Topic

Mathematics

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Exersice # 9.2

1 $\dfrac{dy}{dx} = \dfrac{x^2}{y(1+x^3)}$

$\int y\,dy = \int \dfrac{x^2}{1+x^3}\,dx$

$\dfrac{y^2}{2} = \dfrac{1}{3}\int \dfrac{3x^2}{1+x^3}\,dx$

$\dfrac{y^2}{2} = \dfrac{1}{3}\ln(1+x^3) + c$

$\dfrac{3y^2}{2} = \ln\{1+x^3) + 3c$

$3y^2 = 2\ln(1+x^3) + 6c$

$3y^2 = 2\ln(1+x^3) + c'$ Ans

....

2 $\dfrac{dy}{dx} + y^2 \sin x = 0$

$\dfrac{dy}{dx} = -y^2 \sin x$

$\int \dfrac{dy}{y^2} = -\int \sin x\,dx$

$-\dfrac{y^{-1}}{1} = -(-\cos x) + c \Rightarrow -1/y = \cos x + c$ Ans

3 $\frac{dy}{dx} = 1+x+y^2+xy^2$

$$\frac{dy}{dx} = (1+x) + y^2(x+1)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{dx} = \int (1+x)(1+y^2)$$

$$\int \frac{1}{1+y^2}\, dy = \int (1+x)\, dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1} y = 2x + x^2 + c'$$

4 $\frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$

$$= 2x^2 - 2x - 2 + y - x^2y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{1}{2-y}\, dy = \int (x^2 - x - 1)\, dx$$

$$-\int -\frac{1}{2-y}\, dy = \int (x^2 - x - 1)\, dx$$

$$-\ln(2-y) = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$-\ln(2-y) = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-6\ln(2-y) = 2x^3 - 3x^2 - 6x + 6c$$

$$\ln |2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln c$$

$$\ln |2-y|^{-6} = \ln e^{2x^3-3x^2-6x+6c}$$

$$|2-y|^{-6} = e^{2x^3-3x^2-6x} \cdot e^{6c} \quad \underline{\text{Ans}}$$

<u>5</u> $(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$

$$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$$

$$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2)}\,dx + \frac{1}{y+2}\,dy = 0$$

$$\int \frac{x+1}{x^2+2}\,dx + \int \frac{1}{y+2}\,dy = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2) + \ln c$$

$$y+2 = \frac{c}{\sqrt{x^2+2}} \quad \underline{\text{Ans}}$$

6 $\operatorname{cosec} y \, dx + \sec x \, dy = 0$

$\div$ by $\operatorname{cosec} y \sec x$

$$\Rightarrow \frac{1}{\sec x} dx + \frac{1}{\operatorname{cosec} y} dy = 0$$

$$\Rightarrow \int \frac{1}{\sec x} dx + \int \frac{1}{\operatorname{cosec} y} dy = 0$$

$$\int \cos x \, dx + \int \sin y \, dy = 0$$

$$\sin x - \cos y = c \text{ Ans}$$

7 $y(1+x)dx + x(1+y)dy = 0$

~~let~~ $\div$ by xy

$$\Rightarrow \frac{1+x}{x} dx + \frac{1+y}{y} dy = 0$$

$$\Rightarrow \int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy = 0$$

$$\Rightarrow \ln x + x + \ln y + y = c$$

$$\Rightarrow x + y + \ln(xy) = C \text{ Ans}$$

8 $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}} \quad \text{if} \quad |x|<1, \; |y|<1$$

or $$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = -\sin^{-1} x + c$$

$$y = \sin(c - \sin^{-1} x) \quad \text{Ans}$$

9 $y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$$\int \frac{\sqrt{1+x^2}}{x}\,dx + \int \frac{\sqrt{1+y^2}}{y}\,dy = 0$$

put $\sqrt{1+x^2} = t$, put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$, $1+y^2 = z^2$

