NAME

SHAHZAIB AKRAM

Roll No

29009

Subject

Math

Submitted by

Farhan khalid
Sahib

Exercise 9.2

Q No: 01 $\frac{d}{dx} = \frac{x^2}{y(x)(1+x^2)}$

$\frac{d}{dx} = \frac{x^2}{y(1+x^2)}$

$\frac{d}{dy} = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$

$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + c$

$\frac{3y^2}{2} = \frac{1}{3}\ln(1+x^3) + 3c$

$3y^2 = 2\ln(1+x^3) + 6c$

$3y^2 = 2\ln(1+x^3) + c$

Q No: 02

$\frac{dy}{dx} + y^2 \sin x = 0$

$\frac{dy}{dx} + y^2 \sin x = 0$

$\frac{dy}{dx} = -y^2 \sin x$

$\int \frac{dy}{dx} = -y^2 \sin x$

$y^{-1} = -(-\cos x) + c$

$-\frac{1}{y} = \cos x + c$

QNO: 03

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{dx} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

QNO: 04

$$-by \operatorname{Cosec} y \operatorname{Sec} x$$

$$\frac{1}{\operatorname{Sec} x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\int \cos x\,dx + \int \sin y\,dy = \int 0\,dx$$

$$\sin x - \cos y = c$$

Q NO:- 5

$$(4x + 3y^2)dx + 2xy\,dy = 0$$

$M = 4x + 3y^2$ $\quad N = 2xy$

$My = 0 + 6y$ $\quad Ny = 2y$

$My \neq Nx$ Not exuat

$$\frac{Nx - My}{M} = \frac{2y - 6y}{4x + 3y^2}$$

$$\frac{My - Nx}{N} \cdot \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$$

$$\int \frac{2}{x} dx \quad 2 \ln x \quad \ln x^2$$

If $e = e = e = x^2$

Multply both Side of (1) by I.F

$$(4x^3 + 3y^2x^2)dx + (2x^3y)dy = 0 \quad \text{— (11)}$$

$M_y = N_x \quad \therefore$ Exact Eq

$\int M dx + \int (\text{tarms of } N \; x) \, dy = c$

$\int (4x^3 + 3y^2x^2) dx + Nil = c$

$\cancel{4} = \frac{x^4}{\cancel{4}} + \cancel{3}y^2 \frac{x^3}{\cancel{3}} = c$

$x^4 + y^2x^3 = c$

Q No: 6

$\frac{dy}{dx} = e^2x + y - 1$

$dy = (e^{2x} + y - 1) dx - dy = 0$ —— (i)

$M = e^{2x} + y - 1 \qquad N = -1$

$M_y = 1 \qquad N_x = 0$

$M_y \neq N_x \;\therefore$ is Non exact

$\frac{M_x - M_y}{N} = \frac{1-0}{-1} = -1 = -x^2$

$if = e^{\int -1 dx} = -e^{-x}$

multiply both Side eq (1) by e^{-x}

$e^{-x}(e^{2x} + y - 1) dx - e^{-x} dy = 0$

$(e^x + e^{-x}y) - e^{-x}) dx - e^{-x} dy = 0$ —— (ii)

$M_y = e^x + e^{-x}y - e^{-x} \quad N = e^{-x}$

$M_y = N_x$ —— (1) Exact

$\int (e^x + e^{-x}y - e^{-x}) dx + Nil = c$

$e^x - e^{-x}y + e^{-x} = c$

Q No:1

EXERCISE → 9.3

$(y+2xy)dx + x^3 dy = 0$

$x^2 dy = -(y^2 + 2xy)dx$

$\frac{dy}{dx} = \frac{(y^2+2xy)}{x^2}$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Using (ii) of (iii) in (i)

