

Submit To Fashan Khalid Sab

Roll No 29016

Submit by Jawad Mahyudeen

Subject APPLIED Technology Math

Semester 2nd (evening)

Ex 9.2

(1)

$$Q1, \frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y dy = \frac{x^2}{1+x^3} dx$$

$$\int y dy = \frac{1}{3} \int \frac{3x^2}{1+x^3} dx$$

$$\frac{y^2}{2} = \frac{1}{3} \ln(1+x^3) + C$$

3 dividing by in both side
multiply

$$\frac{3y^2}{2} = 3 \frac{1}{3} \ln(1+x^3) + 3C$$

$$\frac{3y^2}{2} = \ln(1+x^3) + 3C$$

2 multiply by in both side

$$2 \times \frac{3y^2}{2} = 2 \ln(1+x^3) + 6C$$

$$3y^2 = 2 \ln(1+x^3) + 6C$$

(2)

$$\text{Q2: } \frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x \, dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$\frac{-1}{y} = \cos x + c$$

←—————→

$$\text{Q3: } \frac{dy}{dx} = 1+x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x) \, dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2 \tan^{-1} y = 2x + x^2 + c'$$

$$04: (xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$$

$$[x(y+2) + (y+2)]dx + x(x+2)dy = 0$$

$$[(y+2)(x+1)]dx + x(x+2)dy = 0$$

$$\div \text{ by } x(x+2)(y+2)$$

$$\frac{x+1}{x(x+2)} dx + \frac{1}{y+2} dy = 0$$

$$\int \frac{x+1}{x^2+2x} dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2} \int \frac{2x+2}{x^2+2x} dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2} \ln(x^2+2x) + \ln C$$

$$y+2 = \frac{C}{\sqrt{x^2+2x}}$$

$$\text{Q5: } \frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$$

$$= 2x^2 - 2x - 2 + y - x^2y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1) dx$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1) dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + C$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6C}{6}$$

$$-6\ln|2-y| = (2x^3 - 3x^2 - 6x + 6C) \ln e$$

$$\ln|2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6C) \ln e$$

$$\ln|2-y|^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6C}$$

$$|2-y|^{-6} = e^{2x^3 - 3x^2 - 6x} \cdot e^{6C}$$

$$|2-y|^{-6} = C_1 e^{2x^3 - 3x^2 - 6x}$$

(5)

$$Q6: \operatorname{cosec} y \, dx + \sec x \, dy = 0$$

$$\operatorname{cosec} y \, dx + \sec x \, dy = 0$$

$$\div \operatorname{cosec} y \sec x$$

$$\Rightarrow \frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\Rightarrow \int \cos x \, dx + \int \sin y \, dy = \int 0 \, dx$$

$$\sin x - \cos y = C$$

←—————→

$$Q7: y(1+x) \, dx + x(1+y) \, dy = 0$$

$$\Rightarrow y(1+x) \, dx + x(1+y) \, dy = 0$$

$$\div \text{by } xy$$

$$\Rightarrow \frac{1+x}{x} dx + \frac{1+y}{y} dy = 0$$

$$\Rightarrow \int \left(\frac{1}{x} + 1 \right) dx + \int \left(\frac{1}{y} + 1 \right) dy = \int 0 \, dx$$

$$\Rightarrow \ln x + x + \ln y + y = C$$

$$x + y + \ln(x + y) = C$$

$$Q8: y\sqrt{1+x^2} dx + x\sqrt{1+y^2} dy = 0$$

$$y\sqrt{1+x^2} dx + x\sqrt{1+y^2} dy = 0$$

$$\div by xy$$

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x} dx + \int \frac{\sqrt{1+y^2}}{y} dy = \int 0 dx$$

$$\text{Put } \sqrt{1+x^2} = t$$

$$\text{Put } \sqrt{1+y^2} = z$$

$$1+x^2 = t^2$$

$$1+y^2 = z^2$$

$$2x dx = 2t dt$$

$$2y dy = 2z dz$$

$$x dx = t dt$$

$$y dy = z dz$$

Therefore

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x^2} x dx + \int \frac{\sqrt{1+y^2}}{y^2} y dy = \int 0 dx$$

$$\Rightarrow \int \frac{t \cdot t dt}{t^2 - 1} + \int \frac{z \cdot z dz}{z^2 - 1} = c$$

$$\Rightarrow \int \left(\frac{t^2 - 1 + 1}{t^2 - 1} \right) dt + \int \frac{z^2 - 1 + 1}{z^2 - 1} dz = c$$

$$\Rightarrow \int \left(1 + \frac{1}{t^2 - 1} \right) dt + \int \frac{z^2 - 1 + 1}{z^2 - 1} \left(1 + \frac{1}{z^2 - 1} \right) dz = c$$

$$\Rightarrow t + \frac{1}{2} \ln \left(\frac{t-1}{t+1} \right) + z + \frac{1}{2} \ln \left(\frac{z-1}{z+1} \right) = c$$

$$\Rightarrow \sqrt{1+x^2} + \frac{1}{2} \ln \left[\frac{\sqrt{1+x^2} - 1}{\sqrt{1+x^2} + 1} \right] + \sqrt{1+y^2} + \frac{1}{2} \ln \left[\frac{\sqrt{1+y^2} - 1}{\sqrt{1+y^2} + 1} \right] = c$$

