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ASSIGNMENT:

MATH

Exercise No 9.2

Q:1

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$y^2/2 = \frac{1}{3}\ln(1+x^3) + c$$

$$\cancel{6}^{3} \times \frac{y^2}{\cancel{2}} = {}^{2}\cancel{6} \times \frac{1}{\cancel{3}} \ln(1+x^3) + 6c$$

$$3y^2 = 2\ln(1+x^3) + c'$$

Q:2

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int 1/y^2\,dy = -\int \sin x\,dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$-\frac{1}{y} = \cos x + c$$

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Q:3

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1 + x) + y^2(1 + x)$$

$$\frac{dy}{dx} = (1 + y^2)(1 + x))$$

$$\int \frac{1}{1+y^2}\,dy = \int (1 + x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1} y = 2x + x^2 + 2c$$

Q:4.

$$(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$$

$$[x(y+2) + 1(y+2)]dx + [x(x+2)]dy = 0$$

$$[(x+1)(y+2)]dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\int \frac{x+1}{x(x+2)}\,dx + \int \frac{1}{y+2}\,dy = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\ln(x^2 + 2x) + \ln(y+2) + \ln c = 0$$

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$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$$

$$\ln(y+2) = \ln\left(\frac{c}{\sqrt{x^2+2x}}\right)$$

$$y+2 = \frac{c}{\sqrt{x^2+2x}}$$

Q:5

$$\frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$$

$$= 2x^2 - 2x - 2 + y - x^2y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (2-y)(x^2-x-1)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\int \frac{-dy}{2-y} = \int x^2 dx - \int x dx - \int 1 dx$$

$$-\ln(2-y) = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$-6\ln(2-y) = 6\frac{x^3}{3} - 6\frac{x^2}{2} - 6x + 6c$$

$$\ln(2-y)^{-6} = 2x^3 - 3x^2 - 6x + 6x$$

$$\ln(2-y)^{-6} = (2x^3 - 3x^2 - 6x + 6x)\ln e$$

$\ln(2-y)^{-6} = \ln e^{2x^3-3x^2-6x+6c}$

$(2-y)^{-6} = e^{2x^3-3x^2-6x} \cdot e^{c}$

$(2-y)^{-6} = c_1 e^{2x^3-3x^2-6x}$

Q:6

$\operatorname{cosec} y \, dx + \sec x \, dy = 0$

$\div$ by $\operatorname{cosec} y \sec x$

$\frac{1}{\sec x} dx + \frac{1}{\operatorname{cosec} y} dy = 0$

$\int \cos x \, dx + \int \sin y \, dx = 0$

$\sin x - \cos y = c$

Q:7

$y(1+x)dx + x(1+y)dy = 0$

$\div$ by xy

$\frac{1+x}{x} dx + \frac{1+y}{y} dy = 0$

$\int\left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0 \, dy$

$\ln x + x + \ln y + y = c$

$x + y + \ln(xy) = c$

Q: 8

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

$\div$ by xy

$$\frac{\sqrt{1+x^2}}{x}dx + \frac{\sqrt{1+y^2}}{y}dy = 0$$

Put $\sqrt{1+x^2} = t$, $\sqrt{1+y^2} = z$

$$1+x^2 = t^2 \qquad 1+y^2 = z^2$$

$$2x\,dx = 2t\,dt \qquad 2y\,dy = 2z\,dz$$

$$x\,dx = t\,dt \qquad y\,dy = dz$$

Therefore

$$\int\frac{\sqrt{1+x^2}}{x^2}x\,dx + \int\frac{\sqrt{1+y^2}}{y^2}y\,dy = \int 0\,dx$$

$$\int\frac{t}{t^2-1}\cdot t\,dt + \int\frac{z}{z^2-1}z\,dz = c$$

$$\int\frac{t^2}{t^2-1}dt + \int\frac{z^2}{z^2-1}dz = c$$

$$\int\frac{t^2-1+1}{t^2-1}dt + \int\frac{z^2-1+1}{z^2-1}dz = c$$

$$\left(1+\frac{1}{t^2-1}\right)dt + \left(1+\frac{1}{z^2-1}\right)dz = c$$

$$t + \frac{1}{2}\ln\frac{(t-1)}{(t+1)} + z\,\frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$$

$$\sqrt{1+x^2} + \frac{1}{2}\ln\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} + \sqrt{1+y^2} + \frac{1}{2}\ln\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1} = c$$