$v + x\frac{dv}{dx} = \frac{-(v^2e^2 + 2x\,vx)}{x^2}$

$x\frac{dv}{dx} = \frac{e^2(v^2+2v)}{e^2} - v$

$x\frac{dv}{dx} = -(v^2+3v)$

$\int \frac{dv}{v^2+3v} = -\int (v^2+3v)$

$\int \frac{dv}{v^2+3v} = -\int \frac{dx}{x}$

$\int \frac{1}{v(v+3)} dv = -\int \frac{dx}{x}$

$\frac{1}{3}\int \frac{3}{v(v+3)} dv = -\int \frac{dx}{x}$

$\frac{1}{}\ln v - \frac{1}{}\ln(v+3) = \ln x + \ln c$

$\ln\left(\frac{v^{1/3}}{v+3^{1/2}}\right) = \ln\frac{L}{x}$

$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{L}{x}$

Q NO :- 2

$(x^2+y^2+2x)dx + 2y\,dy = 0$

$M = x^2+y^2+2x \qquad N = 2y$

$M_y = 2y \qquad N_x = 0$

$M_y \neq N_x$ is Not exact

$\dfrac{N_x + M_y}{M} = \dfrac{0-2y}{x^2+y^2+2x}$ Not of x dy

If $e^{\int 1 dx} = e^x$

e^x multiply both side.

$e^x(x^2+y^2+2x)dx + e^x(2y)dy = 0$

$\int M dx + \int$ terms of N terms x $dy = c$

$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x\,dx = c$

$x^2e^x - \int 2xe^x dx + e^x y^2 + \int e^x 2x\,dx = c$

Q NO :- 3

$(x^2+y^2)dx - 2xy\,dy = 0$

$M = x^2+y^2 \qquad N = 2xy$

$M_y = 2y \qquad N_x = -2y$

$M_y \neq N_x$ is not exact

$\dfrac{M_x - M_y}{M} = \dfrac{-2y-2y}{x^2+y^2}$

$\dfrac{M_y - M_x}{N} = \dfrac{2y+2y}{-2xy} = \dfrac{-4y}{-2x} = \dfrac{-2}{2}$

$\int \dfrac{-2}{x} dx \; -2\ln x + \ln x^{-2}$

If $e = e = e = x^{-2} = \dfrac{1}{x^2}$

$$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$$

$$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}\,dy = 0$$

$$M_y = \frac{2y}{x^2} \qquad N_x = \frac{-2y}{x^2}$$

$M_y = N_x$ is exact.

Q NO:- 4

$$(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$$

$M = y^4+2y \qquad N = xy^3+2y^4-4x$

$M_y = 4y^3+2 \qquad N_x = y$

$M_y \neq N_x$ is Not exact

$$\frac{N_x - M_y}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} = \frac{3y^3-6}{y(y^3+2)} = \frac{-3}{y}$$

$\int -\frac{3}{1}dy - 3\ln y \quad \ln y^{-3}$

$$\frac{1}{y^3}(y^4+2x)dx + \frac{1}{y^3}\left(x^2y+2y^4-\frac{4x}{y^3}\right)dy = 0$$

Now $M = y+\frac{2}{y^2} \qquad N = x+2y-\frac{4xy}{y^3}$

$M_y = N_x$ is exact

$\int M dx + \int(\text{tanx af N turms})dy = c$

$\int\left(y+\frac{2}{y}\right)dx + \int 2y\,dy = c$

$$xy+\frac{2x}{y^2}+\frac{2y^2}{2}=c$$

$$xy+\frac{2x}{y^2}+y^2=c$$

EXERGISE :- 9.4

Q NO:- $(2xy+y-\tan y)dx+(x^2-x\tan^2 y)+\sec^2 y)dy=0$

$M=2x+y-\tan y \qquad N=x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial m}{\partial y}=2x+1-\sec^2 y \qquad \frac{\partial N}{\partial x}=2x-\tan^2 y+y$

$=2x\tan^2 y$

$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}$

$\int Mdx+\int(\tan x$ of $oNx)dy=c$

$\int(2xy+(-\tan y)dx+\int\sec^2 y\,dy=c$

$\frac{2x^2y}{2}+xy-x\tan y+\tan y=c$

Q NO:- 2: $\frac{(x+y)}{(y-1)}dx=-\frac{1}{2}\frac{(x+1)}{(y-1)}dy=0$

$M=\frac{x+y}{y-1} \qquad N=-\frac{1}{2}\frac{(x+1)^2}{(y-1)^2}$

$N=\frac{-1}{2}\frac{(x^2-2x+1)}{(y-1)^2}$

$\frac{\partial M}{\partial y}=\frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2} \qquad \frac{\partial N}{\partial x}=\frac{-2(2x+y)}{-2(y+1)}$