Ex 9.3

7

Q1, $(x-y)dx + (x+y)dy = 0$

$$(x+y)dy = -(x-y)dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{H. DE } \textcircled{1}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

using $\textcircled{1}$ & $\textcircled{2}$ in $\textcircled{1}$

$$v + x \frac{dv}{dx} = \frac{vx-x}{x+vx}$$

$$\frac{x dv}{dx} = \frac{x(v-1)}{x(1+v)} - v$$

$$= \frac{v-1-v-v^2}{1+v}$$

$$\frac{x dv}{dx} = \frac{-(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1} dv = - \int \frac{dx}{x}$$

$$\frac{1}{2} \int \frac{2v dv}{v^2+1} + \int \frac{dv}{v^2+1} = - \int \frac{dx}{x}$$

$$\frac{1}{2} \ln(v^2+1) + \tan^{-1} v = -\ln x + C$$

Q.2

$$(y^2 + 2xy)dx + x^2 dy = 0$$

$$x^2 dy = -(y^2 + 2xy)dx$$

$$\frac{dy}{dx} = -\frac{(y^2 + 2xy)}{x^2} \quad \text{H.D.E} \quad \textcircled{1}$$

Put $y = vx$ ———— \textcircled{ii}

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{---} \quad \textcircled{iii}$$

using \textcircled{ii} & \textcircled{iii} in $\textcircled{1}$

$$v + x \frac{dv}{dx} = -\frac{(v^2 x^2 + 2x vx)}{x^2}$$

$$x \frac{dv}{dx} = -x^2 \frac{(v^2 + 2v)}{x^2} - v$$

$$x \frac{dv}{dx} = -(v^2 + 3v)$$

$$\int \frac{dv}{v^2 + 3v} = -\int \frac{dx}{x}$$

$$\int \frac{1}{v(v+3)} dv = -\int \frac{dx}{x}$$

$$\frac{1}{3} \int \frac{3}{v(v+3)} dv = -\int \frac{dx}{x}$$

$$\frac{1}{3} \int \frac{v+3-v}{v(v+3)} dv = -\int \frac{dx}{x}$$

$$\frac{1}{3} \int \left(\frac{1}{v} - \frac{1}{v+3} \right) dv = -\int \frac{dx}{x}$$

$$\frac{1}{3} \ln v - \frac{1}{3} \ln(v+3) = -\ln x + \ln C$$

$$\ln \left[\frac{v^{1/3}}{(v+3)^{1/3}} \right] = \ln \frac{C}{x}$$

Antilog

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{C}{x}$$

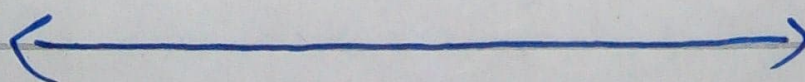
$$x \cdot v^{1/3} = C(v+3)^{1/3}$$

$$x \left(\frac{y}{x} \right)^{1/3} = C \left(\frac{y}{x} + 3 \right)^{1/3}$$

$$x \frac{y^{1/3}}{x^{1/3}} \cdot x^{1/3} = C (y+3x)^{1/3}$$

$$x y^{1/3} = C (y+3x)^{1/3}$$

$$x^3 y = C (y+3x)$$



Q.3. $(x^2 - 3y^2)dx + 2xydy = 0$ (11)

$$2xydx = -(x^2 - 3y^2)dx$$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy} \quad \text{H.D.E} \quad (1)$$

Put $y = vx$ (11)

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad (111)$$

Using (11) & (111) in (1)

$$v + x \frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$x \frac{dv}{dx} = \frac{(3v^2 - 1)x^2}{2vx^2} - v$$

$$x \frac{dv}{dx} = \frac{3v^2 - 1 - 2v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{3v^2 - 1 - 2v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\int \frac{2v}{v^2 - 1} dv = \int \frac{dx}{x}$$

$$\ln(v^2 - 1) = \ln x + \ln c$$

$$\ln\left(\frac{v^2}{x^2} - 1\right) = \ln cx$$

Q.4 $(x^2+3y^2)dz - 2zydy = 0$

$$(x^2+3y^2)dz = 2zydy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{H.O.E} \quad \text{①}$$