$2x\,dx = 2t\,dt$, $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$, $y\,dy = z\,dz$

$$\int \frac{\sqrt{1+x^2}}{x^2}\,x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}\,y\,dy = 0$$

$$\int \frac{t\,.\,t\,dt}{t^2-1} + \int \frac{z\,.\,z\,dz}{z^2-1} = 0$$

$$\int \frac{\quad}{t^2-1}\,dt + \int \frac{\quad}{z^2-1}\,dz$$

$$\int \left(1+\frac{1}{t^2-1}\right)dt + \int \left(1+\frac{1}{z^2-1}\right)dz = 0$$

$$t+\frac{1}{2}\ln\left(\frac{t-1}{t^2-1}\right) + z+\frac{1}{2}\ln\left(\frac{z-1}{z^2-1}\right) = c$$

$$\sqrt{1+x^2}+\frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+\sqrt{1+y^2}+\frac{1}{2}\ln\left(\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right) =$$

Exersice # 9.3

$$(x-y)\,dx + (x+y)\,dy = 0$$

$$(x+y)\,dy = -(x-y)\,dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{--- } i$$

Put $\quad y = Vx$ — ii

$$\frac{dy}{dx} = V + \frac{x\,dv}{dx} \quad \text{---} \quad iii$$

using ii iii, & in i

$$V + x\frac{dv}{dx} = \frac{Vx-x}{x+Vx}$$

$$x\frac{dv}{dx} = \frac{x}{x}\left(\frac{V-1}{V+1}\right) - V \Rightarrow \frac{\cancel{V}-1-\cancel{V}-V^2}{1+V}$$

$$x\frac{dv}{dx} = -\frac{(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{1}{x}\,dx$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + c$$

$$\ln(v^2+1)^{1/2} + \tan^{-1}v + \ln x = c \Rightarrow \ln\sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\frac{y}{x} + \ln x$$

$$\ln\sqrt{y^2+x^2} - \cancel{\ln\sqrt{x^2}} + \tan^{-1}\frac{y}{x} + \cancel{\ln x} = c$$

$$\ln\sqrt{y^2+x^2} + \tan^{-1}\frac{y}{x} = c \text{ Ans}$$

2 $(x^2-3y^2)dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2-3y^2)dx$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{--- } i$$

put $y = vx$, using ii, iii in i

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2x\,vx}$$

$$x\frac{dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v, \quad x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}, \quad \int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$$

$$\frac{y^2-x^2}{x} = cx$$

$$y^2-x^2 = (cx)\,x^2 \quad \underline{\text{Ans}}$$

3 $(x^2+xy+y^2)\,dx - x^2\,dy = 0$

$$(x^2+xy+y^2)\,dx = x^2\,dy.$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{— } \underline{i}$$

Put $y = vx$ — $\underline{ii}$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— } \underline{iii}$$

using ii, iii in i

$$v + x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{(1+v+v^2)\,x^2}{x^2} - v$$

$$\int\frac{dv}{1+v^2} = \int\frac{dx}{x}$$

$$\tan^{-1}v = \ln x + c$$

$$\tan^{-1}\frac{y}{x} = \ln x + c \quad \underline{\text{Ans}}$$

4 $\quad x \sin\left(\frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$

$$\frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{— } \underline{i}$$

put $\quad y = Vx \quad \text{— } \underline{ii}$

$$\frac{dy}{dx} = V + x\frac{dv}{dx} \quad \text{— } \underline{iii}$$

using $\underline{ii}, \underline{iii}$ in $\underline{i}$

$$V + x\frac{dv}{dx} = \frac{Vx \sin \frac{Vx}{x} - x}{x \sin \frac{Vx}{x}}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(V\sin V - 1)}{\cancel{x} \sin V} - V$$

$$x\frac{dv}{dx} = \frac{V \cancel{\sin V} - 1 - V \cancel{\sin V}}{\sin V}$$

$$\int \sin v \, dv = -\int \frac{1}{x} dx$$

$$-\cos V = -\ln x + c, \quad \cos V = \ln x - c$$

$$\cos \frac{y}{x} = \ln x - c \quad \text{Ans}$$

5. $\quad \frac{dy}{dx} = \frac{x+y}{x} \quad \text{— } \underline{i} \qquad y(1) = 1$

