Name:-

Zain Islam

Roll No:-

29028

Assignment

1

Subject

Math

Ex 9.2

Q1) $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

$y\,dy = \frac{x^2}{1+x^3}\,dx$

$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$

$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$

$\frac{3y^2}{2} = \ln(1+x^3)+3c$

$3y^2 = 2\ln(1+x^3)+6c$

$3y^2 = 2\ln(1+x^3)+c'$ Ans

Q2) $\frac{dy}{dx} + y^2 \sin x = 0$

$\frac{dy}{dx} + -y^2 \sin x$

$\int \frac{dy}{y^2} = -\int \sin x\,dx$

$\frac{y^{-1}}{-1} = -(-\cos x)+c$

$-\frac{1}{y} = \cos x + c$ Ans

Q3) $\frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$

$= 2x^2 - 2x - 2 + y - x^2y + xy$

$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$

$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$

$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\, dx$

$-\int \frac{dy}{2-y} = -\int (x^2 - x - 1)\, dx$

$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$

$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$

$-6\ln|2-y| = 2x^3 - 3x^2 - 6x + 6c$

$\ln|2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln e$

$\ln|2-y|^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6c}$

$|2-y|^{-6} = e^{2x^3 - 3x^2 - 6x + 6c}$

$|2-y|^{-6} = e^{2x^3 - 3x^2 - 6x + 6c}$ Ans

Q4) $\frac{dy}{dx} = 1+x+y^2+xy^2$

$\frac{dy}{dx} = (1+x) + y^2(1+x)$

$\frac{dy}{dx} = (1+x)(1+y^2)$

$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$

$\tan^{-1}y = x + \frac{x^2}{2} + c$

$2\tan^{-1}y = 2x + x^2 + c'$ Ans

Q5) $y(1+x)dx + x(1+y)\,dy = 0$

$\div$ by xy

$\frac{(1+x)}{x}dx + \frac{(1+y)}{y}dy = 0$

$\int \left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0\,dx$

$\ln x + x + \ln y + y = c$

$x + y + \ln(xy) = c$ Ans

Q6) $(\sin x + \cos x)\,dy + (\cos x - \sin x)\,dx = 0$

$\div$ by $(\sin x + \cos x)$

$$\int dy + \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)}\,dx = \int 0\,dx$$

$y + \ln(\sin x + \cos x) = c$

$y \ln c + \ln(\sin x + \cos x) = c \ln e$

$\ln e^y + \ln(\sin x + \cos x) = \ln e^c$

$\ln[e^y(\sin x + \cos x)] = \ln e^c$

$e^y(\sin x + \cos x) = c'$

$$e^y = \frac{c'}{\sin x + \cos x} \quad \text{Ans}$$

Q7) $8\cos^2 y\,dx + \operatorname{cosec}^2 x\,dy = 0,\ y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}$

$\div$ by $\cos^2 y \operatorname{cosec}^2 x$

$$\frac{8}{\operatorname{cosec}^2 x}\,dx + \frac{1}{\cos^2 y}\,dy = 0$$

$\int 8 \sin^2 x \, dx + \int \sec^2 y \, dy = \int 0 \, dx$

$4\int 2 \sin^2 x \, dx + \tan y = c$

$4\int (1 - \cos 2x) \, dx + \tan y = c$

$4\left(x - \dfrac{\sin 2x}{2}\right) \tan y = c$

$4x - 2\sin 2x + \tan y = c$

$\tan y = -4x + 2 \sin 2x + c$

$\because y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}$

$\therefore \tan\left(\frac{\pi}{4}\right) - 4\left(\frac{\pi}{12}\right) + 2\sin 2\left(\frac{\pi}{12}\right)$

$1 = -\frac{\pi}{3} + 2 \sin \frac{\pi}{6} + c$

$1 = -\frac{\pi}{3} + 2\left(\frac{1}{2}\right) + c$

$\cancel{1} - \cancel{1} + \frac{\pi}{3} = c \Rightarrow \boxed{c = \frac{\pi}{2}}$