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Exercise No 9.3

1: $(x-y)dx + (x+y)dy = 0$

$$(x+y)dy = -(x-y)dx$$

$$\frac{dy}{dx} = \frac{-(x-y)}{x+y}$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{——— (i)}$$

Put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——— (iii)}$$

Using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(1+v)} - v$$

$$= \frac{\cancel{v} - 1 - \cancel{v} - v^2}{1+v}$$

$$x\frac{dv}{dx} = -\frac{(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1}\, dv = -\int \frac{1}{x} dx$$

$$\frac{1}{2}\int \frac{2v}{v^2+1}dv + \int \frac{1}{v^2+1}\, dv = -\int \frac{1}{x} dx$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + c$$

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$$\ln (v^2+1)^{1/2} + \tan^{-1} v + \ln x = c$$

$$\ln \sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\frac{y}{x} + \ln x = c$$

$$\ln \sqrt{y^2+1} - \cancel{\ln} \sqrt{x^2} + \tan^{-1}\frac{y}{x} + \cancel{\ln x} = c$$

$$\ln \sqrt{y^2+x^2} + \tan^{-1}\frac{y}{x} = c$$

2 $(y^2 + 2xy)dx + x^2 dy = 0$

$$x^2 dy = -(y^2+2xy)dx$$

$$\frac{dy}{dx} = \frac{-(y^2+2xy)}{x^2} \quad \text{———— (i)}$$

Put $y = vx$ ———— (ii)

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \quad \text{———— (iii)}$$

Using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{-(v^2x^2+2xvx)}{x^2}$$

$$x\frac{dv}{dx} = -\frac{\cancel{x^2}(v^2+2v)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = -(v^2+2v) - v$$

$$x\frac{dv}{dx} = -(v^2+3v)$$

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$$\int \frac{dv}{v^2+3v} = -\int \frac{dx}{x}$$

$$\int \frac{1}{v(v+3)}\, dv = -\int \left\{\frac{1}{x}\right\} dx$$

$$\frac{1}{3}\int \frac{3}{v(v+3)}\, dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{(v+3)-v}{v(v+3)}\, dv = -\int \frac{1}{x}\, dx$$

$$\frac{1}{3}\int \frac{1}{v} dv - \int \frac{1}{v+3}\, dv = -\int \frac{1}{x} dx$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(v+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln \frac{c}{x}$$

Antilog

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x\, v^{1/3} = c\,(v+3)^{1/3}$$

$$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x\, \frac{y^{1/3}}{x^{1/3}} \cdot x^{3} = c\,(y+3x)^{1/3}$$

$$x \cdot y^{1/3} = c\,(y+3x)^{1/3}$$

$$x^3 y = c'(y+3x)$$

3. $(x^2-3y^2)dx + 2xydy = 0$

$$2xy\,dy = -(x^2-3y^2)dx$$

$$\frac{dy}{dx} = -\frac{x^2-3y}{2xy} \quad \text{—— (i)}$$

Put $y = Vx$ —— (ii)

$$dy = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

Using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2x^2v}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(3v^2-1)}{\cancel{x^2}\,2v} - v$$

$$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$2v\,dv = \frac{v^2-1}{x}dx$$

$$\int\frac{2v}{v^2-1}dv = \int\frac{1}{x}dx$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = cx$$

$$\boxed{y^2 - x^2 = cx(x)^2}$$

4. $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{------(i)}$$

Put $\quad y = vx \quad$ ------(ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{------(iii)}$$

Using (ii),(iii) in(i)

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v^2)}{\cancel{x^2}\,2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln(cx)$$

$$1 + \frac{y^2}{x^2} = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$\boxed{x^2+y^2 = cx(x^2)}$$

5: $(x^2+xy+y^2)\,dx - x^2dy = 0$

$$(x^2+xy+y^2)\,dx = x^2dy$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{HDE} \;\text{------}\; (i)$$

Put $y = vx$ ------ (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \;\text{------}\; (iii)$$

Using (ii), (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{x^2+x^2v+v^2x^2}{x^2}$$

$$v + x\frac{dv}{dx} = \frac{x^2(1+v+v^2)}{x^2}$$

$$x\frac{dv}{dx} = 1+\cancel{v}+v^2-\cancel{v}$$

$$x\frac{dv}{dx} = 1+v^2$$

$$\int\frac{1}{1+v^2}\,dv = \int\frac{1}{x}\,dx$$

$$\tan^{-1}v\,\cancel{(1+v^2)} = \ln x + \ln c$$

$$\tan^{-1}\frac{y}{x} = \ln cx$$

6: $(x^2+3xy+y^2)dx - x^2dy = 0$

$$(x^2+3xy+y^2)dx = x^2dy$$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \quad \text{----(i)}$$

