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Assigment Mathamatics

Semester 2nd

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Page 2 Assignment Mathamatic

EX No - 9.2

Question No - 1

$$\frac{d}{dx} = \frac{x^2}{y(1+x^2)}$$

$$\frac{d}{dx} = \frac{x^2}{y(1+x^2)}$$

$$\frac{d}{dy} = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$$

$$\frac{3y^2}{2} = \ln(1+x^3)+c$$

$$3y^2 = 2\ln(1+x^2)+c \text{ Ans}$$

Question No - 2

$$\frac{dy}{dx} + y^2\sin x = 0$$

$$\frac{dy}{dx} + y^2\sin x = 0$$

$$\frac{dy}{dx} = -y^2\sin x$$

$$\int \frac{dy}{dx} = -\int \sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x$$

$$\frac{y^{-1}}{-1} = -(-\cos x)+c$$

$$-\frac{1}{y} = \cos x + c \text{ Ans}$$

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Question No-3

$\frac{dy}{dx} = 1+x+y^2+xy^2$

$\frac{dy}{dx} = 1+x+y^2+xy^2$

$\frac{dy}{dx} = (1+x)+y^2(xy^2)$

$\int \frac{dy}{1+y^2} = \int (1+x)dx$

$\tan y^{-1} = x + \frac{x^2}{2} + C$

Question No-4

$\operatorname{cosec} y\, dx + \sec x\, dy = 0$

$\div dy \operatorname{cosec} y + dx \sec x$

$\frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$

$\int \cos x\, dx + \int \sin y\, dy + \int dx$

$\sin x - \cos y + c$ Ans

Question No-5

$y(1+x)dx + x(1+y)dy = 0$

$\div y\ xy$

$\left(\frac{1+x}{x}\right)dx + \frac{(1+y)}{y}dy = 0$

$\int\left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0\, dx$

$\ln x + x + \ln y + y + c$

$x + y + \ln(xy) = c$ Ans

Question No-6

$\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{1-x^2} = 6$

$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{1-x^2}$

$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} y = -\sin^{-1} x + c$

$y = \sin(c - \sin^{-1} x)$

$\frac{dy}{dx} + \frac{\sqrt{y^2-1}}{x^2-1} = 0$

$\frac{dy}{dx} = -\frac{\sqrt{y^2-1}}{x^2-1}$

$\cos^{-1} y - \cos^{-1} x + c$

$y = \cos(c - \cos^{-1} x)$ Ans

Question No-7

$(1+2y^2)dy = y\cos x\, dx \quad y(0) = 1$

$\div$ by y

$\frac{(1+2y^2)}{y}dy = \cos x\, dx$

$\int\left(\frac{1}{y}+2y\right)dy = \int \cos x\, dx$

$\ln y + \frac{2y^2}{2} = \sin x + c$

$\cdot(1) = 1$

$\ln 1 + 1 = 0 + c$

$1 = c$

$\ln y + y^2 = \sin x + 1$

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Question No - 8

$(xy+2x+y+2)dx + (x^2+2x)dy = 0$

$[x(y+2)+(y+2)]\,dx + x(x+2)\,dy = 0$

$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$

$\frac{x+1}{x(x+2)}\,dx + \frac{1}{y+2}\,dy = 0$

$\int \frac{x+1}{x(x+2)}\,dx + \int \frac{dy}{y+2} = 0$

$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$

$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + c$

$y+2 = \frac{c}{x^2+2x}$ Ans

Ex No - 93

question No - 1

$(y^2+2xy)dx + x^2 dy = 0$

$x^2 dy = -(y^2+2xy)dx$

$\frac{dy}{dx} = \frac{-(y^2+2xy)}{x^2}$ HDE ①

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

using (ii) of (iii) in (i)

