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Class EET (2nd smst)

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Assignment No (2)

(Exercise 9.2)

Question ①

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln x(1+x^3) + c$$

$$\frac{3y^2}{2} = \ln x(1+x^3) + 3c$$

$$\frac{3y^2}{2} = \ln x(1+x^3) + 3c$$

$$3y^2 = 2\ln x(1+x^3) + 6c$$

$$3y^2 + 2\ln x(1+x^3) + c$$

Question ②

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x\,dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$\frac{-1}{y} = \cos x + c$$

Question 3

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1} y = 2x + x^2 + c$$

Question 4

$$(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$$

$$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$$

$$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2x)}\,dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln x(y+2) = -\frac{1}{2}\ln x(x^2+2x) + \ln x$$

$$y + 2 = \frac{e}{\sqrt{x^2+2x}}$$

Question ⑤

$$\frac{dy}{dx} = 2x^2\, x \cdot y - x^2 y + xy - 2x \cdot 2$$

$$= 2x^2 - 2x - 2 + y - x^2 y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\, dx$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1)\, dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-6\ln|2-y| = 2x^3 - 3x^2 - 6x + 6C$$

$$\ln|2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln e$$

$$\ln|2-y|^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6c}$$

$$|2-y|^{-6} = e^{2x^3 - 3x^2 - 6x} \cdot e$$

$$|2-y|^{-6} = 2x^3 - 3x^2 - 6x$$

Question ⑥

$\operatorname{cosec} y\, dx + \sec x\, dy = 0$

$\div$ by $\operatorname{cosec} y \sec x$

$$\frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\int \cos x\, dx + \int \sin y\, dy = \int 0\, dx$$

$$\sin x - \cos y = C$$

$\longleftrightarrow$

Question 7

$y(1+x)\,dx + x(1+y)\,dy = 0 \quad | \div \text{by } xy$

$\left(\frac{1+x}{x}\right)dx + \frac{(1+y)}{y}\,dy = 0$

$\int\left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0\,dx$

$\ln x + x \ln y + y$

$x + y + \ln(xy) = c$

Question 8

$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div y \quad xy$

$\int \frac{\sqrt{1+x^2}}{x}\,dx + \int \frac{\sqrt{1+y^2}}{y}\,dy = \int 0\,dx$

put $\sqrt{1+x^2} = t$ put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ $\quad 1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $\quad 2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $\quad y\,dy = z\,dz$

$\int \frac{\sqrt{1+x^2}}{x^2}\,x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}\,y\,dy = \int 0\,dx$

$\int \frac{t \cdot t\,dt}{t^2-1} + \int \frac{z \cdot z\,dx}{z^2-1} = c$

$\int \left(\frac{t^2-1+1}{t^2-1}\right)dt + \int \frac{z^2-1+1}{z^2-1}\,dz = c$

$\int \left(1+\frac{1}{t^2-1}\right)dt + \int\left(1+\frac{1}{z^2-1}\right)dz = c$

$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$

$\sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + \sqrt{1+y^2} + \frac{1}{2}$

$\ln\left(\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right)$

(Exercise 9.3)

Question ①

$$(x-y)dx+(x+y)dy=0$$

$$(x+y)dy=-(x-y)dx$$

$$\frac{dy}{dx}=\frac{y-x}{x+y} \quad \text{--- ①}$$

put $y=vx$ — ②

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad \text{--- ③}$$

$$v+x\frac{dv}{dx}=\frac{vx-x}{x+vx}$$

$$x\frac{dv}{dx}=\frac{\cancel{x}(v-1)}{\cancel{x}(1+v)}-v$$

$$=\frac{\cancel{v}-1-\cancel{v}-v^2}{1+v}$$

$$x\frac{dv}{dx}=-\frac{(v^2+1)}{1+v}$$

$$\int\frac{v+1}{v^2+1}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v\,dv}{v^2+1}+\int\frac{dv}{v^2+1}=-\int\frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1)+\tan^{-1}v=-\ln+c$$