Put $y = vx$ ---- (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{----(iii)}$$

Using (ii),(iii) in (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3x^2v+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v+v^2)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = 1+3v+v^2-v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$x\frac{dv}{dx} = (1+v)^2$$

$$\int\frac{1}{(1+v)^2}dv = \int\frac{1}{x}dx$$

$$\frac{-1}{(1+v)} = \ln x + c$$

$$-\frac{1}{\left(\frac{y}{x}+1\right)} = \ln x + c$$

$$\frac{-1}{\frac{y+x}{x}} = \ln x + c$$

$$\boxed{\frac{-x}{y+x} = \ln x + c}$$

7. $\frac{dy}{dx} = \frac{4y-3x}{2x-y}$ .....

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y} \text{ -----(i) HDE}$$

Put $y = vx$ ------(ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ -----(iii)}$$

Using (ii),(iii) in (i)

$$v + x\frac{dv}{dx} = \frac{4vx-3x}{2x-vx}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x}(4v-3)}{\cancel{x}(2-v)}$$

$$x\frac{dv}{dx} = \frac{4v-3}{2-v} - v$$

$$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$$

$$x\frac{dv}{dx} = \frac{v^2+2v-3}{2-v}$$

$$\int\frac{2-v}{(v+3)(v-1)}dv = \int\frac{1}{x}dx$$

Separable form.

By Partial fractions

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$$(2-v) = A(v-1) + B(v+3)$$

Put $v+3=0 \Rightarrow 5 = -4A \Rightarrow \boxed{A = -5/4}$

Put $v-1=0 \Rightarrow 1 = 4B \Rightarrow B = \boxed{1/4}$

$$\therefore \frac{2-v}{(v+3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v-1)}$$

from iv

$$-\frac{5}{4}\int\frac{dv}{v+3} + \frac{1}{4}\int\frac{dv}{v-1} = \int\frac{dx}{x}$$

$$4\times -\frac{5}{4}\ln(v+3) + 4\times\frac{1}{4}\ln(v-1) = 4\times\ln x + \ln c\times 4$$

$$-5\ln(v+3) + \ln(v-1) = 4\ln cx$$

$$-\ln(v+3)^5 + \ln(v-1) = \ln c^4x^4$$

$$\ln\frac{(v-1)}{(v+3)^5} = \ln c^4x^4$$

Antilog

$$\frac{(y/x - 1)}{(y/x + 3)^5} = c^4x^4$$

$$\frac{\left(\frac{y-x}{x}\right)}{\left(\frac{y+3x}{x}\right)^5} = c^4x^4$$

$$\frac{(y-x)x^5}{(y+3x)^5 x \cdot x^4} = c'$$

$$\frac{y-x}{(y+3x)^5} = c'$$

8. $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\frac{y}{x} - x\right) dx$

$$\frac{dy}{dx} = \frac{y \sin y/x - x}{x \sin y/x} \quad \text{------(i)}$$

Put $y = vx$ ------ (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{-----(iii)}$$

Using (ii),(iii) in (i)

$$v + x\frac{dv}{dx} = \frac{vx \sin\frac{vx}{x} - x}{x \sin \cancel{v x}/\cancel{x}}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x} \sin v} - v$$

$$x\frac{dv}{dx} = \frac{v\cancel{\sin v} - 1 - v\cancel{\sin v}}{\sin v}$$

$$\int \sin v\, dv = \int -1/x\, dx$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x + c$$

$$\boxed{\cos y/x = \ln x - c}$$

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Exercise No 9.4

1. $(3x^2+4xy)dx+(2x^2+2y)dy=0$

$M = 3x^2+4xy \quad , \quad N = 2x^2+2y$

$\frac{\partial M}{\partial y} = 0+4x \qquad \frac{\partial N}{\partial x} = 4x+0$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so given Diff eq. is Exact

Now

$\int M dx + \int (\text{terms of } N \text{ free form } x) dy = c$

$\int (3x^2+4xy)dx + \int 2y\, dy = 0$

$\frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{4}^{2}x^2y}{\cancel{2}} + \frac{\cancel{2}y^2}{\cancel{2}} = c$