$\therefore \tan y = -4x + 2 \sin 2x + \frac{\pi}{3}$ Ans

Q28) $x e^{x^2+y} \, dx = y \, dy$

$x e^{x^2} e^{y} \, dx = y \, dy$

$x e^{x^2} \, dx = y e^{-y} \, dy$

$$\frac{1}{2}\int e^{x^2}(2x)\,dx = \int y e^{-y}$$

$$\because e^{x^2} = t \quad e^{x^2} 2x\,dx = dt$$

$$\int e^{x^2} 2x\,dx = \int dt = t = e^{x^2}$$

So $\frac{1}{2} e^{x^2}$

$$= y \frac{e^{-y}}{-1} - \int 1 \cdot \frac{e^{-y}}{-1}\,dy$$

$$= -ye^{-y} + \int e^{-y}\,dy$$

$$= -ye^{-y} + \frac{e^{-y}}{-1} + c$$

$$\frac{1}{2} e^{x^2} = -ye^{-y} - e^{-y} + c$$

$$e^{x^2} = -2e^{-y}(y+1) + 2c$$

$$e^{x^2} = -2e^{-y}(y+1)\,c' \quad \text{Ans}$$

Ex 9.3

Q1) $(x^2-3y^2)dx + 2xy\,dy = 0$

$2xy\,dy = -(x^2-3y^2)\,dx$

$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy}$ — (i)

Put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (iii)

using (ii)(iii) in (i)

$v + x\frac{dv}{dx} = \frac{3v^2x^2-x^2}{2x\,vx}$

$x\frac{dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$

$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$

$x\frac{dv}{dx} = \frac{v^2-1}{2v}$

$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$

$\ln(v^2-1) = \ln x + \ln c$

$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$

$\frac{y^2-x^2}{x^2} = cx$

$y^2-x^2 = (cx)x^2$ Ans

22) $(x^2+3x+y^2)\,dx \;-x^2\,dy = 0$

$(x^2+3xy+y^2)\,dx = x^2\,dy$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \quad \text{———— (i)}$$

Put $y = vx$ $\;x^2$ ———— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{———— (iii)}$$

using eq (ii) & (iii) Put in eq (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{x^2(1+3v+v^2)}{x^2} - v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\frac{-1}{(v+1)} = \ln x + c$$

$$\frac{-1}{(\frac{y}{x}+1)} = \ln x + c$$

$$\frac{-x}{(x+y)} = \ln x + c \quad \text{Ans}$$

Q3) $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\frac{y}{x} - x\right) dx$

$$\frac{dy}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{---} \ (i)$$

Put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{———} \ (iii)$$

eq (ii) & (iii) Put in eq (i)

$$v + x\frac{dv}{dx} = \frac{vx \sin\frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{x(v \sin v - 1)}{x \sin v} - v$$

$$x\frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v}$$

$$\int \sin v \, dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x - c$$

$$\cos y/x = \ln x - c \ \text{Ans}$$

Q4) $\left(x^3 + y^2\sqrt{x^2+y^2}\right) dx - xy\sqrt{x^2+y^2}\, dy = 0$

$$x^3 + y^2\sqrt{x^2+y^2}\, dx = xy\sqrt{x^2+y^2}\, dy$$

$$\frac{dy}{dx} = \frac{x^3 + y^2\sqrt{x^2+y^2}}{xy\sqrt{x^2+y^2}} \quad \text{———} \ (i)$$