Put $y = vx$ _____ ②

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

using ② & ③ in ①

$$v + x \frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x \frac{dv}{dx} = \frac{x^2(1+3v^2) - v}{x^2 2v}$$

$$\frac{x dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$\frac{x dv}{dx} = \frac{1+v^2}{2v}$$

$$\int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$$

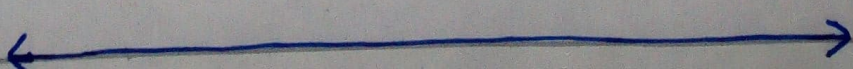
$$\ln(1+v^2) = \ln x + \ln C$$

$$\ln(1+v^2) = \ln Cx$$

$$\left(1 + \frac{y^2}{x^2}\right) = Cx$$

$$\frac{x^2+y^2}{x^2} = Cx$$

$$x^2+y^2 = (Cx) x^2$$



Q.5 $(x^2 + xy + y^2) dx - x^2 dy = 0$

$$(x^2 + xy + y^2) dx = x^2 dy$$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2} \quad \text{H.D.E (i)}$$

Put $y = vx \quad \text{--- (ii)}$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$$

using (ii) & (iii) in (i)

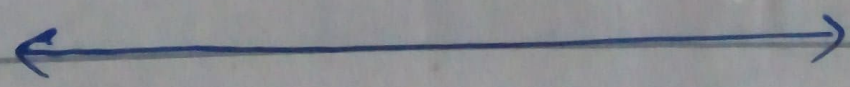
$$v + x \frac{dv}{dx} = \frac{x^2 + x(vx) + v^2 x^2}{x^2}$$

$$\frac{x dv}{dx} = \frac{(1 + v + v^2) x^2}{x^2} - v$$

$$\int \frac{dv}{1 + v^2} = \int \frac{dx}{x}$$

$$\tan^{-1} v = \ln x + C$$

$$\tan^{-1} \left(\frac{y}{x} \right) = \ln x + C$$



$$Q.6 \quad (x^2 + 3xy + y^2) dx - x^2 dy = 0$$

$$(x^2 + 3xy + y^2) dx = x^2 dy$$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} \quad \text{H.D.E} \quad (i)$$

Put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{————— (iii)}$$

using (ii) & (iii) in (i)

$$v + x \frac{dv}{dx} = \frac{x^2 + 3xvx + v^2x^2}{x^2}$$

$$\frac{x dv}{dx} = \frac{x^2(1 + 3v + v^2) - v}{x^2}$$

$$x \frac{dv}{dx} = 1 - 2v + v^2$$

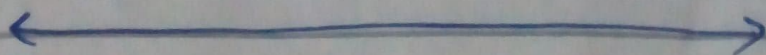
$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\frac{-1}{v+1} = \ln x + C$$

$$\frac{-1}{\left(\frac{y}{x} + 1\right)} = \ln x + C$$

$$\frac{-1}{\frac{y+x}{x}} = \ln x + C$$

$$\frac{-x}{y+x} = \ln x + C$$



$$Q.7 \quad x \sin\left(\frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$$

$$\frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{H.D.F} \quad (1)$$

$$\text{Put } y = vx \quad (2)$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad (3)$$

using (2) & (3) in (1)

$$v + x \frac{dv}{dx} = \frac{vx \sin \frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$$

$$x \frac{dv}{dx} = \frac{x(v \sin v - 1) - v}{x \sin v}$$

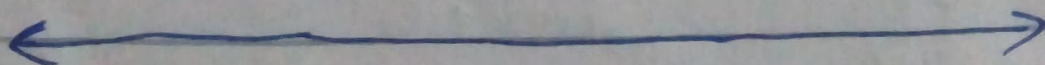
$$x \frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v}$$

$$\int \sin v dv = \int \frac{-dx}{x}$$

$$-\cos v = -\ln x + C$$

$$\cos v = \ln x - C$$

$$\cos \frac{y}{x} = \ln x - C$$



Q.8 $\frac{dy}{dx} = \frac{x+y}{x} \quad \text{--- (i)}$

Put $y = vx \quad \text{--- (ii)}$

$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$

using (ii) & (iii) in (i)