$$\ln(v^2+1)^{1/2}+\tan^{-1}v+\ln=c$$

$$\ln\sqrt{\frac{y^2}{x^2}+1}+\tan^{-1}\left(\frac{y}{x}\right)+\ln x=c$$

$$\ln x\sqrt{y^2+x^2}-\ln x\sqrt{x^2}+\tan^{-1}\left(\frac{y}{x}\right)+\ln x=c$$

$$\ln\sqrt{y^2+x^2}+\tan^{-1}\left(\frac{y}{x}\right)=c$$

Question ②

$$(y^2+2xy)dx+x^2dy=0$$

$$x^2dy=-(y^2+2xy)dx$$

$$\frac{dy}{dx}=-\frac{(y^2+2xy)}{x^2}$$

put $y=vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = -\frac{(v^2 - x^2 + 2xvx)}{x^2}$$

$$x\frac{dv}{dx} = -\cancel{x^2}\frac{(v^2+2x)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = -v^2 + 3v$$

$$\int \frac{dv}{v^2+3v} = -\int \frac{dx}{x}$$

$$\int \frac{1}{v(v+3)}dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(x+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{1/3} = c(v+3)^{1/3}$$

$$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x} + 3\right)^{1/3}$$

$$x\left(\frac{y^{1/3}}{x^{1/3}}\right)x^{1/3} = c(y+3x)^{1/3}$$

$$xy^{1/3} = c(y+3x)^{1/3}$$

$$x^3y = c(y+3x)$$

Question (3)

$$(x^2-3y^2)dx+2xy\,dy=0$$

$$2xy\,dy=-(x^2-3y^2)dx$$

$$\frac{dy}{dx}=\frac{3y^2-x^2}{2xy} \quad \text{--- (i)}$$

put $y=vx$ --- (ii)

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad \text{--- (iii)}$$

using (ii) & (iii) in (i)

$$v+x\frac{dv}{dx}=\frac{3v^2x^2-x^2}{2xvx}$$

$$x\frac{dv}{dx}=\frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}}-v$$

$$x\frac{dv}{dx}=\frac{v^2-1}{2v}$$

$$\int\frac{2v}{v^2-1}dv=\int\frac{dx}{x}$$

$$\ln(v^2-1)=\ln x+\ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right)=\ln cx$$

$$\frac{y^2-x^2}{x^2}=cx$$

$$y^2-x^2=(cx)x^2$$

Question (4)

$$(x^2+3y^2)dx-2xy\,dy=0$$

$$(x^2+3y^2)dx=2xy\,dy$$

$$\frac{x^2+3y^2}{2xy}=\frac{dy}{dx} \quad \text{--- (i)}$$

Put $y=vx$ --- (ii)

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad \text{--- (iii)}$$

using (ii) (iii) in (i)

$$v+x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = x^2\frac{(1+3v^2)}{x^2 2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln cx$$

$$\left(1+\frac{y^2}{v^2}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$x^2+y^2 = (cx)x^2$$

Question (5)

$$(x^2+xy+y^2)\,dx - x^2\,dy = 0$$

$$(x^2+xy+y^2)\,dx = x^2\,dy$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{--- (i)}$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (iii)}$$

using (ii) (iii) in (i)

$$v+x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{(1+v+v^2)\cancel{x^2}}{\cancel{x^2}} - v$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$$\tan^{-1} v = \ln x + C$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + C$$

Question 6

$$(x^2+3xy+y^2)\,dx - x^2\,dy = 0$$

$$(x^2+3xy+y^2)\,dx = x^2\,dy$$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \quad \text{— (i)}$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (iii)}$$

$$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \cancel{x^2}\left(\frac{1+3v+v^2}{\cancel{x^2}}\right) - v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\left(\frac{1}{v+1}\right) = \ln x + C$$

$$\left(\frac{-1}{\frac{y}{x}+1}\right) = \ln x + C$$

$$-\frac{1}{\frac{y+x}{x}} = \ln x + C$$

$$\frac{-x}{(x+y)} = \ln x + C$$

Question ⑦

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y} \quad \text{— (i)}$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (iii)}$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{4vx-3x}{2x-vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(4v-3)}{\cancel{x}(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$$

$$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x} \quad \text{—}$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$$2-v = A(v-1) + B(v+3)$$