Put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{x^3 + v^2x^2\sqrt{x^2+v^2x^2}}{x\,vx\sqrt{x^2+v^2x^2}}$$

$$= \frac{x^2\left(1+v^2\sqrt{1+v^2}\right)}{x^2\,v\sqrt{1+v^2}} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2\sqrt{1+v^2} - v^2\sqrt{1+v^2}}{v\sqrt{1+v^2}}$$

$$x\frac{dv}{dx} = \frac{1}{v\sqrt{1+v^2}}$$

$$\int v\sqrt{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\frac{1}{2}\sqrt{1+v^2}\,(2v)\,dv = \int\frac{dx}{x}$$

$$\frac{1}{2}\left(1+v^2\right)^{3/2} = 3\ln x \; 3c$$

$$\left(1+\frac{y^2}{x^2}\right)^{3/2} = 3\ln x \; 3c$$

$$\left(\frac{x^2+y^2}{x^2}\right)^{3/2} = \ln x^3 + c'$$

$$\frac{\left(x^2+y^2\right)^{3/2}}{x^3} = \ln x^3 + c'$$

$$\left(x^2+y^2\right)^{3/2} = x^3\ln x^3 + c'x^3 \quad \text{Ans}$$

Q5) $(3x^2+9xy+5y^2)dx-(6x^2+4xy)dy=0$

$(6x^2+4xy)dy=(3x^2+9xy+5y^2)\,dx$

$\frac{dy}{dx}=\frac{3x^2+9xy+5y^2}{6x^2+4xy}$

Put $y=vx$ --- ii

$\frac{dy}{dx}=v+x\frac{dv}{dx}$ --- iii

$v+x\frac{dv}{dx}=\frac{3x^2+9x\,vx+5v^2x^2}{6x^2+4x\,vx}$

$x\frac{dv}{dx}=\frac{\cancel{x^2}(3+9v+5v^2)}{\cancel{x^2}(6+4v)}-v$

$x\frac{dv}{dx}=\frac{3+9v+5v^2-6v-4v^2}{6+4v}$

$x\frac{dv}{dx}=\frac{v^2+3v+3}{6+4v}$

$\int\frac{(4v+6)\,dv}{v^2+3v+3}=\int\frac{dx}{x}$

$2\int\frac{(2v+3)\,dv}{v^2+3v+3}=\int\frac{dx}{x}$

$2\cdot\ln(v^2+3v+3)=\ln x+\ln c$

$\ln(v^2+3v+3)^2=\ln cx$

$(v^2+3v+3)^2=cx$

$\left(\frac{y^2}{x^2}+\frac{3y}{x}+3\right)^2=cx$

$$\frac{(y^2+3yx+3x^2)^2}{x^2} = cx$$

$$\frac{y^2+3yx+3x^2}{x^2} = \sqrt{cx}$$

$$y^2+3yx+3x^2 = x^2\sqrt{cx}$$

$$\because y(2) = -6$$

$$\frac{12}{4} = \sqrt{2c}$$

$$(3)^2 = 2c \Rightarrow c = 9 \text{ Ans}$$

Q6) $$\frac{dy}{dx} = -\frac{(4x+3y+15)}{2x+y+7}$$

Put $x = X+h$, $y = Y+k$ $\Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$

$$\frac{dY}{dX} = -\frac{(4X+4h+3Y+3k+15)}{2X+2h+Y+k+7}$$

$$\frac{dy}{dx} = -\frac{4x-3y}{2x+y} \quad \text{—— (i)}$$

Put $y = vx$ —— (ii)

$$\frac{dy}{dx} \; v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

using (ii) & (iii) in (i)

$$v + x\frac{dv}{dx} = -\frac{4x-3vx}{2x+\ldots}$$

$$x\frac{dv}{dx} = -\frac{x(4+3v)}{x(2+v)} - v$$

$$= \frac{-4-3v-2v-v^2}{2+v}$$

$$x\frac{dv}{dx} = -\frac{(v^2+5v+4)}{2+v}$$

$$\int \frac{(v+2)\,dv}{v^2+5v+4} = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{dv}{(v+1)} + \frac{2}{3}\int \frac{dv}{v+4} = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln(v+1) + \frac{2}{3}\ln(v+4) = -\ln x + \ln c$$

$$\frac{1}{2}\ln\left[(v+1)(v+4)^2\right] = \ln\left(\frac{c}{x}\right)$$

$$\ln\left[\left(\frac{y}{x}+1\right)\left(\frac{y}{x}+4\right)^2\right]^{1/3} = \ln\left(\frac{c}{x}\right)$$