$v + x \frac{dv}{dx} = \frac{x+vx}{x}$

$\frac{xdv}{dx} = \frac{x(1+v)}{x} - v$

$\frac{xdv}{dx} = 1$

$\int dv = \int \frac{dx}{x}$

$v = \ln x + C$

$\frac{y}{x} = \ln x + C$

$\because y(1) = 1$

$\frac{1}{1} = \ln 1 + C$

$1 = 0 + C$

So $\frac{y}{x} = \ln x + 1$

$y = x \ln x + x$

$y = x [\ln x + 1] \quad \text{Ans}$



EX 9.4

Q No 1:- Solve $(3x^2 + 4xy)dx + (2x + 2y)dy = 0$

$$M = 3x^2 + 4xy, \quad N = 2x^2 + 2y$$

$$\frac{\partial M}{\partial y} = 0 + 4x, \quad \frac{\partial N}{\partial x} = 4x + 0$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{So given differ is Exact}$$

Now, $\int M dx + \int (\text{term of } N \text{ free from } x) dy = C$

$$\text{Now, } \int M dx + \int (\text{term of } N \text{ free from } x) dy = C$$

$$\int (3x^2 + 4xy) dx + \int 2y dy = C$$

$$\frac{3x^3}{3} + \frac{4x^2y}{2} + \frac{2y^2}{2} = C$$

$$x^3 + 2x^2y + y^2 = C$$

○ ————— ○

Q No 2:-

$$(2xy + y - \tan y) dx + (x^2 - x \tan^2 y + \sec^2 y) dy = 0$$

$$M = 2xy + y - \tan y, \quad N = x^2 - x \tan^2 y + \sec^2 y$$

$$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y, \quad \frac{\partial N}{\partial x} = 2x - \tan^2 y$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{So giving differ is Exact}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (2xy + y - \tan y) dx + \int \sec^2 y dy = C$$

$$\frac{2x^2 y}{2} + xy - x \tan y + \tan y = C$$

$$x^2 y + xy - x \tan y + \tan y = C$$

Q No 3:-

$$\frac{(x+y)}{(y-1)} dx - \frac{1}{2} \left(\frac{x+1}{y-1} \right)^2 dy = 0$$

$$M = \frac{x+y}{y-1}, \quad N = -\frac{1}{2} \left(\frac{x+1}{y-1} \right)^2$$

$$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1) - (x+y)(1)}{(y-1)^2}, \quad N = -\frac{1}{2} \left(\frac{x^2 + 2x + 1}{(y-1)^2} \right)$$

$$= \frac{\cancel{y-1} - x - \cancel{y}}{(y-1)^2}, \quad \frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \because \text{given differ is Exact} = \frac{-x-1}{(y-1)^2}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int \left(\frac{x+y}{y-1} \right) dx + \int -\frac{1}{2(y-1)^2} dy = C$$

$$\frac{1}{y-1} \int (x+y) dx + \left(-\frac{1}{2} \right) \left(\frac{-1}{y-1} \right)^{-2} dy = C$$

$$\frac{1}{y-1} \left(\frac{x^2}{2} + xy \right) + \left(-\frac{1}{2} \right) \left(\frac{-1}{y-1} \right) = c$$

$$\frac{x^2 + 2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2 + 2xy + 1 = c'(y-1) \text{ Ans.}$$

QNO4:-

$$\frac{dy}{dx} = - \frac{(ax+hy)}{hx+by}$$

$$(hx+by) dy = -(ax+hy) dx$$

$$(ax+hy) dx + (hx+by) dy = 0$$

$$M = ax+hy, \quad N = hx+by$$

$$\frac{\partial M}{\partial y} = a+h, \quad \frac{\partial N}{\partial x} = h$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ Hence Exact diffx}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$$

$$\int (ax+hy) dx + \int by dy = c$$

$$\frac{ax^2}{2} + hxy + \frac{by^2}{2} = c$$

$$ax^2 + 2hxy + by^2 = c'$$

Q No 5:-

$$(1 + \ln xy) dx + (1 + \frac{x}{y}) dy = 0$$

$$M = 1 + \ln xy, N = 1 + \frac{x}{y}$$
$$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy} \cdot x, \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$$

$$\frac{\partial M}{\partial y} = \frac{1}{y}, \frac{\partial N}{\partial x} = \frac{1}{y}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ Hence Exact Differ}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$$

$$\int (1 + \ln xy) dx + \int 1 \cdot dy = c$$

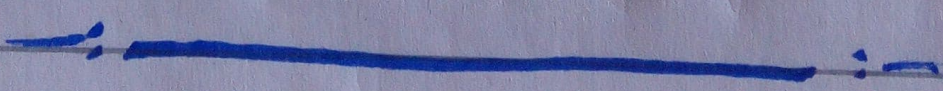
$$\int dx + \int \ln xy dx + \int dy = c$$

$$x + \left[\ln xy \cdot (x) - \int \frac{1}{xy} \cdot y \cdot x dx \right] + y = c$$