$v + \frac{dv}{dx} = -\frac{(v^2x^2+2xvx)}{x^2}$

$x\frac{dv}{dx} = \frac{e^x(v^2+2v)}{e^x}$

$x\frac{dv}{dx} = -(v^2+3v)$

$\int \frac{dv}{v^2+3v} = -\int \frac{dx}{x}$

$\int \frac{1}{v(v+3)} dv = -\int \frac{dx}{x}$

$\frac{1}{3}\int \frac{1}{v} - \frac{1}{v+3} dv = -\int \frac{dx}{x}$

$\frac{1}{3}\ln v - \frac{1}{3}\ln(v+3) = \ln x + \ln c$

$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$

$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$

$x \cdot v^{1/3} = c(v+3)^{1/3}$

$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$

$x \frac{y^{1/3}}{x^{1/3}} x^{1/3} = c(y+3x)^{1/3}$

$x y^{1/3} = c(y+3x)^{1/3}$

$xy = c(y+3x)$

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Question No-2

$(x^2-3y^2)dx+2xy\,dy=0$

$2xy\,dx=-(x^2-3y^2)dx$

$\frac{dy}{dx}=\frac{3y^2-x^2}{2xy}$ ...(i)

Put $y=vx$

$\frac{dy}{dx}=v+x\frac{dv}{dx}$ ...(ii)

using (i) (ii) (i)

$v+x\frac{dv}{dx}=\frac{3v^2x^2-x^2}{2xvx}$

$x\frac{dv}{dx}=\frac{(3v^2-1)x^2}{2vx^2}-v$

$x\frac{dv}{dx}=\frac{3v^2-1-2v^2}{2v}$

$x\frac{dv}{dx}=\frac{v^2-1}{2v}$

$\int\frac{2v}{v^2-1}dv=\int\frac{dx}{x}$

$\ln(v^2-1)=\ln x+\ln c$

$\frac{y^2-x^2}{x^2}=cx$

$y^2-x^2=(cx)x^2$ Ans

Question No 3

$(x^2+xy+y^2)dx-x^2dy=0$

$(x^2+xy+y^2)dx=x^2dy$

$\frac{dy}{dx}=x^2+xy+y^2$

Putting the value y

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{x^2+x(vx)+v^2x^2}{x^2}$

$x\frac{dv}{dx}=\frac{(1+v+v^2)x^2}{x^2}-v$

$\int\frac{dv}{1+v^2}=\int\frac{dx}{x}$

$\tan^{-1}v=\ln x+c$

$\tan^{-1}(\frac{y}{x})=\ln x+c$ Ans

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Question No-4

$x\sin\left(\frac{y}{x}\right)dy = y\left(y\sin \frac{y}{x} - x\right)dx$

$\frac{dy}{dx} = \frac{y\sin\frac{y}{x} - x}{x\sin\left(\frac{y}{x}\right)}$

Put the value y

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

using (iii) (ii) (i)

$v + x\frac{dv}{dx} = \frac{vx\sin\frac{vx}{x} - x}{x\sin\left(\frac{vx}{x}\right)}$

$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x}\sin v} - v$

$x\frac{dv}{dx} = \frac{v\cancel{\sin v} - 1 - v\cancel{\sin v}}{\sin v}$

$\int \sin v\, dv = \int -\frac{dx}{x}$

$-\cos v = \ln x + c$

$\cos v = \ln x + c$

$\cos\frac{v}{x} = \ln x\ c$ Ans

Question No-5

$\frac{dy}{dx} = x + y$

Putting the value $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

using (ii) (iii) (i)

$v + x\frac{dv}{dx} = \frac{x + vx}{x}$

$x\frac{dv}{dx} = \frac{\cancel{x}(1+v)}{\cancel{x}} - v$

$x\frac{dv}{dx} = 1$

$\int dv = \int \frac{dx}{x}$

$v = \ln x + c$

$\therefore y(1) = 1$

$\therefore \frac{1}{1} = \ln 1 + c$

$1 = 0 + c$

so $\frac{y}{x} = \ln x + 1$

$y = x\ln x + x = x(\ln x + 1)$ Ans

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Question No-6

$(2x-5y)dx+(4x-y)dy = y(1)=4$

$(4x-y)dy = (2x-5y)dx$

$\frac{dy}{dx} = \frac{5y-2x}{4x-y}$

Putting the value $y=vx$

$\frac{dy}{dx} = \frac{v+x\,dv}{dx}$

Solving (i)(ii)(i)