$\boxed{x^3+2x^2y+y^2=c}$

2. $(2xy+y\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$

$M = 2xy+y-\tan y \; , \; N = x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial y} = 2x+1-\sec^2 y \; , \; \frac{\partial N}{\partial x} = 2x-\tan^2 y+0$

$= 2x+\tan^2 y$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Now

$\int M dx + \int (\text{term of } N \text{ free form } x) dy = c$

$\int (2xy+y-\tan y)dx + \int \sec^2 y\, dy = c$

$\frac{2x^2y}{2} + xy - x\tan y + \tan y = c$

$\boxed{x^2y+xy-x\tan y+\tan y=c}$

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3. $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$M = \frac{x+y}{y-1}$, $N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}$, $N = -\frac{1}{2}\frac{x^2+2x+1}{(y-1)^2}$

$= \frac{y-1-x-y}{(y-1)^2}$, $\frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$

$= \frac{-\cancel{2}(x+1)}{\cancel{2}(y-1)^2}$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\therefore$ given diff eq is Exact

$= \frac{-x-1}{(y-1)^2}$

Now

$\int M dx + \int (\text{term of } N \text{ free from } x) dy = c$

$\int \frac{x+y}{y-1} dx + -\frac{1}{2}\int \frac{1}{(y-1)^2} dy = c$

$\frac{1}{y-1}\int x+y \, dx + -\frac{1}{2}\int (y-1)^{-2} dy = c$

$\frac{1}{y-1}\left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$

$$\boxed{\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c}$$

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4. $\frac{dy}{dx} = -\frac{ax+hy}{hx+by}$

$(hx+by)\,dy = -(ax+hy)\,dx$

$(hx+by)\,dy + ax+hy\,dx = 0$

$M = ax+hy$, $N = hx+by$

$\frac{\partial M}{\partial y} = 0+h$, $\frac{\partial N}{\partial x} = h$

$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = c$

$\int ax+hy\,dx + \int by\,dy = c$

$\frac{ax^2}{2} + hxy + by^2/_2 = c$

$\boxed{ax^2 + 2hxy + by^2 = c'}$

5 $(1+\ln xy)\,dx + (1+\frac{x}{y})\,dy = 0$

$M = 1+\ln xy$, $N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = 0+\frac{1}{xy}\cdot x$ $\quad \frac{\partial N}{\partial x} = 0+\frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = c$

$\int (1+\ln xy)\,dx + \int 1\,dy$

$x + [\ln xy \cdot x - \int \frac{1}{xy}\cdot y \cdot x\,dx] + y = c$

$x + x\ln xy - \int dx + y = 0$

$$x + x\ln xy - x + y = c$$

$$\boxed{x\ln xy + y = c}$$

6. $\dfrac{y\,dx + x\,dy}{1-x^2y^2} + x\,dx = 0$

$$\frac{y\,dx}{1-x^2y^2} + \frac{x\,dy}{1-x^2y^2} + x\,dx = 0$$

$$\left(x + \frac{y}{1-x^2y^2}\right)dx + \frac{x\,dy}{1-x^2y^2} = 0$$

$$M = x + \frac{y}{1-x^2y^2}$$

$$\frac{\partial M}{\partial y} = 0 + \frac{(1-x^2y^2).1 - y(-2x^2y)}{(1-x^2y^2)^2}$$

$$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2}$$

$$= \frac{1+x^2y^2}{(1-x^2y^2)^2}$$

$$N = \frac{x}{1-x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} =$$

$$\frac{\partial N}{\partial x} = \frac{1+x^2y^2}{(1-x^2y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = C$$

$$\int \left(x + \frac{y}{1-x^2y^2}\right) dx + 0 = C$$

$$\int x\,dx + \int \frac{y}{1-x^2y^2}\,dx = C$$

$$\frac{x^2}{2} + \int \frac{y/y^2}{\frac{1}{y^2} - \frac{x^2y^2}{y^2}} = C$$

$$\frac{x^2}{2} + \frac{1}{y} \int \frac{dx}{\left(\frac{1}{y^2}\right)^2 - x^2} = C$$

$$\frac{x^2}{2} + \frac{1}{y} \int \frac{dx}{\left(\frac{1}{y^2}\right)^2 - x^2} = C$$