Put $v+3=0 \Rightarrow +5 = -4A \Rightarrow \boxed{A = \frac{-5}{4}}$

Put $v-1=0 \Rightarrow 1 = 4B \Rightarrow \boxed{B = \frac{1}{4}}$

$$\frac{2-v}{(v+3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v-1)}$$

$$-\frac{5}{4}\int\frac{dv}{v+3} + \frac{1}{4}\int\frac{dv}{v-1} = \int\frac{dx}{x}$$

$$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-1) = \ln x + \ln c$$

$$-\ln(v+3)^5 + \ln(v-1) = 4\ln cx$$

$$\ln\frac{v-1}{(v+3)^5} = \ln c^4 x^4$$

Antilog $$\frac{\left(\frac{y}{x}-1\right)}{\left(\frac{y}{x}+3\right)^5} = c^4 x^4$$

$$\frac{(y-x)\,x^5}{x(y+3x)x^4} = c'$$

$$\frac{y-x}{(y+3x)^5} = c'$$

Question ⑧

$$x\sin\left(\frac{y}{x}\right)dy = \left(y\sin\frac{y}{x} - x\right)dx$$

$$\frac{dy}{dx} = \frac{y\sin\frac{y}{x} - x}{x\sin(y/x)} \quad \text{— (i)}$$

put

$$y = vx \quad \text{— (ii)}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (iii)}$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{vx\sin\frac{vx}{x} - x}{x\sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x}\sin v} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v\sin v} - 1 - \cancel{v\sin v}}{\sin v}$$

$$\int \sin v\, dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x - c$$

$$\cos\frac{y}{x} = \ln x - c$$

<——>

Non HomogeNious Equation

① $\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$

Put $x = x+h$

$\frac{dy}{dx} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x+h+3(y+k)-5}{x+h-(y+k)-1}$

$\frac{dy}{dx} = \frac{x+3y}{x-y}$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x+3vx}{x-vx}$

$x\frac{dv}{dx} = \frac{\cancel{x}(1+3v)}{\cancel{x}(1-v)}$

$x\frac{dv}{dx} = \frac{(1-v)^2}{1-v}$

$\int \frac{1-v}{(1-v)^2} dv = \int \frac{dx}{x}$

$\int \frac{1\,dv}{(1+v)^2} - \int \frac{v\,dv}{(1+v)^2} = \int \frac{dx}{x}$

(14)

(2) $\frac{dy}{dx} = \frac{-(4x+3y+15)}{2x+y+7}$

put $x = X+h$

$y = Y+k$

$\frac{dY}{dX} = \frac{-(4X+4h+3Y+3k+15)}{2X+2h+Y+k+7}$

$\frac{dY}{dX} = \frac{-4X-3Y}{2X+Y}$ — (i)

put $Y = VX$ — (ii)

$\frac{dY}{dX} = V+X\frac{dV}{dX}$ — (iii)

using (ii) (iii) in (i)

$V+X\frac{dV}{dX} = \frac{-4X-3VX}{2X+VX}$

$X\frac{dV}{dX} = \frac{-\cancel{X}(4+3V)}{\cancel{X}(2+V)} - V$

$= \frac{-4-3V-2V-V^2}{2+V}$

$\frac{XdV}{dX} = \frac{-(V^2+5V+4)}{2+V}$

$\int \frac{(V+2)\,dV}{V^2+5V+4} = -\int \frac{dX}{X}$

$\frac{1}{3}\int \frac{dV}{V+1} + \frac{2}{3}\int \frac{dV}{V+4} = -\int \frac{dX}{X}$