$v+x\frac{dv}{dx} = \frac{5vx-2x}{4x-vx}$

$= \frac{\cancel{x}(5v-2)}{\cancel{x}(4-v)} - v$

$= \frac{5v-2-4v+v^2}{4-v}$

$x\frac{dv}{dx} = \frac{v^2+v-2}{4-v}$

$\int \frac{4-v}{v^2+v-2}dv = \int \frac{dx}{x}$

$\int \frac{(4-v)dv}{(v-1)(v+2)} = \int \frac{dx}{x}$

$\int \left[\frac{dv}{v-1} - \frac{2}{v+2}\right] dv = \int \frac{dx}{dx}$

$\ln(v-1) - 2\ln(v+2) = \ln x + \ln c.$

$\ln\left[\frac{v-1}{v+2}\right] - \ln cx$

$\frac{y-x}{x(y+2x)^2} \quad \frac{1}{x} = c$

$y(1)=4$

$\frac{4-1}{4+2^2} = c$

$c = \frac{3}{36} = \frac{1}{12}$

$\frac{y-x}{y+2x} = \frac{1}{12} = (y-x)(4+2x$

$\frac{4-v}{(v-1)(v+2)} = \frac{A}{v-1} + \frac{B}{v+2}$

$4-v = A(v+2)+B(v-1)$

$v+2=0 \quad v=-2$

$B=-2$

$v-1=0 \quad v=1$

$A=1$

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Question No-7

$\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$

Putting the value = z

$3-4\frac{dy}{dx} = \frac{dz}{dx}$

$3-\frac{dz}{dx} = 4\frac{dy}{dx}$

$\frac{1}{4}(3-\frac{dz}{dx}) = \frac{dy}{dx}$

using (i) (ii) (i)

$\frac{1}{4}(3-\frac{dz}{dx}) = \frac{z-2}{z-3}$

$3-\frac{(4z-8)}{z-3} = \frac{dz}{dx}$

$\frac{3z-9-4z+8}{z-3} = \frac{dz}{dx}$

$-\frac{(1+z)}{z-3} = \frac{dz}{dx}$

$-dx = \frac{(z-3)}{1+z}dz$

$\frac{z-3-1+1}{1+z} = -dx$

$\int \frac{(z+1)-4}{1+z}dz = -\int dx$

$\int 1-\frac{4}{1+z}\, dx = -\int dx$

$= -4\ln(1+z) = -x+c$

$(3x-4y) - 4\ln(1+3x-4y)\, x + c$ A

Question No-8

$\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$

Putting value y-x=z

$\frac{dy}{dx} - 1 = \frac{dz}{dx}$

$\frac{dy}{dx} = \frac{1+dz}{dx}$

Solving (i) (ii) (i)

$1+\frac{dz}{dx} = \frac{z+1}{z+5}$

$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$

$= \frac{z+1-z-5}{z+5}$

$\frac{dz}{dx} = \frac{-4}{z+5}$

$\int (z+5)dz = -4\int dx$

$\frac{z^2}{2} + 5z = -4x+c$

$z^2 + 10z = -8x+2c$

$(y-x^2)\ 10(y-x) = -8x+c'$

$(y-x^2) + 10(y-x) + 8x = c$

$(y-x^2) + 10x + 8x + c =$

$(y-x^2) + 10y - 2x = c$ Ans

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Ex No - 9.4

Question No - 1

$(2xy + y - \tan)dx + (x^2 - x\tan y + \sec^2 y)dy = 0$

$M = 2xy + y - \tan y \;,\; N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y \quad \frac{dN}{dx} = 2x - \tan^2 y + 0$

$= 2x - \tan^2 y$

$\frac{\partial M}{\partial y} \quad \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{tarix of } \alpha N x)\, dy = c$

$\int (2xy + \quad - \tan y)\, dx + \int \sec^2 y\, dy = c$

$\frac{2x^2 y}{2} + xy - x\tan y + \tan y + c$ Ans

Question No - 2

$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

$M = 3x^2 + 4xy \quad , \quad N = 2x^2 + 2y$

$\frac{\partial M}{\partial y} = 0 + 4x \qquad \frac{\partial N}{\partial x} = 4x + 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So given diff eq is exact

now $\int M dx + \int (\tan \text{ of } N \text{ are form})\, dy = c$

$(3x^2 + 4xy)\, dx + \int 2y\, dy = c$

$3\frac{x^3}{3} + \frac{4x^2 y}{2} + \frac{2y^2}{2} = c$

$3\frac{x^3}{3} + \frac{4x^2 y}{2} + \frac{2y^2}{2} = c$

Question No-3

$$\frac{(x+y)}{y-1}dx - \frac{1}{2}\frac{(x+1)^2}{y-1}dy = 0$$

$$M = \frac{x+y}{y-1} \quad N = -\frac{1}{2}\frac{(x+1)}{y-1}dy = 0$$

$$\frac{\partial M}{\partial y} = (y-1)(0+1)-(x+y)(1) \;, \quad N = -\frac{1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$$