Put $y = Vx \text{ — } \underline{i}$, $\quad \frac{dy}{dx} = V + x\frac{dv}{dx} \quad \text{— } \underline{iii}$

using $\underline{ii}, \underline{iii}$ in $\underline{i}$

$$V + x\frac{dv}{dx} = \frac{x + Vx}{x}, \Rightarrow x\frac{dv}{dx} = \frac{x(1+V)}{x} - V$$

$$x\frac{dv}{dx} = 1 \Rightarrow \int dv = \int \frac{dx}{x}$$

$$V = \ln x + C, \quad \frac{y}{x} = \ln x + C \quad \underline{\text{Ans}}$$

6 $(3x^2+9xy+5y^2)\,dx-(6x^2+4xy)\,dy=0, \quad y(2)=-6$

$$(6x^2+4xy)\,dy=(3x^2+9xy+5y^2)\,dx$$

$$\frac{dy}{dx}=\frac{3x^2+9xy+5y^2}{6x^2+4xy} \quad \text{— } \underline{i}$$

put $y=vx$, — $\underline{ii}$ $\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$ — $\underline{iii}$

using ii & $\underline{iii}$ in $\underline{i}$

$$v+x\frac{dv}{dx}=\frac{3x^2+9xvx+5v^2x^2}{6x^2+4xvx}$$

$$x\frac{dv}{dx}=\frac{\cancel{x^2}(3+9v+5v^2)}{\cancel{x^2}(6+4v)}-v$$

$$x\frac{dv}{dx}=\frac{3+9v+5v^2-6v-4v^2}{6+4v}$$

$$x\frac{dv}{dx}=\frac{v^2+3v+3}{6+4v}$$

$$\int\frac{4v+6}{v^2+3v+3}\,dv=\int\frac{dx}{x}, \Rightarrow 2\int\frac{2v+3}{v^2+3v+3}\,dv=\int\frac{dx}{x}$$

$$2\ln(v^2+3v+3)=\ln x+c, \Rightarrow \ln(v^2+3v+3)^2=\ln xc$$

$$(v^2+3v+3)^2=cx \Rightarrow \left(\frac{y^2}{x^2}+3\frac{y}{x}+3\right)^2=cx$$

$$\frac{(y^2+3yx+3x^2)^2}{x^2}=\sqrt{cx}$$

$$y^2+3yx+3x^2=x^2\sqrt{cx} \qquad \because y(2)=6$$

$$\frac{12}{4}=\sqrt{2c}$$

$$3^2=2c \Rightarrow \quad 3x^2+3xy+y^2=x^2\sqrt{\frac{9}{2}x}$$

$$(3)^2= \qquad 3x^2+3xy+y^2=\frac{9}{\sqrt{2}}x^{2+\frac{1}{2}}$$

$$\boxed{c=\frac{9}{2}} \qquad \sqrt{2}(3x^2+3x+y^2)=3x^{5/2}$$

$$2(3x^2+3xy+y^2)=9x^5$$

$$v=\ln x+c, \quad \frac{y}{x}=\ln x \ldots$$

7 $\frac{dy}{dx} = \cancel{5y+3}\ \frac{5y-2x}{4x-y}$ — i

put $y = Vx$ — ii $\frac{dy}{dx} = V + x\frac{dv}{dx}$ — iii

using ii & iii in i

$$V + x\frac{dv}{dx} = \frac{5Vx - 2x}{4x - Vx}$$

$$V + x\frac{dv}{dx} = \frac{5Vx - 2x}{4x - Vx}$$

$$= \frac{x(5V-2)}{x(4-V)} - V$$

$$x\frac{dv}{dx} = \frac{5V - 2 - 4V + V^2}{4-V} \Rightarrow x\frac{dv}{dx} = \frac{V^2+V-2}{4-V}$$

$$\int \frac{4-V}{V^2+V-2}\,dV = \int \frac{dx}{x}$$

By partial fraction

$$\int \frac{4-V}{(V-1)(V+2)}\,dV = \int \frac{dx}{x}$$

$$\int \left(\frac{dv}{V-1} - \frac{2}{V+2}\right) dv = \int \frac{dx}{x}$$

$$\ln(V-1) - 2\ln(V+2) = \ln x + \ln c$$

$$\Rightarrow \ln \left(\frac{V-1}{(V+2)^2}\right) = \ln cx$$

Applying $\frac{\frac{y}{x} - 1}{\left(\frac{y}{x}+2\right)^2} = cx,$ $\frac{\frac{y-x}{x}}{\frac{(y+2x)}{x}} = c$