$$\frac{x^2}{2} + \frac{1}{y}\left[\frac{1}{2(1/y)} \ln\left|\frac{\frac{1}{y} + x}{\frac{1}{y} - x}\right|\right] = C$$

$$\frac{x^2}{2} + \frac{1}{2} \ln\left(\frac{1+xy}{1-xy}\right) = C$$

$$\boxed{x^2 + \ln\left(\frac{1+xy}{1-xy}\right) = C'}$$

7 $(6xy + 2y^2 - 5)\,dx + (3x^2 + 4xy - 6)\,dy = 0$

$M = 6xy + 2y^2 - 5 \qquad N = 3x^2 + 4xy - 6$

$\frac{\partial M}{\partial y} = 6x + 4y \qquad \frac{\partial N}{\partial x} = 6x + 4y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M dx + \int (\text{term of } N \text{ free from } x) dx = c$

$$\int (6xy + 2y^2 - 5) dy + \int -6 dy = c$$

$$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$$

$$\boxed{3x^2 + 2xy^2 - 5x - 6y = c}$$

8 $(y\sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = c$

$M = y\sec^2 x + \sec x \tan x$, $N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x$ $\qquad \frac{\partial N}{\partial x} = \sec^2 x$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M dx + \int (\text{term of } N \text{ free from } x) dy = c$

$(y\sec^2 x + \sec x \tan x)dx + \int 2y \, dy = c$

$$\boxed{y \tan x + \sec x + y^2 = c}$$

Exercise No 9.5

1. $(xy^2+y)dx - xdy = 0$ ---- (i)

$M = xy^2+y \qquad N = -x$

$M_y = 2xy+1 \qquad N_x = -1$

$M_y \neq N_x \therefore$ Non exact

$$\frac{M_y - N_x}{N} = \frac{2xy+1-1}{-x}$$

$$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$$

$$= \frac{-2\cancel{(1+xy)}}{y\cancel{(1+xy)}} = -\frac{2}{y}$$

$$e^{-\int \frac{2}{y}dy} = e^{-2\ln} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Multiply both Sides of eq (i) by I.F $= \frac{1}{y^2}$

$$\frac{1}{y^2}(xy^2+y)dx - \frac{x}{y^2}dy = 0$$

$$\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0 \text{ --- (ii)}$$

Now $M = x+\frac{1}{y} \qquad N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2} \qquad N_x = -\frac{1}{y^2}$

$M_y = N_x$ is exact Diff eq

So $\int M dx \int$ (term of N free from x) $dy = c$

$$\int \left(x + \frac{1}{y}\right) dx + \text{Nil} = c$$

$$\boxed{\frac{x^2}{2} + \frac{x}{y} = c}$$

2 $(x^2 + x - y)\,dx + x\,dy = 0$

$$(x^2 + x - y)\,dx + x\,dy = 0 \quad \text{----(i)}$$

$M = x^2 + x - y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$M_y \neq N_x$ Non Exact Diff eq.

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = -\frac{2}{x}$ fn of x alone

Multiply both sides of eq i by I.F $= \frac{1}{x^2}$

$$\frac{1}{x^2}(x^2 + x - y)\,dx + \frac{x}{x^2}\,dy$$

$$\left(1 + \frac{1}{x} - \frac{y}{x^2}\right)dx + \frac{1}{x}\,dy = 0 \quad \text{--(ii)}$$

Now

$M = 1 + \frac{1}{x} - \frac{y}{x^2} \qquad N = \frac{1}{x}$

$M_y = -\frac{1}{x^2} \qquad N_x = -\frac{1}{x^2}$

$M_y = N_x \therefore$ ii is Exact Diff Eq.

So

$$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$$

$$\left(\frac{1}{1}+\frac{1}{x}-\frac{y}{x^2}\right)dx + Nil = c$$

$$\boxed{x + \ln x + yx = c}$$

3. $dy + \left(\frac{y - \sin x}{x}\right)dx = 0$

$$dy + \left(\frac{y - \sin x}{x}\right)dx = 0 \text{ ---- (i)}$$

$M = \frac{y - \sin x}{x}$ $\quad N = 1$

$M_y = \frac{1}{x} - 0$ $\quad N_x = 0$

Diff

$M_y \neq N_x$ i is non Exact eq.