$\frac{1}{3}\ln(V+1) + \frac{2}{3}\ln(V+4) = -\ln X + \ln c$

(3) $(3y-7x-3)dx+(7y-3x-7)dy=0$

$(7y-3x-7)dy = -(3y-7x-3)dx$

$$\frac{dy}{dx} = \frac{-3y+7x+3}{7y-3x-1}$$

Put $x = x+h$, $y = y+k$ $\Big] = \frac{dy}{dx} = \frac{dy}{dx}$

$$\therefore \frac{dy}{dx} = \frac{-3(y+k)+7(x+h)+3}{7(y+k)-3(x+h)-7}$$

$$\frac{dy}{dx} = \frac{-3y+7x}{7y-3x} \quad \text{--- (i)}$$

put $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (iii)}$$

using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{-3vx+7x}{7vx-3x}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(-3v+7)}{\cancel{x}(7v-3)} - v$$

$$x\frac{dv}{dx} = \frac{-\cancel{3v}+7-7v^2+\cancel{3v}}{7v-3}$$

$$\int \frac{7v-3}{7(1-v^2)}dv = \int \frac{dx}{x}$$

$$\frac{2}{7}\int \frac{dv}{1-v} - \frac{5}{7}\int \frac{dv}{1+v} = \int \frac{dx}{x}$$

$$-\frac{2}{7}\ln(1-v) - \frac{5}{7}\ln(1+v) = \ln x + \ln c$$

$$\frac{1}{7}\ln\left[(1-\frac{y}{x})^{-2}(1+\frac{y}{x})^{-5}\right] = 7\ln cx$$

$$\ln\left[\left(\frac{x-y}{x}\right)^{-2}\left(\frac{x+y}{x}\right)^{-5}\right] = \ln c^7x^7$$

(4) $\frac{x-2y+5}{2x+y-1}$

put $x = X+h$

$y = Y+k$

$$\therefore \frac{dy}{dx} = \frac{X+h-2Y-2k+5}{2X+2h+Y+k-1}$$

$$\frac{dy}{dx} = \frac{x-2y}{2x+y} \text{ --- (i)}$$

Put $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ --- (iii)}$$

$$v + x\frac{dv}{dx} = \frac{x-2vx}{2x+vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(x-2v)}{\cancel{x}(2+v)} - v$$

$$\frac{x\,dv}{dx} = \frac{1-2v-2v-v^2}{2+v}$$

$$\int \frac{2+v}{1-4v-v^2}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}(\ln(v^2+4v-1) = -\ln x + \ln c$$

$$\ln\sqrt{v^2+4v-1} = \ln\frac{c}{x}$$

5 $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$

Put $3x-4y=z$ — (ii)

$3-4\frac{dy}{dx} = \frac{dz}{dx}$

$3-\frac{dz}{dx} = 4\frac{dy}{dx}$

$\frac{1}{4}\left(3-\frac{dz}{dx}\right) = \frac{dy}{dx}$ — (iii)

using (ii) (iii) in (i)

$\frac{1}{4}\left(3-\frac{dz}{dx}\right) = \frac{z-2}{z-3}$

$3-\frac{dz}{dx} = \frac{4z-8}{z-3}$

$3-\frac{(4z-8)}{z-3} = \frac{dz}{dx}$

$\frac{3z-9-4z+8}{z-3} = \frac{dz}{dx}$

$\frac{-(1+z)}{z-3} = \frac{dz}{dx}$

$-dx = \frac{z-3}{1+z}dz$

$\frac{z-3-1+1}{1+z} = -dz$

(6) $\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$ — (i)

Put $y-x = z$ — (ii)

$\frac{dy}{dx} - 1 = \frac{dz}{dx}$

$\frac{dy}{dx} = 1 + \frac{dz}{dx}$ — (iii)

using (ii) + (iii) in (i)

$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$

$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$

$= \frac{z+1-z-5}{z+5}$

$\frac{dz}{dx} = \frac{-4}{z+5}$

$\int (z+5)\,dz = -4\int dx$

$\frac{z^2}{2} + 5z = -4x + C$

$\frac{z^2+10z}{2} = -4x + C$

$z^2 + 10z = -8x + 2C$

$(y-x)^2 + 10(y-x) = -8x + c'$

Exact Equation

$(3x^2+4xy)dx+(2x^2+2y)dy=0$

$M=3x^2+4xy \; , \; N=2x^2+2y$

$\frac{\partial M}{\partial y}=0+4x \quad \frac{\partial N}{\partial x}=4x+0$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

