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Assignment

1

Subject

Math

Ex 9.2

$$Q1) \frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y dy = \frac{x^2}{1+x^3} dx$$

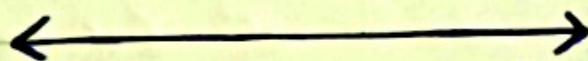
$$\int y dy = \frac{1}{3} \int \frac{3x^2}{1+x^3} dx$$

$$\frac{y^2}{2} = \frac{1}{3} \ln(1+x^3) + C$$

$$\frac{3y^2}{2} = \ln(1+x^3) + 3C$$

$$3y^2 = 2 \ln(1+x^3) + 6C$$

$$3y^2 = 2 \ln(1+x^3) + C' \quad \text{Ans}$$



$$Q2) \frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} + y^2 \sin x$$

$$\int \frac{dy}{y^2} = - \int \sin x dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + C$$

$$-\frac{1}{y} = \cos x + C \quad \text{Ans}$$



$$23) \frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$$

$$= 2x^2 - 2x - 2 + y - x^2y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1) dx$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1) dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + C$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6C}{6}$$

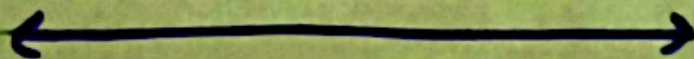
$$-6 \ln|2-y| = 2x^3 - 3x^2 - 6x + 6C$$

$$\ln|2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6C) \ln C$$

$$\ln|2-y|^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6C}$$

$$|2-y|^{-6} = e^{2x^3 - 3x^2 - 6x + 6C}$$

$$|2-y|^{-6} = e^{2x^3 - 3x^2 - 6x + 6C} \quad \text{Ans}$$



$$24) \frac{dy}{dx} = 1+x+y^2+xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x) dx$$

$$\tan^{-1}y = x + \frac{x^2}{2} + C$$

$$2 \tan^{-1}y = 2x + x^2 + C' \text{ Ans}$$



$$25) y(1+x)dx + x(1+y)dy = 0$$

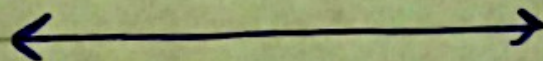
$$\div \text{ by } xy$$

$$\frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy = 0$$

$$\int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy = \int 0 dx$$

$$\ln x + x + \ln y + y = C$$

$$x+y + \ln(xy) = C \text{ Ans}$$



$$Q6) (\sin x + \cos x) dy + (\cos x - \sin x) dx = 0$$

$$\div \text{ by } (\sin x + \cos x)$$

$$\int dy + \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx = \int 0 dx$$

$$y + \ln(\sin x + \cos x) = C$$

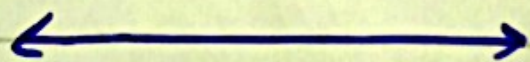
$$y \ln C + \ln(\sin x + \cos x) = C \ln e$$

$$\ln e^y + \ln(\sin x + \cos x) = \ln e^C$$

$$\ln[e^y (\sin x + \cos x)] = \ln e^C$$

$$e^y (\sin x + \cos x) = C'$$

$$e^y = \frac{C'}{\sin x + \cos x} \quad \text{Ans}$$



$$Q7) 8 \cos^2 y dx + \csc^2 x dy = 0, y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}$$

$$\div \text{ by } \cos^2 y \csc^2 x$$

$$\frac{8}{\csc^2 x} dx + \frac{1}{\cos^2 y} dy = 0$$

$$\int 8 \sin^2 x \, dx + \int \sec^2 y \, dy = \int 0 \, dx$$

$$4 \int 2 \sin^2 x \, dx + \tan y = C$$

$$4 \int (1 - \cos 2x) \, dx + \tan y = C$$

$$4 \left(x - \frac{\sin 2x}{2} \right) + \tan y = C$$

$$4x - 2 \sin 2x + \tan y = C$$

$$\tan y = -4x + 2 \sin 2x + C$$

$$\therefore y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}$$

$$\therefore \tan\left(\frac{\pi}{4}\right) = 4\left(\frac{\pi}{12}\right) + 2 \sin 2\left(\frac{\pi}{12}\right) + C$$