$$\left(\frac{y+x}{x}\right)\frac{(y+4x)^2}{x^2} = \frac{c^3}{x^3}$$

$$(y+x)(y+4x^2)\frac{\cancel{x^3}}{\cancel{x^3}} = c'$$

$$\cancel{(y+1)}\ (y+1+x+3)(y+1+4x+12) = c'$$

$$(x+y+4)(4x+y+13) = c' \quad \text{Ans}$$

Q1) $\dfrac{dy}{dx} = \dfrac{y-x+1}{y-x+5}$ --- (i)

Put $y-x = z$ (ii)

$\dfrac{dy}{dx} - 1 = \dfrac{dz}{dx}$

$\dfrac{dy}{dx} = 1 + \dfrac{dz}{dx}$ --- (iii)

$1 + \dfrac{dz}{dx} = \dfrac{z+1}{z+5}$

$\dfrac{dz}{dx} = \dfrac{z+1}{z+5} - 1$

$= \dfrac{z+1-z-5}{z+5}$

$\dfrac{dz}{dx} = \dfrac{-4}{z+5}$

$\int (z+5)\, dz = -4\int dx$

$\dfrac{z^2}{2} + 5z = -4x + c$

$\dfrac{z^2+10z}{2} = -4x + c$

$z^2 + 10z = -8x + 2c$

$(y-x)^2 + 10(y-x) = -8x + c'$

$(y-x)^2 + 10(y-x) + 8x = c'$

$(y-x)^2 + 10y - 10x + 8x = c'$

$(y-x)^2 + 10y - 2x = c'$ Ans

Ex # 9.4

Q1) $(2xy+y-\tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy$

$M = 2xy+y-\tan y, \quad N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x+1-\sec^2 y, \quad \frac{\partial N}{\partial x} = 2x - \tan^2 y + 0$

$= 2x - \tan^2 y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int (2xy+y-\tan y)\,dx + \int \sec^2 y\,dy = c$

$\frac{2x^2y}{2} + xy - x\tan y + \tan y = c$

$x^2 + xy - x\tan y + \tan y = c$

Q2) $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$M = \frac{x+y}{y-1} \quad N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$

$N = -\frac{1}{2}\left(\frac{x^2+2x+1}{(y-1)^2}\right)$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2} \qquad \frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$

$= \frac{y-1-x-y}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$

$\int M\,dx + \int$ terms of N free (from x) $dy = c$

$$\int \frac{(x+y)}{y-1}\,dx + \int -\frac{1}{2(y-1)^2}\,dy = c$$

$$\left(\frac{1}{y-1}\right)\int (x+y)\,dx + \left(-\frac{1}{2}\right)\int (y-1)^{-2}\,dy = c$$

$$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2}+xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2+2xy+1 = c'(y-1) \quad \text{Ans}$$

Q3) $(6xy+2y^2-5)\,dx + (3x^2+4xy-6)$

$M = 6xy+2y^2-5$, $N = 3x^2+4xy-6$

$$\frac{\partial M}{\partial y} = 6x+4y \qquad \frac{\partial N}{\partial x} = 6x+4y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M\,dx + \int$ (terms of N free from x) $dy = c$

$$\int (6xy+2y^2-5)\,dx + \int -6\,dy = c$$

$$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$$

$$3x^2y + 2xy^2 - 5x - 6y = c \quad \text{Ans}$$

Q4) $(y\sec^2 x + \sec x \tan x)\,dx + (\tan x + 2y)\,dy = 0$

$M = y\sec^2 x + \sec x \tan x, \quad N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x \qquad \frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\cancel{\int \frac{M\,dx}{\partial y} = \frac{\partial N}{\partial x}}$

$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int (y\sec^2 x + \sec x \tan x)\,dx + \int 2y\,dy = c$