$\because y(1) = 4$

$\frac{4-1}{(4+2)^2} = c$

$\frac{3}{36} = \frac{1}{12}$

$12(y-x) = (y+2x)^2$ Ans

$V = \ln x + c,\ \frac{y}{x} = \ln x \ldots$

Exercise # 9.4

1. $(3x^2+4xy)dx+(2x^2+2y)dy=0$

Let $M=3x^2+4xy$, $N=2x^2+2y$

$\frac{\partial M}{\partial y}=0+4x$, $\frac{\partial N}{\partial x}=4x+0$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ so given Diff Eq

Now $\int M\,dx+\int(\text{tan of } N \text{ from } x)\,dy=C$

$\int(3x^2+4xy)dx+\int 2y\,dy=C$

$\frac{3x^3}{3}+\frac{4x^2y}{2}+\frac{2y^2}{2}=C$

$x^3+2x^2y+y^2=C$ Ans

2 $(2xy+y-\tan y)dx+(x^2+x\tan^2 y+\sec^2 y)dy=0$

$M=2xy+y-\tan y$, $N=x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial y}=2x+1-\sec^2 y$, $\frac{\partial N}{x}=2x-\tan^2 y+0$

$=2x-\tan^2 y$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\int M\,dx+\int(\text{tan of } N \text{ from } x)\,dy=C$

$\int(2xy+y-\tan y)dx+\int \sec^2 y\,dy=C$

$\frac{2x^2y}{2}+xy-x\tan y+\tan y=C$

$x^2y+xy-x\tan y+\tan y=C$ Ans

$\ln x+C$, $\frac{y}{x}=\ln\ldots$

3 $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$M = \frac{x+y}{y-1}$, $N = -\frac{1}{2}\left(\frac{(x+1)^2}{(y-1)^2}\right)$

$N = -\frac{1}{2}\,\frac{x^2+2x+1}{(y-1)^2}$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}$, $\frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$

$= \frac{y-1-x-1}{(y-1)^2}$, $= \frac{-x-1}{(y-1)^2}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\because$ given diff is Exact

$\int M dx + \int (\text{term of } N \text{ from } x)\, dy = c$

$\int \left(\frac{x+y}{y-1}\right)dx + \int -\frac{1}{2(y-1)^2}\, dy = c$

$\left(\frac{1}{y-1}\right)\int (x+y)dx + -\frac{1}{2}\int (y-1)^{-2} dy = c$

$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{y-1}\right) = c$

$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$

$x^2+2xy+1 = c'(y-1)$ Ans

4 $\frac{dy}{dx} = -\left(\frac{ax+hy}{hx+by}\right)$

$(hx+by)\,dy = -(ax+hy)\,dx$

$(ax+hy)\,dx + (hx+by)\,dy = 0$

$M = ax+hy$ $N = hx+by$

$\frac{\partial M}{\partial y} = 0+h$ $\frac{\partial N}{\partial x} = h$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact Diff Eq.

$V = \ln x + c$, $\frac{y}{x} = \ln x \ldots$

$\int (ax + hy)dx + \int by\, dy = c$

$\frac{ax^2}{2} + hxy + b\frac{x^2}{2} = c$

$ax^2 + 2hxy + by^2 = c'$ Ans

5 $(1+ \ln xy)\, dx + (1+ \frac{x}{y})\, dy = 0$

$M = 1 + \ln xy \qquad N = 1 + \frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy} \cdot x \quad , \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Exact Diff eq.

$\int M dx + \int (\text{term of free } x)\, dy = c$

$\int (1+ \ln xy) + \int 1 \cdot dy = c$

$\int dx + \int 1 \ln xy\, dx + \int dy = c$

$x + x \ln xy - \int dx + y = c$

$\cancel{x} + x \ln xy - \cancel{x} + y = c$

$x \ln xy + y = c$ Ans

6 $(6xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6) = 0$

$M = 6xy + 2y^2 - 5 \quad , N = 3x^2 + 4xy - 6$

$\frac{\partial M}{\partial y} = 6x + 4y \qquad \frac{\partial N}{\partial x} = 6x + 4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact Diff eq.