$$= \frac{y-1-x-1}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial x} \quad \text{equal}$$

$$\int M dx + \int (\text{tanx of N term } x)\, dy = c$$

$$\int \frac{(x+y)}{(y-1)} dx + \int \frac{-1}{2(y-1)} dy = c$$

$$\left(\frac{1}{y-1}\right)\int (x+y)\,dx + \left(\frac{-1}{2}\right)\int (y-1)^{-2} dy = c$$

$$\frac{1}{(y-1)}\left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)}$$

$$x^2 - 2xy + 1 = c'(y-1)$$

Q No:- 3

$$\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$$

$$(hx+by)dy = (-ax+hy)\,dx$$

$$(ax+hy)dx + (hx+by)dy = 0$$

$$M = ax+hy \qquad N = hx+by$$

$$\frac{\partial M}{\partial y} = 0+h \qquad \frac{\partial N}{\partial x} = h$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M dx + \int (\text{terms of N term } x)\, dy = c$$

$\int (ax+hy)dx + \int by\,dy = c$

$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c'$

QNO:-4

$(1+\ln x)dx + (1+\frac{x}{y})dy = 0$

$M = 1 + \ln xy \qquad N = 1 + \frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy}x \qquad \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence exist

$\int M dx + \int (\text{terms of } N \text{ terms } x)\,dy = c$

$\int dx + \int 1 \ln xy\, dx + \int dy = c$

$x + [\ln xy\,(x) - \int \frac{1}{xy} y \cdot x\, dx] + y = c$

$x + x\ln xy - \int dx + y = c$

$\cancel{x} + x\ln xy - \cancel{x} + y = c$

$x\ln xy + y = c$

QNO:-5

$(\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = c$

$M = y(\cos x + 2xe^y \qquad N = \sin + x^2e^y - 1$

$\frac{dx}{dy} = \cos x + 2xe \quad \frac{dN}{dx} = \cos x + 2xe^y$

$\frac{dM}{dy} = \frac{dN}{dx}$ Hence exist

$\int (y\cos x + 2xe^y)dx + \int -1\, dy = c$

$$y - \sin + x \frac{x^2 e^y}{y} - y = c$$

QNo:-6

$(y \cdot \sec^2 x + \sec x \tan x) + (\tan x + 2y) dy$

$M = y \sec^2 + \sec x \tan x = N \tan + 2y$

$\frac{dM}{dy} = \sec^2 x \qquad \frac{dN}{dx} = \sec^2 x$

$\frac{dM}{dy} = \frac{dN}{dx}$ Hence exist

$\int M dx + \int$ terms N terms $x) dy = c$

$\int (y \sec^2 + \sec x \tan x) dx + \int 2y \, dy = c$

$y \tan x + \sec x + y^2 = c$

EXERCISE: 9.5

QNo:1

$(x^2 + x - y) dx + x \, dy = 0$

$M = x^2 + x - y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$

$\int \frac{-2}{x} dx - 2 \ln x \ \ln x^{-2}$

$\frac{1}{x^2}(x^2 + x - y) dx + \frac{x}{x^2} dy = 0$

$\left(1 + \frac{1}{x^2} \frac{-y}{x^2}\right) dx + \frac{1}{x} dy = 0$

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Now

$M = 1 + \frac{1}{x} - \frac{y}{x^2}$ $\quad N = \frac{1}{x}$

$\int (1 + \frac{1}{x} - \frac{y}{x^2}) dx$ Nil $= c$

$x + \ln x + \frac{y}{x} = c$

Q No - 02.

$dy + \frac{(y - \sin x)}{x} dx = 0.$

$M = \frac{y - \sin x}{x}$ $\quad N = 1.$

$M_y = \frac{1}{x} = 0$ $\quad N_x = 0$

$M_y = \frac{1}{x} = 0$ $\quad N_x = 0$

$M_y \neq N_x$ is not exact Now

$\frac{M_y - N_x}{N} = \frac{1/x - 0}{1} = \frac{1}{x}$

$if = e^{\int \frac{1}{x} dx} = e^{\ln x} = x.$

Multiplying both sides of eq — (1)

$x\, dy + \frac{(y - \sin x)}{x} dx = 0$ — (ii)

$M = y - \sin x$ $\quad N = x$

$M_y = 1$ $\quad N_x = 1$

$M_y = N_x$ is exact.