$y \cdot \tan x + \sec x + y^2 = c$ Ans

5) $(y\cos x + 2xe^y)\,dx + (\sin x + x^2 e^y - 1)\,dy$

$M = y\cos x + 2xe^y, \quad N = \sin x + x^2 e^y - 1$

$\frac{\partial M}{\partial y} = \cos x + 2xe^y \quad \frac{\partial N}{\partial x} = \cos x + 2xe^y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int (y\cos x + 2xe^y)\,dx - \int -1\,dy = c$

$$y\sin x + \frac{\cancel{2}x^2e^y}{\cancel{2}} - y = c$$

$$y\sin x + x^2e^y - y = c$$ Ans

7) Solve the internal value Problem

$(2xy-3)\,dx + (x^2+4y)\,dy = 0,$

$M = 2xy-3 \qquad N = x^2+4y$

$\frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2x$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M\,dx + \int(\text{terms of } N \text{ free from } x)\,dy = c$

$\int(2xy-3)\,dx + \int 4y\,dy = c$

$\frac{2x^2y}{2} - 3x + \frac{4y^2}{2} = c$

$x^2y - 3x + 2y^2 = c$

$\because y(1) = 2$

$\therefore 2 - 3 + 8 = c$

$\underline{\overline{7 = c}}$

Hence $x^2y - 3x + 2y^2 = 7$

8) $(2x\cos y + 3x^2y)dx + (x^3 - x^2\sin y - y)dy$

$M = 2x\cos y + 3x^2 y \qquad N = x^3 - x^2\sin y - y$

$\frac{\partial M}{\partial y} = 2x(-\sin y) + 3x^2 \qquad \frac{\partial N}{\partial x} = 3x^2 - 2x\sin y = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N \text{ force from } x)\, dy = c$

$\int (2x\cos y + 3x^2 y)dx + \int -y\, dx = c$

$2\frac{x^2}{2}\cos y + \frac{3x^3}{3}y - \frac{y^2}{2} = c$

$x^2\cos y + x^3 y - \frac{y^2}{2} = c$

$\because y(0) = 2$

$0 + 0 - \frac{4}{2} = c$

$-2 = c$

$x^2\cos y + x^3 y - \frac{y^2}{2} = -2$ Ans

Exercise : 9.5

Q.1

$$(xy^2+y)dx - xdy = 0$$

Solution:

$$(xy^2+y)dx - xdy = 0 \quad \text{——— ①}$$

$M = xy^2 + y$ $\qquad N = -x$

$M_y = 2xy + 1$ $\qquad N_x = -1$

$\because M_y \neq N_x$ $\qquad \therefore$ No Exact

$$\frac{M_y - N_x}{N} = \frac{2xy+1+1}{-x}$$ nothing of x alone

$$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$$

$$= \frac{-2(1+xy)}{y(xy+1)} = \frac{-2}{y}$$

$$\therefore I.F = e^{-\int \frac{2}{y} dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Multiply both sides of eq ① by $IF = \frac{1}{y^2}$

$$\frac{1}{y^2}(xy^2+y)dx - \frac{x}{y^2}dy = 0$$

$$\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0 \quad \text{——— (ii)}$$

Now $M = x + \frac{1}{y}$ $\qquad N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2}$ $\qquad N_x = \frac{-1}{y^2}$

$\because M_y = N_x$ $\qquad \therefore$ Exact Diff Eq

So $\int M dx + \int (\text{term of } N \text{ free from } x)\, dy = c$

$$\int\left(x+\frac{1}{y}\right)dx + \text{Nil} = c$$

$$\frac{x^2}{2} + \frac{x}{y} = c \quad \text{Ans}$$

Q.5

$$(y^4+2y)\,dx + (xy^3+2y^4-4x)\,dy = 0 \quad \text{--- (I)}$$

$M = y^4+2y$ $\qquad$ $N = xy^3+2y^4-4x$

$M_y = 4y^3+2$ $\qquad$ $N_x = y^3-4$

$M_y \neq N_x$ $\quad \therefore$ (I) is Non Exact Diff Eq

$$\frac{N_x - M_y}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} = \frac{-3y^3-6}{y(y^3+2)} = \frac{-3}{y}$$

$$I.F = e^{\int -\frac{3}{y}dy} = e^{-3\ln y} = e^{\ln y^{-3}} = y^{-3} = \frac{1}{y^3}$$

$$\frac{1}{y^3}(y^4+2y)\,dx + \frac{1}{y^3}(xy^3+2y^4-4x)\,dy = 0$$

$$\left(y+\frac{2}{y^2}\right)dx + \left(x+2y-\frac{4x}{y^3}\right)dy = 0 \quad \text{--- (II)}$$