$$x + x \ln xy - \int dx + y = c$$

$$\cancel{x} + x \ln xy - \cancel{x} + y = c$$

$$x \ln xy + y = c$$



Q No 7:-

$$M (6xy + 2y^2 - 5) dx + (3x^2 + 4xy - 6) dy$$

$$M = 6xy + 2y^2 - 5, N = 3x^2 + 4xy - 6$$

$$\frac{\partial M}{\partial y} = 6x + 4y, \quad \frac{\partial N}{\partial x} = 6x + 4y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Hence Exact Differential}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (6xy + 2y^2 - 5) dx + \int -6 dy = C$$

$$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = C$$

$$3x^2y + 2xy^2 - 5x - 6y = C$$

Q No 8:-

$$(x \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy = 0$$

$$M = x \sec^2 x + \sec x \tan x, N = \tan x + 2y$$

$$\frac{\partial M}{\partial y} = \sec^2 x, \quad \frac{\partial N}{\partial x} = \sec^2 x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Hence Exact Differ.}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (x \sec^2 x + \sec x \tan x) dx + \int 2y dy = C$$

$$x \tan x + \sec x + y^2 = C$$

Ex 9.5.

(22)

Solve by Finding an I.F

Q No 1:-

$$(xy^2 + y) dx - x dy = 0 \quad \text{--- (1)}$$

$$M = xy^2 + y, \quad N = -x$$

$$M_y = 2xy + 1, \quad N_x = -1$$

$\therefore M_y \neq N_x$ Non Exact

$$\frac{M_y - N_x}{N} = \frac{2xy + 1 + 1}{-x} \quad \text{Not fun of } x, \text{ at } x/y$$

$$\frac{N_x - M_y}{M} = \frac{-1 - 2xy - 1}{xy^2 + y}$$

$$= \frac{-2(1 + xy)}{y(xy + 1)} = -\frac{2}{y}$$

$$\therefore \text{I.F} = e^{\int \frac{-2}{y} dy} = e^{-2 \ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Multiplying both sides of eq (1) by

$$\text{I.F} = \frac{1}{y^2}$$

$$\frac{1}{y^2} (xy^2 + y) dx - \frac{x}{y^2} dy = 0$$

$$\left(\frac{x+1}{y} \right) dx - \frac{x}{y^2} dy = 0 \quad \text{--- (ii)}$$

Now

$$M = x + \frac{1}{y}, \quad N = -\frac{x}{y^2}$$

$$M_y = -\frac{1}{y^2}, \quad N_x = -\frac{1}{y^2}$$

$\therefore M_y = N_x \therefore$ Exact Differ

So,

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$$

$$\int \left(x + \frac{1}{y}\right) dx + N dy = c$$

$$\frac{x^2}{2} + \frac{x}{y} = c$$

Q.2 $(x^2 + x - y) dx + x dy = 0$ — ①

$$M = x^2 + x - y, \quad N = x$$

$$M_y = -1, \quad N_x = 1$$

$M_y \neq N_x \therefore$ Non Exact Diff Eq

$$\frac{M_y - N_x}{N} = \frac{-1 - 1}{x} = \frac{-2}{x}$$

$$\therefore \text{I.F.} = e^{\int \frac{-2}{x} dx} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

Multiply both sides of eq by I.F. = $\frac{1}{x^2}$

$$\frac{1}{x^2} (x^2 + x - y) dx + \frac{x}{x^2} dy = 0$$

$$\left(1 + \frac{1}{x} - \frac{y}{x^2}\right) dx + \frac{1}{x} dy = 0 \quad \text{--- ②}$$

$$\text{Now, } M = 1 + \frac{1}{x} - \frac{y}{x^2}$$


$$N = \frac{1}{x}$$

$$M_y = N_x \quad \therefore \textcircled{1} \text{ is Exact Diff'l eq.}$$

So,

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int \left(1 + \frac{1}{x} - \frac{y}{x^2}\right) dx + \text{Nil} = C$$
$$x + \ln x + \frac{y}{x} = C$$



$$\text{Q.3 } dy + \left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{--- } \textcircled{1}$$

$$M = \frac{y - \sin x}{x}$$

$$N = 1$$

$$M_y = \frac{1}{x} - 0$$

$$N_x = 0$$

$$M_y \neq N_x \quad \therefore \text{is Non Exact Diff'l Eq}$$

Now,

$$\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = -\frac{1}{x}$$

$$\text{I.F} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Multiplying both sides of eq $\textcircled{1}$ by I.F = x

$$x dy + x \left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{--- } \textcircled{11}$$

$$M = y - \sin x$$

$$N = x$$

$$M_y = 1$$

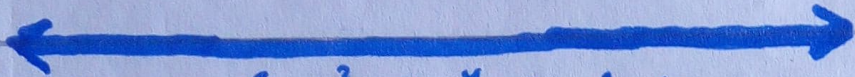
$$N_x = 1$$

$M_y = N_x \therefore$ is Exact Diff'l Eq.