Now $\int M dx+\int$ (terms of N from x) $dy=c$

$\int(3x^2+4xy)dx+\int 2y\,dy=c$

$3\frac{x^3}{3}+\frac{4x^2y}{2}+\frac{2y^2}{2}=c$

$x^3+2x^2y+y^2=c$

Question ②

$(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$

$M=2xy+y-\tan y \; , \; N=x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial y}=2x+1-\sec^2 y \; ; \; \frac{\partial N}{\partial x}=2x-\tan^2 y+0$

$=2x-\tan^2 y$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\int M dx+\int$ (terms of N force for x) $dy=c$

$\int(2xy+y-\tan y)dx+\int\sec^2 y\,dy=c$

$\frac{2x^2y}{2}+xy-x\tan y+\tan y=c$

$x^2y+xy-x\tan y+\tan y=c$

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Question (3)

$$\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$$

$M = \frac{x+y}{y-1}$. $N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}$ $N = -\frac{1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$

$= \frac{\cancel{y}-1-x\cancel{-y}}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ∴ given by ...

$\int M dx + \int (\text{terms of } N \text{ free from } x)dy = c$

$\int \cancel{Mdx}\left(\frac{x+y}{y-1}\right)dx + \int -\frac{1}{2}\frac{dy}{(y-1)^2} = c$

$\left(\frac{1}{y-1}\right)\int (x+y)dx + \frac{-1}{2}\int (y-1)\,dy = c$

$\frac{1}{(y-1)}\left(\frac{x^2}{2}+xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$

$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$

$x^2+2xy+1 = c'(y-1)$

Question (4)

$$\frac{dy}{dx} = \frac{-(ax+hy)}{hx+by}$$

$(hx+by)dy = -(ax+hy)dx$

$(ax+hy)dx + (hx+by)dy = 0$

$M = ax+hy$ $N = hx+by$

$\frac{\partial M}{\partial y} = 0+h$ $\frac{\partial N}{\partial x} = h$

∴ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N$

$\int (ax+hy)dx + \int by\, dy = c$

$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c$

Question (5)

$(1+\ln xy)dx + (1+\frac{x}{y})dy = 0$

$M = 1+\ln xy \qquad N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy}\cdot x \qquad \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int$(terms of N force from x)$dy = c$

$\int (1+\ln xy)dx + \int 1 \cdot dy = c$

$\int dx + \int 1 \ln xy\, dx + \int dy = c$

$x + [\ln xy\cdot(x) - \int \frac{1}{xy}\cdot y \cdot x\, dx] + y = c$

$x + x\ln xy - \int dx + y = c$

$x + x\ln xy - x + y = c$

$x\ln xy + y = c$

Question 6

$$\frac{y\,dx + x\,dy}{1-x^2y^2} + x\,dx = 0$$

$$\frac{y\,dx}{1-x^2y^2} + \frac{x\,dy}{1-x^2y^2} + x\,dy = 0$$

$$\left(\frac{x+y}{1-x^2y^2}\right)dx + \frac{x\,dy}{1-x^2y^2} = 0$$

$$M = \frac{x+y}{1-x^2y^2}$$

$$\frac{\partial M}{\partial y} = \frac{0 + (1-x^2y^2).1 - y(-2x^2y)}{(1-x^2y^2)^2}$$

$$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{1-x^2y^2}$$

$$N = \frac{x}{1-x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2) - x(-2xy^2)}{(1-x^2y^2)^2}$$

$$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int (\text{terms of } N \ldots x)\,dy = C$$

