Q \cdot I.F\, dx + c$$

$$\int d(y(x^2+1)) = \int \frac{4x^2}{(x^2+1)}(x^2+1)\,dx + c$$

$$y(x^2+1) = \frac{4x^3}{3} + c$$

$$3y(x^2+1) = 4x^3 + c'$$