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (y - \sin x) dx = C$$

$$xy + \cos x = C$$



Q.4 $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0$ — (1)

$$M = y^4 + 2y$$

$$N = xy^3 + 2y^4 - 4x$$

$$M_y = 4y^3 + 2$$

$$N_x = y^3 - 4$$

$M_y \neq N_x \therefore$ (1) is Non Exact Diff'l eq.

$$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3 + 2)} = -\frac{3}{y}$$

$$\text{I.F.} = e^{\int -\frac{3}{y} dy} = e^{-3 \ln y} = e^{\ln y^{-3}} = y^{-3} = \frac{1}{y^3}$$

Multiplying by $\frac{1}{y^3}$

$$\frac{1}{y^3} (y^4 + 2y) dx + \frac{1}{y^3} (xy^3 + 2y^4 - 4x) dy = 0$$

$$\left(y + \frac{2}{y^2} \right) dx + \left(x + 2y - \frac{4x}{y^3} \right) dy = 0 \text{ — (11)}$$

Now,

$$M = y + \frac{2}{y^2}$$

$$N = x + 2y - \frac{4x}{y^3}$$

$$M_y = 1 - \frac{4}{y^3}$$

$$N_x = 1 + 0 - \frac{4}{y^3}$$

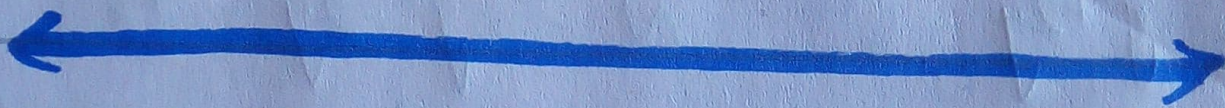
$\therefore My = Nx$ is Exact Diff Eq.

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int \left(y + \frac{2x}{y^2} \right) dx + \int 2y dy = C$$

$$xy + \frac{2x}{y} + \frac{2y^2}{2} = C$$

$$xy + \frac{2x}{y} + y^2 = C$$



$$Q.5 \quad (x^2 + y^2 + 2x)dx + 2ydy = 0 \quad \text{--- (i)}$$

$$M = x^2 + y^2 + 2x \quad / \quad N = 2y$$

$$M_y = 2y \quad / \quad N_x = 0$$

$M_y \neq N_x \quad \therefore$ is Non Exact Diffd Eq.

$$\frac{N_x - M_y}{M} = \frac{0 - 2y}{x^2 + y^2 + 2x}$$

$$\frac{M_y - N_x}{N} = \frac{2y - 0}{2y} = 1 = x'$$

$$I.F = e^{\int 1 \cdot dx} = e^x$$

Multiplying both sides of eq (i) by I.F = e^x

$$e^x (x^2 + y^2 + 2x)dx + e^x (2y)dy = 0 \quad \text{--- (ii)}$$

$$M = e^x (x^2 + y^2 + 2x) \quad N = e^x 2y$$

$$M_y = e^x 2y \quad N_x = e^x 2y$$

$M_y = N_x \quad \therefore$ is Exact Diffd eq.

$$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x) dy = 0$$

$$\int e^x (x^2 + y^2 + 2x) dx + Nil = C$$

$$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x dx = C$$

$$\int x^2 e^x - \int 2x e^x dx + e^{2x} + \int e^x 2x dx = 0$$

$$\cdot (x^2 + y^2) e^x = C \quad \underline{\underline{\text{Ans}}}$$

Q.6 $(x^2+y^2)dx - 2xy dy = 0$ ——— ①

$$M = x^2+y^2, \quad N = -2xy$$

$$M_y = 2y, \quad N_x = -2y$$

$M_y \neq N_x \therefore$ is Non Exact Diff'l Eq.

$$\frac{N_x - M_y}{M} = \frac{-2y - 2y}{x^2+y^2}$$

$$\frac{M_y - N_x}{N} = \frac{-2y + 2y}{-2xy} = \frac{0}{-2xy} = \frac{-2}{x}$$

$$I.F = e^{\int \frac{-2}{x} dx} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