Now $\frac{M_y - N_x}{N} = \frac{1/x - 0}{1} = \frac{1}{x}$

$I.F = e^{\int 1/x\, dx} = e^{\ln x} = x$

Multiplying both sides of eq i by $I.F = x$

$M = y - \sin x$ $\quad N = x$

$M_y = 1$ $\quad N_x = 1$

$M_y = N_y$ ii is Exact Diff eq.

Now

$\int M dx + \int(\text{term of } N \text{ free from } x)dy = c$

$(y - \sin x)dx = c$

$$\boxed{xy + \cos x = c}$$

4. $y(2xy+e^x)dx - e^x dy = 0$

$y(2xy+e^x)dx - e^x dy = \ldots\ldots (i)$

$M = 2xy^2 + e^x y \qquad N = -e^x$

$M_y = 4xy + e^x \qquad N_x = -e^x$

~~$M_y = 4xy + e^x$~~

$M_y \neq N_x$ i is non exact eq.

$\frac{M_y - N_x}{N} = \frac{4xy + e^x + e^x}{-e^x}$ Not fn x alone.

$\frac{N_x - M_y}{M} = \frac{-e^x - 4xy - e^x}{2xy^2 + ye^x} = \frac{-2(e^x - 2xy)}{y(2xy+e^x)}$

$= -2/y$ fn of y alone.

$I.F = e^{\int -2/y\, dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \boxed{\frac{1}{y^2}}$

Multiply both sides of i by $I.F = 1/y^2$

$\left(\frac{1}{y^2} 2x + \frac{e^x}{y}\right)dx - \frac{1}{y^2} e^x dy = 0 \ldots (ii)$

$\left(2x + \frac{e^x}{y}\right)dx - \frac{e^x}{y^2} dy = 0 \ldots (ii)$

$M = 2x + \frac{e^x}{y} \qquad N = -\frac{e^x}{y^2}$

$M_y = 0 + \left(-\frac{e^x}{y^2}\right) \qquad N_x = -\frac{e^x}{y^2}$

$M_y = N_x$ ∴ ii is exact Diff eq.

Now $\int M dx + \int (\text{term of } N \text{ free from } x) dy = C$

$\int \left(y + \frac{2}{y^2}\right) dx + \int 2y\, dy = C$

$$\boxed{xy + \frac{2x}{y^2} + y^2 = C}$$

5. $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$ —(i)

$M = y^4+2y \qquad N = xy^3+2y^4-4x$

$M_y = 4y^3+2 \qquad N_x = y^3-4$

$M_y \neq N_x \therefore$ i is non exact Diff eq.

$$\frac{N_x-M_y}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} = \frac{-3y^3-6}{y(y^3+2)} = -\frac{3}{y}$$

$$I.F = e^{\int -\frac{3}{y}dy} = e^{-3\ln y} = e^{\ln y^{-3}} = y^{-3} = \boxed{\frac{1}{y^3}}$$

x. by $1/y^3$ $\quad \frac{1}{y^3}(y^4+2y)dx + \frac{1}{y^3}(xy^3+2y^4-4x)dy=0$

$$\left(y+\frac{2}{y^2}\right)dx + \left(x+2y-\frac{4x}{y^3}\right)dy = 0 \quad \text{—(ii)}$$

Now

$M = y+\frac{2}{y^2} \qquad N = x+2y-\frac{4x}{y^3}$

$M_y = 1-\frac{4}{y^3} \qquad N_x = 1+0-\frac{4}{y^3}$

$\because M_y = N_x \therefore$ ii is Exact Difl eq

$\int M dx + \int (\text{term of } N \text{ free from } x)dy = c$

$$\left(y+\frac{2}{y^2}\right)dx + \int 2y\,dy = c$$

$$\boxed{xy + \frac{2x}{y^2} + y^2 = c}$$

6) $(x^2+y^2+2x)dx + 2y\,dy = 0$

$(x^2+y^2+2x)dx + 2y\,dy = 0$ ---- (i)

$M = x^2+y^2+2x$ $\quad N = 2y$

$M_y = 2y$ $\quad N_x = 0$

$M_y \neq N_x$ non Exact Diff Eq.

$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2x}$ Not fn of y only.

$\frac{M_y - N_x}{N} = \frac{2y-0}{2y} = 1 = x$ fn of x only

$I.F = e^{\int 1 \cdot dx} = e^x$

Multiply both sides of eq i by $I.F = e^x$

$e^x(x^2+y^2+2x)dx + e^x(2y)dy = 0$ ---- (ii)

$M = (e^x x^2+y^2+2x)$ $\quad N = e^x 2y$

$M_y = e^x 2y$ $\quad N_x = e^x 2y$

$M_y = N_x$ $\therefore$ ii is Exact Diff eq.