Now $M = y+\frac{2}{y^2}$ $\qquad$ $N = x+2y-\frac{4x}{y^3}$

$M_y = 1-\frac{4}{y^3}$ $\qquad$ $N_x = 1+0-\frac{4}{y^3}$

$\because M_y = N_x$ $\quad \therefore$ (II) is Exact Diff Eq

$$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$$

$$\int \left(y+\frac{2}{y^2}\right)dx + \int 2y\,dy = c$$

$$xy + \frac{2x}{y^2} + 2\frac{y^2}{2} = c$$

$$xy + \frac{2x}{y^2} + y^2 = c \quad \text{Ans}$$

Q.2

$$(x^2+x-y)dx + x\,dy = 0 \quad \text{———} \ (1)$$

Solution:

$M = x^2+x-y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$M_y \neq N_x \qquad \therefore$ Non Exact Diff Eq

$\dfrac{M_y - N_x}{N} = \dfrac{-1-1}{x} = \dfrac{-2}{x}$

$\therefore I.F = e^{\int -2/x\,dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \dfrac{1}{x^2}$

Multiply both sides of eq (1) by I.F $= \dfrac{1}{x^2}$

$\dfrac{1}{x^2}(x^2+x-y)\,dx + \dfrac{x}{x^2}\,dy = 0$

$$\left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + \frac{1}{x}\,dy = 0 \quad \text{———} \ (ii)$$

Now $M = 1+\dfrac{1}{x}-\dfrac{y}{x^2} \qquad N = \dfrac{1}{x}$

$M_y = \dfrac{1}{x^2} \qquad N_x = -\dfrac{1}{x^2}$

$M_y = N_x \quad \therefore$ (ii) is Exact Diff Eq

So

$\int M\,dx + \int (\text{terms of } N \text{ free } x)\,dy = c$

$\int \left(1+\dfrac{1}{x}-\dfrac{y}{x^2}\right)dx + Nil = c$

$x + \ln x + \dfrac{y}{x} = c$

Q.8

$$(4x+3y^2)dx + 2xy\,dy = 0 \quad \text{--- (1)}$$

$M = 4x + 3y^2$ $\qquad N = 2xy$

$M_y = 0 + 6y$ $\qquad N_x = 2y$

$M_y \neq N_x$ $\quad \therefore$ (1) is Non Exact Diff Eq.

$$\frac{N_x - M_y}{M} = \frac{2y - 6y}{4x + 3y^2}$$

$$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$$

$$\therefore I.F = e^{\int \frac{2}{x} dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

Multiply both sides of (1) by I.F $= x^2$

$$(4x^3 + 3y^2x^2)dx + (2x^3y)dy = 0 \quad \text{--- (2)}$$

$M_y = 6yx^2$ $\qquad N_x = 6x^2y$

$M_y = N_x$ $\quad \therefore$ (2) is Exact Diff Eq.

$$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x)\, dy = c$$

$$\int (4x^3 + 3y^2x^2)dx + \text{null} = c$$

$$\frac{\cancel{4}x^4}{\cancel{4}} + \cancel{3}y^2\frac{x^3}{\cancel{3}} = c$$

$$x^4 + y^2x^3 = c \quad \text{Ans}$$

Q.4

$$y(2xy+e^x)\,dx - e^x\,dy = 0$$

Solution:

$$(2xy^2+e^x y)\,dx - e^x\,dy = 0 \quad \text{--- (1)}$$

$M = 2xy^2 + e^x y$ $\qquad N = -e^x$

$M_y = 4xy + e^x$ $\qquad N_x = -e^x$

$M_y \neq N_x$ $\therefore$ (1) is Non Exact Diff Eq.