$$\int \left(x + \frac{y}{1-x^2y^2}\right)dx + Nil = C$$

$$\int x\,dx + \int \frac{y\,dx}{1-x^2y^2} = C$$

$$\frac{x^2}{2} + \int \frac{y/y^2}{\frac{\frac{1}{y^2}-x^2y^2}{y^2}}\,dx = C$$

$$\frac{x^2}{2} + \frac{1}{y}\int \frac{dx}{\left(\frac{1}{y}\right)^2 - x^2} = C$$

$$\frac{x^2}{2} + \frac{1}{y}\left[\frac{1}{2}\left(\frac{1}{y}\right)\ln\left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right] = C$$

$$\frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right| = C$$

$$x^2 + \ln\left|\frac{1+xy}{1-xy}\right| = C$$

Question (7)

$(6xy+2y^2-5)\,dx+(3x^2+4xy-6)=0$

$M=6xy+2y^2-5\ ,\quad N=3x^2+4xy-6$

$\frac{\partial M}{\partial y}=6x+4y \qquad \frac{\partial N}{\partial x}=6x+4y$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Hence Exact Diff. eq

$\int M\,dx+\int (\text{terms of } N \ldots x)\,dy=c$

$\int(6xy+2y^2-5)\,dx+\int -6\,dy=c$

$\frac{6x^2y}{2}+2xy^2-5x-6y=c$

$3x^2y+2xy^2-5x-6y=c$

Question (8)

$(y\sec^2x+\sec x\tan x)\,dx+(\tan x+2y)\,dy=0$

$M=y\sec^2x+\sec x\tan x\ ,\quad N=\tan x+2y$

$\frac{\partial M}{\partial y}=\sec^2x \quad \frac{\partial N}{\partial x}=\sec^2x$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\int M\,dx+\int(\text{terms of } N \ldots x)\,dy=c$

$\int(y\sec^2+\sec x\tan x)\,dx+\int 2y\,dy=c$

$y\tan x+\sec x+y^2=c$

(Exercise 9.5)

NON Exact Equation

$(xy^2+y)\,dx - x\,dy = 0$ — (1)

$M = xy^2+y$ $\quad N = -x$

$M_y = 2xy+1$ $\quad N_x = -1$

$\because M_y \neq N_x$ $\quad \therefore$ Non Exact

$\frac{M_y - N_x}{N} = \frac{2xy+1+1}{-x}$

$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$

$= \frac{-2(1+xy)}{y(xy+1)} = -\frac{2}{y}$

$-\int \frac{2}{y}\,dy = -2\ln y$

$\frac{1}{y}(xy^2+y)\,dx - \frac{x}{y^2}\,dy = 0$

$(x+\frac{1}{y})\,dx - \frac{x}{y^2}\,dy = 0$ (ii)

Now $M = x+\frac{1}{y}$ $\quad N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2}$ $\quad N_x = -\frac{1}{y^2}$

$\because M_y = N_x$

$\int M\,dx + \int$ (terms of N ...

$\int (x+\frac{1}{y})\,dx + \text{Nil} = c$

$\frac{x^2}{2} + \frac{x}{y} = c$

Question (2)

$(x^2+x-y)dx + x\,dy = 0$ — (1)

$M = x^2+x-y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$M_y \neq N_x \quad \therefore$ Non Exact

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$

$\int \frac{-2}{x}dx \quad -2\ln x \quad \ln x^{-2}$

Multiply both sides of Equation (1)

$\frac{1}{x^2}(x^2+x-y)dx + \frac{x}{x^2}dy = 0$

$(1+\frac{1}{x} - \frac{y}{x^2})dx + \frac{1}{x}dy = 0$ — (11)

Now $M = 1+\frac{1}{x}+\frac{y}{x^2} \qquad N = \frac{1}{x}$

$M_y = -\frac{1}{x^2} \qquad N_x = -\frac{1}{x^2}$

$M_y = N_x$

So

$\int M\,dx + \int (\text{terms of } N \text{ -- } x)\,dy = c$

$\int (1+\frac{1}{x} - \frac{y}{x^2})dx + Nil = c$

$x + \ln x + \frac{y}{x} = c$