$\therefore \int M dx + \int(\text{term of } N \text{ free from } x)dy = c$

$\int e^x(x^2+y^2+2x)dx + \text{Nil} = c$

$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x\,dx = c$

$x^2e^x - \cancel{\int 2xe^x dx} + e^x y^2 + \cancel{\int e^x 2x\,dx} = c$

$(x^2+y^2)e^x = c$

$$\boxed{(x^2+y^2)e^x = c}$$

7. $(x^2+y^2)dx - 2xy\,dy = 0 \quad \cdots (i)$

$(x^2+y^2)dx - 2xy\,dy = 0 \quad \cdots (i)$

$M = x^2+y^2 \qquad N = -2xy$

$M_y = 2y \qquad N_x = -2y$

$M_y \neq N_x$ is Non Exact Diff eq.

$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = -\frac{4y}{2xy} = -\frac{2}{x}$

$I.F = e^{\int -2/x\,dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

Multiply both sides of eq by $I.F = \frac{1}{x^2}$

$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)\,dy = 0$

$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}\,dy = 0 \quad \cdots (ii)$

$M = 1+\frac{y^2}{x^2} \qquad N = -\frac{2y}{x}$

$M_y = 2y/x^2 \qquad N_x = 2y/x^2$

$M_y = N_x$ is Exact eq.

$\therefore \int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$(1+y^2/x^2)dx + \text{Nil} = c$

$$\boxed{x - \frac{y^2}{x} = c}$$

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8. $(4x+3y^2)dx + 2xy\,dy = 0$

$(4x+3y^2)dx + 2xy\,dy = 0$ --- (i)

$M = 4x+3y^2 \qquad N = 2xy$

$M_y = 0+6y \qquad N_x = 2y$

$M_y \neq N_x$ is non Exact Diff eq.

$\frac{N_x - M_y}{M} = \frac{2y-6y}{4x+3y^2}$ not fn for y alone.

fn of x alone.

$\frac{M_y - N_x}{N} = \frac{6y-2y}{2xy\,\cancel{4x+3y^2}} = \frac{4y}{2xy} = \frac{2}{x}$

$\therefore I.F = e^{\int \frac{2}{x} dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

Multiply both sides of i by $I.F = x^2$

$(4x^3 + 3y^2x^2)dx + 2x^3y\,dy = 0$ ... (ii)

$M_y = 6yx^2 \qquad N_x = 6x^2y$

$M_y = N_x$ is Exact eq.

$\therefore \int M dx + \int (\text{term of } N \text{ free from } x) dy = C$

$\int (4x^3 + 3y^2x^2) dx + \text{Nil} = C$

$\cancel{4}\frac{x^4}{\cancel{4}} + \cancel{3}y^2\frac{x^3}{\cancel{3}} = C$

$$\boxed{x^4 + y^2x^3 = C}$$

Exercise No 9.6

1. $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$ LDE in y

$I.F = e^{\int P dx} = e^{\int \frac{2x+1}{x} dx} = e^{\int (2+\frac{1}{x}) dx}$

$= e^{2x + \ln x} + e^{2x} e^{\ln x} = e^{2x} x$

$\therefore$ Sol is given by $\int d(y \times I.F) = \int Q \times I.F dx + c$

$\Rightarrow \int d(y e^{2x} x) = e^{-2x} \cdot e^{2x} x dx + c$

$\Rightarrow y e^{2x} x = \int x dx + c$

$x y e^{2x} = \frac{x^2}{2} + c$

2. $\frac{dy}{dx} + \frac{3}{x} y = 6x^2$

$I.F = e^{\int P dx} = e^{\int 3/x dx} = e^{3 \ln x} = e^{\ln x^3} = x^3$

$\therefore$ Sol is given by $\int d(y \times I.F) = \int Q \times I.F dx + c$

$\Rightarrow \int d(yx^3) = \int 6x^2 \cdot x^3 dx + c$

$yx^3 = \int 6x^5 dx + c$

$yx^3 = \cancel{6} \frac{x^6}{\cancel{6}} + c$

$\boxed{x^3 y = x^6 + c}$

3. $\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$ (LDE in y)