Multiplying both sides of eq/① by I.F = $\frac{1}{x^2}$

$$\frac{1}{x^2} (x^2+y^2)dx - \frac{1}{x^2} (2xy) dy = 0$$

$$\left(1 + \frac{y^2}{x^2}\right)dx - \frac{2y}{x} dy = 0 \quad \text{————— ②}$$

$$M = 1 + \frac{y^2}{x^2}, \quad N = \frac{-2y}{x}$$

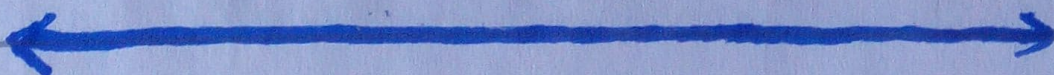
$$M_y = \frac{2y}{x^2}, \quad N_x = \frac{2y}{x^2}$$

$M_y = N_x \therefore$ ② is Exact Diff'l Eq.

$$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x) dy = 0$$

$$\int \left(1 + \frac{y^2}{x^2}\right) dx + N \cdot 0 = C$$

$$x - \frac{y^2}{x} = C \quad \underline{\underline{\text{Ans}}}$$



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Q.7 $(y^2 + xy)dx - x^2 dy = 0$

$$\frac{dy}{dx} = \frac{y^2 + xy}{x^2} \quad \text{H.D.E} \quad \text{--- (i)}$$

Put $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$$

$$v + x \frac{dv}{dx} = \frac{v^2 x^2 + x vx}{x^2}$$

$$v + x \frac{dv}{dx} = \frac{x^2(v^2 + v)}{x^2}$$

$$x \frac{dv}{dx} = v^2 + v - v$$

$$x \frac{dv}{dx} = \frac{dx}{x}$$

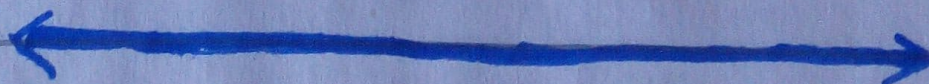
$$\int v^2 dv = \int \frac{dx}{x}$$

$$\frac{-1}{v} = \ln x + C$$

$$0 = \ln x + \frac{1}{v} + C$$

$$= \ln x + \frac{x}{y} + C \quad = y = vx$$

$$\frac{y}{x} = v$$



Q.8

$$y - x \frac{dy}{dx} = x + y \frac{dy}{dx}$$

$$y - x = \frac{dy}{dx} (x + y)$$

$$\frac{dy}{dx} = \frac{y - x}{x + y} \quad \text{H.D.E} \quad \text{--- (i)}$$

Put, $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$$

$$v + x \frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x \frac{dv}{dx} = \frac{x(v-1) - v}{x(1+v)}$$

$$x \frac{dv}{dx} = \frac{v-1-v}{1+v}$$

$$\int \frac{(1+v)}{1+v^2} dv = - \int \frac{dx}{x}$$

$$\int \frac{dv}{1+v^2} + \int \frac{v dv}{1+v^2} = - \int \frac{dx}{x}$$

$$\tan^{-1} v + \frac{1}{2} \int \frac{2v dv}{1+v^2} = - \ln x$$

$$\tan^{-1} v + \frac{1}{2} \ln(1+v^2) + \ln x = C$$

$$\tan^{-1} v + \ln(1+v^2)^{1/2} + \ln x = C$$

$$\tan^{-1} \frac{y}{x} + \ln\left(1 + \frac{y^2}{x^2}\right)^{1/2} + \ln x = C$$

Ex. 9.6 :-

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Q No 1 :-

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x} \text{ L.O.E is } y$$

$$\text{I.F} = e^{\int P dx} = e^{\int \frac{2x+1}{x} dx} = e^{\int \left(\frac{2x}{x} + \frac{1}{x}\right) dx}$$

$$\text{I.F} = e^{2x + \ln x} = e^{2x} \cdot e^{\ln x} = e^{2x} \cdot x$$

∴ Sol is given by $\int d(y \times \text{I.F}) = \int \rho \times \text{I.F} dx + C$

$$\Rightarrow y e^{2x} x = \int x dx + C$$

$$x y e^{2x} = \frac{x^2}{2} + C$$

Q No 2 :-

$$\frac{dy}{dx} + \frac{3}{x} y = 6x^2$$

$$\text{I.F} = e^{\int P dx} = e^{\int \frac{3}{x} dx} = e^{3 \ln x} = e^{\ln x^3} = x^3$$