$\int M dx + \int (\text{ten of } N \text{ free } x)\, dx = 0$

$(6xy + 2y^2 - 5)dx + \int 6\,dy = c$

$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = C$

$3x^2y + 2xy^2 - 5x - 6y = c$ Ans

7 $(y\sec^2x + \sec x\tan x)dx + (\tan x + 2y)dy = 0$

$M = y\sec^2x + \sec x\tan x$; $N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x$ $\frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int(\text{term of } N \text{ from } x)dy = c$

$\int(y\sec^2x + \sec x\tan x)\,dx + \int 2y\,dy = c$

$y\tan x + \sec x + y^2 = C$ = Ans

8. $(y\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$

$M = y\cos x + 2xe^y$, $N = \sin x + x^2e^y - 1$

$\frac{\partial M}{\partial y} = \cos x + 2xe^y$, $\frac{\partial N}{\partial x} = \cos x + 2xe^y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact Diff.

$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = c$

$\int(y\cos x + 2xe^y)\,dx + \int -1\,dy = c$

$y\sin x + \frac{2xe^x}{x} - y = c$

$y\sin x + x^2e^y - y = c$ Ans

Exercise # 9.5

1. $(x^2+x-y)\,dx + x\,dy = 0$

$M = x^2+x-y \quad , \quad N = x$

$M_y = -1 \quad , \quad N_x = 1$

$M_y \neq N_x$ Non Exact Diff Equation.

$\dfrac{M_y - N_x}{N} = \dfrac{-1-1}{x} = \dfrac{-2}{x}$ of x alone

$\therefore I.f = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}}$

$= x^{-2} \Rightarrow \dfrac{1}{x^2}$

Multiplying both sides of eq ① by $I.f = \frac{1}{x^2}$

$\dfrac{1}{x^2}(x^2+x-y)dx + \dfrac{x}{x^2}dy = 0$

$(1+\dfrac{1}{x} - \dfrac{y}{x^2})dx + \dfrac{1}{x}dy = 0$ — (ii)

Now $M = 1+\dfrac{1}{x} - \dfrac{y}{x^2} \qquad N = \dfrac{1}{x}$

$M_y = -\dfrac{1}{x^2} \quad , \quad N_x = -\dfrac{1}{x^2}$

$M_y = N_x$ now Exact eq.

$\int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = 0$

$\int(1+\dfrac{1}{x} - \dfrac{y}{x^2})dx + \text{Nil} = C$

$x + \ln x + \dfrac{y}{x} = C$ Ans

2. $dy + \left(\frac{y-\sin x}{x}\right) dx = 0$

$M = \frac{y-\sin x}{x}$, $N = 1$

$M_y = \frac{1}{x} - 0$ $N_x = 0$

$M_y \neq N_x$ is non Exact Diff eq.

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$

$I.f = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$ ✓

'Xing' both sides of eq (1) by $I.f = x$

$x\,dy + \frac{x(y-\sin x)}{x} dx = 0$

$M = y - \sin x$ $N = x$

$M_y = 1$ $N_x = 1$

$M_y = N_x$ Exact Diff eq.

$\int M\,dx + \int (\text{term of } x)\,dy = 0$

$\int (y - \sin x)\,dx = c$

$xy + \cos x = c$ Ans

3. $(x^2 + y^2 + 2x)\,dx + 2y\,dy = 0$ — i

$M = x^2 + y^2 + 2x$ $N = 2y$

$M_y = 2y$ $N_x = 0$

$M_y \neq N_x$ is non Exact Diff Equation

$\frac{N_x - M_y}{M} = \frac{0 - 2y}{x^2+y^2+2x} = 1 = x$ Not fx of y on

$\frac{M_y - N_x}{N} = \frac{2y - 0}{2y} = 1 = x^0$ fx of x only

$I.F = e^{\int 1.dx} = e^x$

Multiplying both sides of eq (1) by $I.F = e^x$

$e^{x}(x^2+y^2+2x) + e^{x}(2y)dy = 0$ — (i)

$M = e^{x}(x^2+y^2+2x)$ $\quad$ $N = e^{x}2y$

$M_y = e^{x}2y$ $\quad$ $N_x = e^{x}2y$

$M_y = N_x$ Exact Diff Eq.