$$= \frac{y-1-x-y}{(y-1)^2} \;, \quad \frac{\partial N}{\partial x} = -\frac{(2x+2)}{2(y-1)^2}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \because \text{ given differ eq}$$

$$\int M dx + \int (\text{term of } N \text{ free from } x)\, dy = c$$

$$\int \frac{x+y}{y-1}dx + \int -\frac{1}{2}\frac{dy}{(y-1)^2} = c$$

$$\left(\frac{1}{y-1}\right)\int (x+y)\,dx + \left(-\frac{1}{2}\right)\int (y-1)^{-2}dy = c$$

$$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2}+xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2+2xy+1 = c'(y-1) \quad \text{Ans}$$

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Question No 4

$$\frac{dy}{dx} = \left(\frac{ax+hy}{hx+by}\right)$$

$(hx+by) = -(ax+hy)dx$

$(ax+hy)dx + (hx+by)dy = 0$

$M = ax+hy \quad N = hx+by$

$\frac{\partial M}{\partial y} = 0+h \quad \frac{\partial N}{\partial x} = h$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exacting

$\int M dx + \int (\text{ter of free from } x) dy = c$

$\int (ax+hy)dx + \int by \, dy = c$

$\frac{ax^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c$

Question-No-5

$(1+\ln xy)dx + \left(1+\frac{x}{y}\right)dy = 0$

$M = 1+\ln xy \quad N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy} \cdot x \quad \frac{\partial N}{\partial N} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exacting

$\int M dx + \int (\text{tran of } N \text{ free from } x) dy = 0$

$\int (1+\ln xy)dx + \int 1 dx = c$

$\int dx + \int \ln xy \, dx + \int dy = c$

$x + \left[\ln xy \cdot (x) - \int \frac{1}{xy} \cdot y \, x dx\right] + y = 0$

$x + x\ln xy - \int dx + y = c$

$\cancel{x} + x\ln xy - \cancel{x} + y$

$x\ln xy + y = c$

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Question No-6

$\frac{ydx + xdy}{1-x^2y^2} + xdx = 0$

$\frac{ydx}{1-x^2y^2} + \frac{xdy}{1-x^2y^2} + xdx = 0$

$\left(x + \frac{y}{1-x^2y^2}\right)dx + \frac{xdy}{1-x^2y^2} = 0$

$M = x + \frac{y}{1+x^2y^2}$

$\frac{\partial M}{\partial y} = 0 + \frac{(1-x^2y^2)1 - y(-2x^2y)}{(1-x^2y^2)^2}$

$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)} = \frac{1+x^2y^2}{1-x^2y^2}$

$N = \frac{x}{1-x^2y^2}$

$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2)1 - x(-2xy^2)}{(1-x^2y^2)^2}$

$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{(1-x^2y^2)^2}$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{term of N free x})dy = c$

$\int \left(x + \frac{y}{1-x^2y^2}\right)dx + N i l = c$

$\int x dx + \int y \frac{dx}{1-x^2y^2} = c$

$\frac{x^2}{2} + \int \frac{\frac{y}{y^2}}{\frac{1}{y^2} - \frac{x^2y^2}{y^2}} dx = c$

$\frac{x^2}{2} + \frac{1}{y}\left(\frac{1}{2(1/y)} \ln\left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right) = c$

$\frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right|$

$x^2 + \ln\left(\frac{1+xy}{1-xy}\right) = c$ Ans

Question No-7

$(6xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6)$

$M = 6xy + 2y^2 - 5,$

$\frac{\partial M}{\partial y} = 6x + 4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$N = 3x^2 + 4xy - 6$

$\frac{\partial N}{\partial x} = 6x + 4y$

$\int M dx + \int (\text{term } N \text{ free from } x)$

$\int (6xy + 2y^2 - 5)dx + \int -6dy = c$

$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$

$3x^2y + 2xy^2 - 5x - 6y = c$

Question No-8

$(y\sec^2 x + \sec x \tan x)dx + (\tan x + 2y) = 0$

$M = y\sec^2 x + \sec x \tan x$

$\frac{\partial M}{\partial y} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$N = \tan x + 2y$