∴ Sol is giving by $\int d(x \times \text{I.F}) = \int \rho \times \text{I.F} dx + C$

$$\Rightarrow \int d(x^3) = \int 6x^2 \cdot x^3 dx + C$$

$$\Rightarrow x^3 = \int 6x^5 dx + C$$

$$x^3 y = \frac{6x^6}{6} + C$$

$$x^3 y = x^6 + C$$

Q No. 3 :-

$$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$$

$$\text{I.F} = e^{\int P dx} = e^{\int \frac{1}{x \ln x} dx} = e^{\int \frac{dx}{x \ln x}} = e^{\ln(\ln x)}$$

$$\text{I.F} = \ln x$$

Solve is given by $\int d(y \times \text{I.F}) = \int \phi \times \text{I.F} dx + c$

$$\Rightarrow \int d(y \ln x) = \int \frac{3x^2}{\ln x} \ln x dx + c$$
$$y \ln x = \frac{3x^3}{3} + c$$

$$y = \frac{x^3 + c}{\ln x}$$

----- :-

Q No 4 :-

$$\cos^3 x \frac{dy}{dx} + y \cos x = \sin x$$

$$\frac{dy}{dx} + \frac{y \cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \sec^2 x y = \sec^2 x \tan x$$

$$\text{I.F} = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$$

Solve is given by $\int d(y \times \text{I.F}) = \int \phi \times \text{I.F} dx + c$

$$\Rightarrow \int d(y e^{\tan x}) = \int \sec^2 x \tan x e^{\tan x} dx + c$$

$$\Rightarrow y e^{\tan x} = \int \frac{e^t + 1}{t} dt + c \quad \begin{matrix} \tan x = t \\ \sec^2 x dx = dt \end{matrix}$$

$$= te^t - \int 1 \cdot e^t dt + c$$

$$= te^t - e^t + c$$

$$y e^{\tan x} = e^t (t-1) + c$$

$$y e^{\tan x} = e^{\tan x} (\tan x - 1) + c$$

$$y = \tan x - 1 + c e^{-\tan x}$$

Q No 5 :-

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$$\text{I.F} = e^{\int P dx} = e^{\int 3 dx} = e^{3x}$$

Solve given by $\int d(y \times \text{I.F}) = \int Q \times \text{I.F} dx + c$

$$\Rightarrow \int d(y e^{3x}) = \int 3x^2 e^{-3x} e^{3x} dx + c$$

$$y e^{3x} = x^3 + c$$

$$y = e^{-3x} (x^3 + c)$$

Q No 6 :-

$$\frac{ndy}{dn} + (1 + n \cot n) y = n$$

$$\frac{dy}{dn} + \left(\frac{1}{n} + \cot n\right) y = 1$$

$$\int P dn \quad \int \left(\frac{1}{n} + \cot n\right) dn$$

$$\text{I.F} = e \quad = e \quad = e^{\int \frac{1}{n} + \cot n dn} = e^{\ln n + \ln \sin n}$$

$$I.F = e^{\int n(x \sin x)} = x \sin x$$

Sol is given by $\int d(Y \times I.F) = \int Q \times I.F dx + c$

$$\Rightarrow \int d(Y \times \sin x) = \int x \sin x dx + c$$

$$Y \times \sin x = x(-\cos x) - \int 1 \cdot (-\cos x) dx$$

$$= x(-\cos x) + \int \cos x dx$$

$$Y \times \sin x = -x \cos x + \sin x + c$$

$$Y = -\cot x + \frac{1}{x} + \frac{c}{x} \operatorname{cosec} x$$

Q. NO 7 :-

$$(n+1) \frac{dy}{dx} - ny = e^x (n+1)^{n+1}$$

$$\frac{dy}{dx} - \frac{n}{(n+1)} y = e^x (n+1)^n$$

$$I.F = e^{\int P dx} = e^{-\int \frac{n}{n+1} dx} = e^{-n \ln(n+1)} = e^{\ln(n+1)^{-n}}$$

$$I.F = (n+1)^{-n} = \frac{1}{(n+1)^n}$$

Solve is given by $\int d(Y \times I.F) = \int Q \times I.F dx + c$

$$\Rightarrow \int d\left(y \cdot \frac{1}{(n+1)^n}\right) = \int e^x (n+1)^n \frac{1}{(n+1)^n} dx + c$$

$$\frac{y}{(n+1)^n} = e^x + c$$

$$y = (e^x + c)(n+1)^n$$

Q No: 8,

$$(x^2+1) \frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \left(\frac{2x}{x^2+1} \right) y = \frac{4x^2}{x^2+1} \text{ (LDE in } y \text{)}$$

$$\text{I.F} = e^{\int \left(\frac{2x}{x^2+1} \right) dx} = e^{\ln(x^2+1)} = \underline{\underline{x^2+1}}$$

Sol is given by

$$\int d(Y \cdot \text{I.F}) = \int 0 = \text{I.F} dx + c$$

$$\int d(Y(x^2+1)) = \int \frac{4x^2}{(x^2+1)} (x^2+1) dx + c$$

$$Y(x^2+1) = \frac{4x^3}{3} + c$$

$$3Y(x^2+1) = 4x^3 + c'$$