$I.F = e^{\int P dx} = e^{\int \frac{1}{x \ln x} dx} = e^{\int \frac{\frac{dx}{x}}{\ln x}}$

$I.F = e^{\ln(\ln x)} = \ln x$

Sol is given by $\int d(Y \times I.F) = \int Q \times I.F\,dx + c$

$$\Rightarrow \int d(Y \ln x) = \int \frac{3x^2}{\cancel{\ln x}} \cancel{\ln x}\, dx + c$$

$$Y \ln x = \cancel{3}\frac{x^3}{\cancel{3}} + c$$

$$Y \ln x = x^3 + C$$

$$\boxed{Y = \frac{x^3 + C}{\ln x}}$$

4. $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$ (LDE in y)

$$I.F = e^{\int P dx} = e^{\int 3 dx} = \boxed{e^{3x}}$$

Sol is given by $\int d(Y \times I.F) = \int Q \times I.f\,dx + c$

$$\int d(Ye^{3x}) = \int 3x^2 \cancel{e^{-3x}} \cancel{e^{3x}}\, dx + c$$

$$Ye^{3x} = x^3 + c$$

$$\boxed{Ye = e^{-3x}(x^3 + C)}$$

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5. $\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \sec^2 x\, y = \sec^2 x \tan x \text{ LDE in } y$$

$$I.F = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + c$

$$\Rightarrow \int d(y \times e^{\tan x}) = \int \sec^2 x \times \tan x \cdot e^{\tan x} dx + c$$

$$\Rightarrow y e^{\tan x} = \int \underset{II}{e^t} \underset{I}{t}\, dt + c$$

$$= t e^t - e^t + c$$

$$y e^{\tan x} = e^t (t - 1) + c$$

$$e^{-\tan x}\, y e^{\tan x} = e^{\tan x}(\tan x - 1) + c e^{-\tan x}$$

$$y = e^{\tan x}$$

$$\boxed{y = (\tan x - 1) + c e^{-\tan x}}$$

6. $x\frac{dy}{dx} + (1 + x\cot x) y = x$ LDE in y

$$\frac{dy}{dx} = (1/x + \cot x) y = 1 \text{ LDE in } y$$

$$I.F = e^{\int P dx} = e^{\int (1/x + \cot x) dx} = e^{\ln x + \sin \ln x}$$

$$I.F = e^{\ln(x \sin x)} = x \sin x$$

Sol is given by $\int d(y \times I.F) = Q \times I.F\, dx + c$

$\Rightarrow \int d(Y \times \sin x) = \int x \sin x \, dx + C$

$Y \times \sin x = x(-\cos x) - \int 1(-\cos x)dx) + C$

$Y \times \sin x = x(-\cos x) + \int \cos x \, dx + C$

$Y \times \sin x = -x\cos x + \sin x + C$

$$\boxed{Y = -\cot x + \frac{1}{x} + \frac{C}{x}\operatorname{cosec} x}$$

7. $(x+1)\frac{dy}{dx} - ny = e^x (x+1)^{n+1}$

$\frac{dy}{dx} - \frac{n}{(x+1)} y = e^x (x+1)^n$ LDE in Y

$I.F = e^{\int P dx} = e^{-\int \frac{n}{x+1} dx} = e^{-n \ln(x+1)} = e^{\ln(x+1)^{-n}}$

$= (x+1)^{-n} = \frac{1}{(x+1)^n}$

Sol is given by $d(Y \times I.F)dx = Q \, I.f \, dx + C$

$\Rightarrow \int d\left(Y \cdot \frac{1}{(x+1)^n}\right) = \int e^x \cancel{(x+1)^n} \cdot \frac{1}{\cancel{(x+1)^n}} dx + C$

$\frac{Y}{(x+1)^n} = e^x + C$

$$\boxed{Y = (e^x + C)(x+1)^n}$$

8. $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx} + \left(\frac{2xy}{x^2+1}\right) = \frac{4x^2}{x^2+1}$ LDE in y

$I.F = e^{\int\left(\frac{2x}{x^2+1}\right)dx} = e^{\ln(x^2+1)} = \boxed{x^2+1}$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + C$

$\Rightarrow \int d(y(x^2+1)) = \frac{4x^2}{\cancel{(x^2+1)}}\cancel{(x^2+1)}\,dx + C$

$y(x^2+1) = \frac{4x^3}{3} + C$

$$\boxed{3y(x^2+1) = 4x^3 + C'}$$

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