$\int e^{x}(x^2+y^2+2x)dx + Nil = c$

$\int e^{x}x^2\,dx + \int e^{x}y^2\,dx + \int e^{x}2x\,dx = c$

$x^2e^{x} - \int 2xe^{x}dx + e^{x}y^2 + \int e^{x}2x\,dx = c$

$(x^2+y^2)e^{x} = c$ Ans

4. $(x^2+y^2)dx - 2xy\,dy = 0$ — (i)

$M = x^2+y^2$ $\quad$ $N = -2xy$

$M_y = 2y$ $\quad$ $N_x = -2y$

$M_y \neq N_x$ $\because$ (i) is Non Exact Diff Eq.

$\frac{N_x - M_y}{M} = -\frac{2y-2y}{x^2+y^2}$ not for y only

$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = -\frac{2}{x}$ fn for x only.

$I.F = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = e^{-2} = \frac{1}{x^2}$ ✓

xing both sides of eq (i) by I.F $= \frac{1}{x^2}$

$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$

$(1+\frac{y^2}{x^2})dx - \frac{2y}{x}dy = 0$ — (ii)

$M = 1+\frac{y^2}{x^2}$ $\quad$ $N = -\frac{2y}{x}$

$M_y = \frac{2y}{x^2}$ $\quad$ $N_x = \frac{2y}{x^2}$

$M_y = N_x$ Exact Diff eq.

$\int M\,dx + \int(\text{term of } N \not{x})\,dy = 0$

$\int(1 + \frac{y^2}{x^2})\,dx + \text{Nil} = c$

$x - \frac{y^2}{x} = c$ Ans

5. $(4x + 3y^2)\,dx + 2xy\,dy = 0$

$M = 4x + 3y^2$ $\quad$ $N = 2xy$

$M_y = 0 + 6y$ $\quad$ $N_x = 2y$

$M_y \ne N_x$ $\quad$ $\therefore$

$\frac{N_x - M_y}{M} = \frac{2y - 6y}{4x + 3y^2}$

$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$

$(4x^3 + 3y^2x^2)\,dx + (3x^3y)\,dy = 0$

$M_y = 6yx^2$ $\quad$ $N_x = 6x^2y$

$M_y = N_x$ Exact Diff eq.

$\int M\,dx + \int(\text{term of } N \text{ from } x)\,dy = 0$

$\int(4x^3 + 3y^2x^2)\,dx + \text{Nil} = c$

$\frac{\cancel{4}x^4}{\cancel{4}} + \frac{\cancel{3}y^2x^3}{\cancel{3}} = c$

$x^4 + y^2x^2 = c$ Ans

6. $(3xy+y^2)dx + (x^2+xy)dy = 0$ — i

$M = 3xy+y^2$ $\quad\quad N = x^2+xy$

$M_y = 3x+2y$ $\quad\quad N_x = 2x+y$

$M_y \neq N_x$

$\dfrac{N_x - M_y}{M} = \dfrac{2x+y-3x+2y}{3xy+y^2} = \dfrac{-x+3y}{y(3x+y)}$ Not for y.

$\dfrac{M_y - N_x}{N} = \dfrac{3x+2y-2x-y}{x^2+y} = \dfrac{x+y}{x(x+y)} = \dfrac{1}{x}$

$I.f = e^{\int \frac{1}{x}dx} = e^{\ln} = x$

xing by 1 (i) If $= x$

$M = 3x^2+xy^2$ $\quad\quad N = x^3+x^2y$

$M_y = 3x^2+2xy$ $\quad\quad N_x = 3x^2+2xy$

$M_y = N_x$ is Exact eq.

$\int M dx + \int(\text{term of } N)\, dy = c$

$(3x^2y+xy^2)dx + \text{Nil} = c$

$\dfrac{3x^3y}{3} + \dfrac{x^2y^2}{2} = c$

$x^3y + \dfrac{x^2y^2}{2} = c$ Ans.

Excrise # 9.6.