$\frac{\partial N}{\partial x} = \sec^2 x$

$\int M dx + \int (\text{term of N free from x})dy = c$

$\int (y\sec^2 x + \sec x \tan x)dx + \int 2y dy = c$

$y \tan x + \sec x + y^2 = c$ Ans

Ex 9.5

Solve by finding an I.F

Question No-1

$(xy^2+y)dx - x\,dy = 0$ —①

$M = xy^2+y \qquad N = -x$

$M_y = 2xy+1 \qquad N_x = -1$

$\therefore M_y \neq N_x \therefore$ Non Exact

$\frac{M_y - N_x}{N} = \frac{2xy+1+1}{-x}$

$\frac{N_x - M_y}{M} = \frac{-1-2y-1+1}{-x}$

$= \frac{-2(1+xy)}{y(xy+1)} = \frac{-2}{y}$

$-\int \frac{2}{y}dy = -2\ln y \quad \ln y^{-2}$

$I.F = e = e = e = y^{-2} = \frac{1}{y^2}$

multiplay both side of eq ① by $I.F = \frac{1}{y^2}$

$\frac{1}{y^2}(xy^2+y)dx - \frac{x}{y^2}dy = 0$

2 $\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0$ —①

Now $M = x+\frac{1}{y} \qquad N = \frac{-x}{y^2}$

$M_y = -\frac{1}{y^2} \qquad N_x = -\frac{1}{y^2}$

$\therefore M_y = N_x$

$\int M dx + \int (\tan \text{ of } N \text{ free from}) dy = c$

$\int \left(x+\frac{1}{y}\right)dx + Nil = c$

$\frac{x^2}{2} + \frac{x}{y} = c$ Ans

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Question No-3

$(x^2+x-y)dx + x\,dy = 0$ —— ①

$M = x^2+x-y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$M_y \neq N_x$

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$

$I.f = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

Multiply both sides of eq(1) I.f $= 1/x^2$

$1/x^2 (x^2+x-y)dx + x/x^2\, dy = 0$

$(1+\frac{1}{x}-\frac{y}{x^2})dx + 1/x\, dy = 0$

Now $M = 1+1/x - \frac{y}{x^2} \qquad N = 1/x$

$M_y = -1/x^2 \qquad N_x = -1/x^2$

$M_y = N_x$

$\int M dx + \int (\text{term of } N \text{ free from } x)\, dy = c$

$\int (1+1/x - y/x^2)dx + \text{Nil} = c$

$x + \ln x + \frac{y}{x} = c$

Question No-4

$dy + (\frac{y-\sin x}{x})dx = 0$

$M = \frac{y-\sin x}{x} \qquad N = 1$

$M_y = 1/x - 0 \qquad N_x = 0$

$M_y \neq N_x$

Now $\frac{M_y - N_x}{N} = \frac{1/x - 0}{1} = 1/x$

$If = e^{\int 1/x} = e^{\ln x} = \boxed{x}$

multiplying both sides of eq①

$M = y - \sin x \qquad N = x$

$M_y = 1 \qquad N_x = 1$

$M_y = N_x$

$\int M dx + \int (\text{term of } N \text{ free } (x)) = c$

$\int (y - \sin x)dx = c$

$xy + \cos x = c$

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Question No-5

$y(2xy+e^x)dx - e^x dy = 0$

$(2xy^2+e^x y)dx - e^x dy = 0$ —— ①

$M = 2xy^2+e^x y \qquad N = -e^x$

$M_y = 4xy + e^x \qquad N_x = -e^x$

$M_y \neq N_x$

$\frac{M_y - N_x}{N} = \frac{4xy+e^x+e^x}{-e^x}$

$\frac{N_x - M_y}{M} = \frac{-e^x-4xy-e^x}{2xy^2+ye^x} = \frac{-2e^x-4xy}{y(2xy+e^x)}$