$$\frac{M_y - N_x}{N} = \frac{4xy + e^x + e^x}{-e^x}$$

$$\frac{N_x - M_y}{M} = \frac{-e^x - 4xy - e^x}{2xy^2 + ye^x} = \frac{-2e^x - 4xy}{y(2xy+e^x)}$$

$$= \frac{-2(e^x+2xy)}{y(2xy+e^x)} = \frac{-2}{y}$$

$$I.F = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Multiply both sides of (1) by $I.F = \frac{1}{y^2}$

$$\frac{1}{y^2}(2xy^2+e^x y)\,dx - \frac{1}{y^2}e^x\,dy = 0$$

$$\left(2x + \frac{e^x}{y}\right)dx - \frac{e^x}{y^2}\,dy = 0 \quad \text{--- (2)}$$

$M = 2x + \frac{e^x}{y}$ $\qquad N = \frac{-e^x}{y^2}$

$M_y = 0 + \left(-\frac{e^x}{y^2}\right)$ $\qquad N_x = \frac{-e^x}{y^2}$

$M_y = N_x$ $\therefore$ (2) is Exact Diff Eq.

$$\int M\,dx + \int (\text{Elements of } N \text{ free from } x)\,dy = c$$

$$\int \left(2x + \frac{e^x}{y}\right)dx + \text{Nill} = C$$

$$x^2 + \frac{e^x}{y} = C \quad \text{Ans}$$

Signature:

Q.6

$$(x^2+y^2+2x)dx+2y\,dy=0 \quad \text{--- (1)}$$

$M = x^2+y^2+2x$ $\qquad N = 2y$

$M_y = 2y$ $\qquad N_x = 0$

$M_y \neq N_x$ $\quad \therefore$ (1) is Non Exact Diff Eq

$$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2x}$$

$$\frac{M_y - N_x}{N} = \frac{2y-0}{2y} = 1 = x$$

$$I.F = e^{\int 1\,dx} = e^x$$

Multiply both sides of eq (1) by $I.F = e^x$

$$e^x(x^2+y^2+2x)dx + e^x(2y)dy = 0 \quad \text{--- (2)}$$

$M = e^x(x^2+y^2+2x)$ $\qquad N = e^x 2y$

$M_y = e^x 2y$ $\qquad N_x = e^x 2y$

$M_y = N_x$ $\quad \therefore$ (2) is Exact Diff Eq

$$\therefore \int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$$

$$\int e^x(x^2+y^2+2x)\,dx + Nil = c$$

$$\int e^x x^2\,dx + \int e^x y^2\,dx + \int e^x 2x\,dx = c$$

$$x^2 e^x - \cancel{\int 2x e^x\,dx} + e^x y^2 + \cancel{\int e^x 2x\,dx} = c$$

$$(x^2+y^2)e^x = c$$

Q.3

$$\frac{dy}{} + \left(\frac{y - \sin x}{x}\right)dx = 0 \quad \text{--- ①}$$

Solution:

$M = \left(\frac{y - \sin x}{x}\right) dx \qquad N = 1$

$M_y = \frac{1}{x} - 0 \qquad N_x = 0$

$M_y \neq N_x \quad \therefore$ ① is Non Exact Eq

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$

$I.F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Multiplying both sides of eq ① by I.F = x

$x\,dy + \cancel{x}\left(\frac{y - \sin x}{\cancel{x}}\right) dx = 0$ --- ②

$M = y - \sin x \qquad N = x$

$M_y = 1 \qquad N_x = 1$

$M_y = N_x \quad \therefore$ ② is Exact Diff Eq

$\int M dx + \int (\text{terms of } N \text{ free from } x)\, dy = c$

$\int (y - \sin x)\, dx = C$

$xy + \cos x = C$ Ans