1. $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$

I.F $e^{\int P dx} = e^{\int \left(\frac{2x+1}{x}\right) dx} = e^{\int (2 + \frac{1}{x}) dx}$

$= e^{2x + \ln x} = e^{2x} e^{\ln x} = e^{2x} x$

sol is given by $\int d(Y \times I.F) = \int Q \times I.F\, dx$

$\Rightarrow \int d(y e^{2x} x) = \int e^{-2x} e^{2x} x\, dx + C$

$\int y e^{2x} x = \int x\, dx + C$

$= x y e^{2x} = \frac{x^2}{2} + C$ Ans

2. $\frac{dy}{dx} + \frac{3}{x} y = 6x^2$

$I.F = e^{\int P dx} = e^{\frac{3}{x} dx} = e^{3 \ln} = e^{\ln x^3} = x^3$

$\Rightarrow \int d(yx^3) = \int 6x^2 \cdot x^3 dx + c$

$\Rightarrow yx^3 = 6x^5\, dx + c$

$\Rightarrow yx^3 = \frac{6x^6}{6} + c$

$\Rightarrow x^3 y = x^6 + c$ Ans

3. $\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$ (LDE in Y)

$I.F = e^{\int P dx} = e^{\int \frac{1}{x \ln x}} = e^{\int \frac{dx/x}{\ln x}}$

$I.F = e^{\ln(\ln x)} = \ln x$

$\Rightarrow \int d(y \ln x) = \int \frac{3x^2}{\ln x} \ln x\, dx + c$

$y \ln x = \frac{3x^3}{3} + c$, $y = \frac{x^3 + c}{\ln x}$ Ans

4. $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$ (LDE in y)

I.F $e^{\int P dx} = e^{\int 3dx} = e^{3x}$ ✓

sol is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + c$

$\Rightarrow \int d(ye^{3x}) = \int 3x^2 e^{-3x} e^{3x}\, dx + c$

$ye^{3x} = x^3 + c$

$y = e^{-3x}(x^3 + c)$ Ans

5. $(x+1)\frac{dy}{dx} - ny = e^x (x+1)^{n+1}$

$\frac{dy}{dx} - \frac{n}{(x+1)} y = e^x (x+1)^n$ ✓

$= e^{\ln (x+1)^{-n}}$

$I.F = (x+1)^{-n} = \frac{1}{(x+1)^n}$

$\Rightarrow \int d\left(y \cdot \frac{1}{(x+1)^n}\right) = \int e^x \cancel{(x+1)^n} \frac{1}{\cancel{(x+1)^n}} dx + c$

$\frac{y}{(x+1)^n} = e^x + c$

$y = (e^x + c)(x+1)^n$ Ans

6. $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right) y = \frac{4x^2}{x^2+1}$

$I.F = e^{\int \left(\frac{2x}{x^2+1}\right) dx} = e^{\ln(x^2+1)} = x^2+1$ ✓

$\Rightarrow \int d\left[y(x^2+1)\right] = \int \frac{4x^2}{(x^2+1)} (x^2+1)\, dx + c$

$y(x^2+1) = \frac{4x^3}{3} + c$

$3y(x^2+1) = 4x^3 + c'$ Ans

7. $\frac{dy}{dx} = \frac{1}{e^y - x}$

$\frac{dx}{dy} = e^y - x$ Reciprocal

$\frac{dx}{dy} + x = e^y$ (LDE in x)

$I.F = e^{\int 1.dy} = e^y$

sol is given $\int d \int x . I.F = \int Q \times I.F \, dy + c$

$\Rightarrow \int d(xe^y) = \int e^y e^y dy + c$

$\Rightarrow xe^y = \int e^{2y} dy + c$

$x = \frac{1}{e^y}\left(\frac{e^{2y}}{2} + c\right)$

$x = \frac{e^y}{2} + ce^{-y}$ Ans

8. $x\frac{dy}{dx} + 2y = \sin x$

$\frac{dy}{dx} + \frac{2}{x}y = \frac{\sin x}{x}$ LDE in y

$IF = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

sol is given by $\int d(y \times I.F) = \int (Q \times I.F) dx + c$

$\Rightarrow \int d(yx^2) = \int \frac{\sin x}{x} x^2 dx + c$

$yx^2 = \int x \sin x \, dx + c$

$yx^2 = x(-\cos x) - \int 1.(-\cos x) dx + c$

$y = \frac{1}{x^2}(-\cos x + \sin x + c)$ Ans

$v = \ln x + c$, $\frac{y}{x} = \ln x \ldots$