$= \frac{-2(e^x+2xy)}{y(2xy+e^x)} = -2/y$

$I.f = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \boxed{1/y^2}$

multiplying both sides ① by I.f $= 1/y^2$

$\frac{1}{y^2}(2xy^2+e^x)dx - \frac{1}{y^2}e^x dy = 0$

$(2x+\frac{e^x}{y})dx - \frac{e^x}{y^2}dy = 0$ —— ②

$M = 2x+\frac{e^x}{y} \qquad N = \frac{-e^x}{y^2}$

$M_y = 0 + (\frac{-e^x}{y^2} \qquad N_x = \frac{-e^x}{y^2}$

$M_y = N_x$

$\int M dx + \int (\text{term } N \text{ free from } x) dy = c$

$\int (2x+\frac{e^x}{y})dx + \int 2y\,dy = c$

$x^2 + \frac{e^x}{y} = c \text{ Ans}$

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Question No-6

$(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$ — ①

$M = y^4+2y \qquad N = xy^3+2y^4-4x$

$M_y = 4y^3+2 \qquad N_x = y^3-4$

$M_y \neq N_x$

$$\frac{N_x - M_y}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} = \frac{-3y^3-6}{y(y^3+2)} = -3/y$$

$$I.f = e^{\int -3/y\,dy} = e^{-3\ln y} = e^{\ln y^{-3}} = y^{-3} = \boxed{1/y^3}$$

$\frac{1}{y^3}(y^4+2y)dx + 1/y^3(xy^3+2y^4-4x)dy = 0$

$(y + 2/y^2)dx + (x+2y - \frac{4x}{y^3})dy = 0$ — (ii)

NOW $M = y + 2/y^2 \qquad N = x+2y - \frac{4x}{y^3}$

$M_y = 1 - \frac{4}{y^3} \qquad N_x = 1+0-\frac{4}{y^3}$

$M_y = N_x$

$\int M dx + \int(\text{tan of } N \text{ free from } x)\, dy = c$

$\int(y + 2/y^2)dx + \int 2y\,dy = c$

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$

$xy + \frac{2x}{y^2} + y^2 = c$ Ans

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Ex No-9.6

Question No -1

$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$

$\int p\,dy$

$I.f = e^{\int p\,dy} \cdot e^{\int \frac{2x+1}{x}dx} = e^{\int (\frac{x+1}{x})dx}$

$= e^{2x + \ln x} = e^{2x} e^{\ln x} = e^{2x}x$

$\therefore$ Solve is given by $\int (y \times I.f) = \int Q.I.f + c)$

$= \int d(ye^{2x}x) = \int e^{-2x} e^{2x} x\,dx + c$

$\Rightarrow ye^{2x}x = \int x\,dx + c$

$\Rightarrow xye^{2x} = \frac{x^2}{2} + c$ Ans

Question No-2

$\frac{dy}{dx} + \frac{3}{x}y = 6x^2$

$I.f = e^{\int p\,dx} = e^{\int \frac{3}{x}dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

$\therefore$ Solve is given by $\int d(y \times I.f) = \int Q \times I.f\,dx + c$

$\Rightarrow \int d(yx^3) = \int 6x^2 x^2\,dx + c$

$\Rightarrow yx^3 = \int 6x^5\,dx + c$

$\Rightarrow yx^3 = \frac{6x^6}{6} + c$

$\Rightarrow x^3y = x^6 + c$ Ans

Question No-3

$\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x^2}{\ln x}$

$I.f = e^{\int p\,dx} = e^{\int \frac{1}{x\ln x}dx} = e^{\int \frac{1/x}{\ln x}}$

$I.f = e^{\ln(\ln x)} = \boxed{\ln x}$

Sol in by $\int d(y \times I.f) = \int Q \times I.f\,dx + c)$

$\Rightarrow \int d(y\ln x) = \int \frac{3x^2}{\ln x} \ln x\,dx + c$

$\Rightarrow y\ln x = 3x^3 + c$

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Question No-4

$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$

$I.f = e^{\int P dx} = e^{\int 3 dx} = e^{3x}$

Solve by $\int d(y \times I.f) = \int Q \times I.f \, dx + c$

$\Rightarrow \int d(ye^{3x}) = \int 3x^2 e^{-3x} e^{3x} dx + c$

$ye^{3x} = x^3 + c$

$y = e^{-3x}(x^3 + c)$ Ans

Question No-5

$\cos^2 x \frac{dy}{dx} + y\cos x = \sin x$

$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$

$\frac{dy}{dx} + \sec^2 x y = \sec^2 x \tan x$

$\int P dx \quad \int \sec^2 x dx = e^{\tan x}$

Solve by $\int d(y \times I.f) = \int Q \times I.f \, dx + c$

$\Rightarrow \int d(y \times e^{\tan x}) = \int \sin^2 x \tan x e^{\tan x} dx + c$