$\int M\,dx + \int (\text{terms of } N \text{ terms})\,dy = c$

$\int (y - \sin x)\,dx = c$

$xy + \cos x = c$

Q No - 03.

$(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

$M = y^4 + 2y \quad N = xy^3 + 2y^4 - 4x$

$M_y = 4y^3 + 2 \quad N_x = y^3 - 4$

$M \neq N_x$ is not exact.

$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 - 2y} = \frac{-3y^3 - 6}{y(y^3+2)} = \frac{-3}{y}$

$\int \frac{-3}{y}\,dy - 3 \ln y \quad \ln y^{-3}$

if $e = e = e \quad y^{-3} = \frac{1}{y^3}$

$\frac{1}{y^3}(y^4 + 2y)\,dx + \frac{1}{y^3}(x^3 y + 2y^4 - 4x)\,dy = 0$

Now $M = y + \frac{2}{y^2}$ $\quad N = x + 2y - \frac{4x}{y^3}$

$M_y = \frac{1-4}{y^3}$ $\quad N_x = 1 + 0 - \frac{4}{y^3}$

$M_y = N_x$ is exact.

$\int M dx + \int (\text{tan } x \text{ of } N \text{ terms } x) dy = c$.

$\int (y + \frac{2}{y}) dx + \int 2y \, dy = c$.

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$

$xy + \frac{2x}{y^2} + y^2 = c$.

Q No: 4

$(2xy - 3)dx + (x^2 + 4y)dy = 0$

$M = 2xy - 3 \qquad N = x^2 + 4y$

$\frac{dM}{dy} = 2x \qquad \frac{dN}{dx} = 2x$

$\frac{dM}{dN} = \frac{dN}{dx}$ Hence exact

$\int M dx + \int (\text{term of } N \text{ term } x) dy = c$

$\int (2xy - 3) dx + \int 4y \, dy = c$

$$\frac{2x^2y}{2} - 3x + \frac{4y^2}{2} = c$$

$$x^2y - 3x + 2y^2 = c$$

$$\because 4(1) = 2$$
$$\because 2 - 3 + 8 = c$$
$$\because 7 = c$$

Hence $x^2y - 3x + 2y^2 = 7$

Exercise 9.6

QNo:-1

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-1}x$$

$$\int P\,dx \int \left(\frac{2x+1}{x}\,dx\right) \left(x + \frac{1}{x}\right)dx$$

$$IF = e = e = e$$

$\because$ Solve is given by $\int d(y \times I.F) = \int q \times I.F$

$$= \int d\left(ye^{2x}x\right) = \int e^{-2x} - e^{2x}\,x\,dx + c$$

$$= ye^{2x}x \int x\,dx + c$$

QNo:- 2

$$\frac{dy}{dx} + \frac{3}{x}y = 6x^2$$

$\int P dx \int \frac{3}{x} dx \quad 3 \ln x$

$I.F = e = e = e = x^3$

Solve is given $\int d(y \times I.F) = \int q \times I.F$

$= \int d(yx^3) = \int 6x^2 \cdot x^3 \cdot dx + c$

$yx^3 = \int 6x^5 dx + c$

$yx^3 = \frac{6x^6}{6} + c$

$x^3 y = x^6 - c$

QNo:- 3

$$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$$

$\int P dx \int \frac{1}{x \ln x} dx \int \frac{\frac{dx}{x}}{\ln x}$

$I.F = e = \ln x$

Solve is given by $\int d \int y_x\, I.F) = \int Q\, I$

$$\int d(y \ln x) = \int \frac{3x^2}{\ln s} \ln x \, dx + c$$

$$y \ln x = 3\frac{x^3}{3} + c$$

$$y = \frac{x^3 + c}{\ln x}$$

Q No:- 4

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$$I.F\ e^{\int P dx} = e^{\int 3 dx} = e^{3x}$$

Solve given by $\int d(y \times I.F) = \int q \times I.F$

$$\int d(ye^{3x}) = \int 3x^2 e^{-3x} e^{3x} dx + e$$

$$ye^{3x} = x^3 + c$$

$$y = e^{-3x}(x + c)$$