$\Rightarrow ye^{\tan x} = \int e^t t \, dt + c \quad \sec^2 x dx = dt$

$= te^t - \int 1 \cdot e^t dt + c$

$= te^t - e^t + c$

$ye^{\tan x} = e^t (t-1) + c$

$ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$

$y = (\tan x - 1) + ce^{-\tan x}$

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Question No-7

$(x^2+y^2+2x)\,dx + 2y\,dy = 0$ — ①

$M = x^2+y^2+2x \qquad N = 2y$

$M_y = 2y \qquad N_x = 0$

$M_y \neq N_x$

$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2x}$

$\frac{M_y - N_x}{N} = \frac{2y-0}{2y} = 1 \times x$

$I.f = e^{\int 1.dx} = e^x$

Multiplying both sides eq ① by $I.f = \boxed{e^x}$

$e^x(x^2+y^2+2x)dx + e^x(2y\,dy = 0$ — ②

$M = e^x(x^2+y^2+2x) \quad N = e^x 2y$

$M_y = e^x 2y \qquad N_x = e^x 2y$

$M_y = N_x$

$\int M dx + \int$ (term of N free from x) $dy = c$

$\int e^x(x^2+y^2+2x)\,dx + \text{Nil} = c$

$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x dx = c$

$x^2 e^x - \int \cancel{2x} a^x dx + e^x y^2 + \int \cancel{e^x 2x} dx = c$

$(x^2+y^2)e^x = c$ Ans

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Question No-8

$(x^2+y^2)dx - 2xy\,dx = 0$ —①

$M = x^2+y^2 \qquad N = -2xy$

$My = 2y \qquad Nx = -2y$

$My \neq Nx$

$\frac{Nx - My}{M} = \frac{-2y-2y}{x^2+y^2}$

$\frac{My - Nx}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = -2/x$

$If \cdot e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \boxed{1/x^2}$

multiplying both sides of eq① by $If = 1/x^2$

$\frac{1}{x^2}(x^2+y^2)dx - 1/x^2 (2xy)dy = 0$

$(1+y^2)dx - \frac{2y}{x}dy = 0$

$M = \frac{1+y^2}{x^2}, \quad N = \frac{-2y}{x}$

$My = \frac{2y}{x^2} \qquad Nx = +\frac{2y}{x^2}$

$My = Nx$

$\therefore \int M dx + \int (\text{term } N \text{ free from } x)$

$\int \frac{1+y^2}{x^2} dx + Nil = c$

$\frac{x-y^2}{x} = c$ Ans

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Question No - 8

$x\frac{dy}{dx} + (1 + x\cot x)y = x$

Solution

$\frac{dy}{dx} + (1/x + \cot x)y = 1$

$= e^{\int P dx} = e^{\int (\frac{1}{x} + \cot x) dx} = e^{\ln x + \ln \sin x}$

$I.F. = e^{\ln(\sin x)} \Rightarrow x \sin x$

Sol — by $\int d(v \times I.f) = \int Q \times I.f dx + c$

$\Rightarrow \int d(yx \sin x) = \int x \sin x dx + c$

$y \sin x = x(-\cos x) - \int 1 \cdot (-\cos x dx)$

$= x(-\cos x) + \int \cos x dx$

$y x \sin x = -x \cos x + \sin x + c$

$y = \cot x + \frac{1}{x} + \frac{c}{x} \cos x$ Ans

Question No - 7

$(x+1)\frac{dy}{dx} - xy = e^x(x+1)^{x+1}$

$\frac{dy}{dx} - \frac{x}{(x+1)} y = e^x (x+1)^x$

$I.f = e^{\int P dx} = e^{\int (\frac{x}{x+1}) dx} = e^{-x \ln x (x+1)} = e^{\ln (x+1)^{-x}}$

$I.f = (x+1)^{-x} = \frac{1}{(x+1)^x}$

$Sol — \int d(y \times I.f) = \int Q \times I.f dx + c$

$\Rightarrow \int d(y \frac{1}{x+1^x}) = \int e^x (x+1^x \frac{1}{x+1^x} dx + c$

$\frac{y}{(x+1)^x} = e^x + c$

$y = (e^x + c)(x+1)^x$ Ans