$$1 = -\frac{\pi}{3} + 2 \sin \frac{\pi}{6} + C$$

$$1 = -\frac{\pi}{3} + 2\left(\frac{1}{2}\right) + C$$

$$1 - 1 + \frac{\pi}{3} = C \Rightarrow \boxed{C = \frac{\pi}{3}}$$

$$\therefore \tan y = -4x + 2 \sin 2x + \frac{\pi}{3} \quad \text{Ans}$$

$$\text{Q8) } x e^{x^2+y} \, dx = y \, dy$$

$$x e^{x^2} e^y \, dx = y \, dy$$

$$x e^{x^2} \, dx = y e^{-y} \, dy$$

$$\frac{1}{2} \int e^{x^2} (2x) dx = \int y e^y$$

$$\therefore e^{x^2} = t \quad e^{x^2} 2x dx = dt$$

$$\int e^{x^2} 2x dx = \int dt = t = e^{x^2}$$

$$\text{So } \frac{1}{2} e^{x^2} = y \frac{e^y}{-1} - \int 1 \cdot \frac{e^y}{-1} dy$$

$$= -y e^y + \int e^y dy$$

$$= -y e^y + \frac{e^y}{-1} + C$$

$$\frac{1}{2} e^{x^2} = -y e^y - e^y + C$$

$$e^{x^2} = -2 e^y (y+1) + 2C$$

$$e^{x^2} = -2 e^{-y} (y+1) C' \quad \text{Ans}$$



Ex 9.3

$$2) (x^2 - 3y^2) dx + 2xy dy = 0$$

$$2xy dy = -(x^2 - 3y^2) dx$$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy} \quad \text{--- (i)}$$

$$\text{Put } y = vx \quad \text{--- (ii)}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$$

using (ii) (iii) in (i)

$$v + x \frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$x \frac{dv}{dx} = \frac{(3v^2 - 1)x^2}{2vx^2} - v$$

$$x \frac{dv}{dx} = \frac{3v^2 - 1 - 2v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

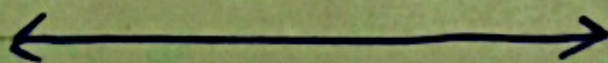
$$\int \frac{2v}{v^2 - 1} dv = \int \frac{dx}{x}$$

$$\ln(v^2 - 1) = \ln x + \ln C$$

$$\ln\left(\frac{y^2}{x^2} - 1\right) = \ln Cx$$

$$\frac{y^2 - x^2}{x^2} = Cx$$

$$y^2 - x^2 = (Cx) x^2 \quad \text{Ans}$$



$$22) (x^2 + 3xy + y^2) dx - x^2 dy = 0$$

$$(x^2 + 3xy + y^2) dx = x^2 dy$$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} \quad \text{--- (i)}$$

$$\text{Put } y = vx \quad \text{--- (ii)}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$$

using eq (ii) & (iii) Put in eq (i)

$$v + x \frac{dv}{dx} = \frac{x^2 + 3xvx + v^2x^2}{x^2}$$

$$x \frac{dv}{dx} = \frac{x^2(1 + 3v + v^2)}{x^2} - v$$

$$x \frac{dv}{dx} = 1 + 2v + v^2$$

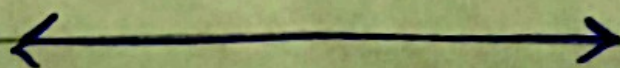
$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\frac{-1}{(v+1)} = \ln x + C$$

$$\frac{-1}{\left(\frac{y}{x} + 1\right)} = \ln x + C$$

$$\left(\frac{y}{x} + 1\right)$$

$$\frac{-x}{(x+y)} = \ln x + C \quad \text{Ans}$$



$$Q3) x \sin\left(\frac{y}{x}\right) dy = (y \sin\frac{y}{x} - x) dx$$

$$\frac{dy}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad (i)$$

Put $y = vx$ (ii)

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad (iii)$$

eq (ii) & (iii) Put in eq (i)

$$v + x \frac{dv}{dx} = \frac{vx \sin\frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$$

$$x \frac{dv}{dx} = \frac{x(v \sin v - 1) - v}{x \sin v}$$

$$x \frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v}$$

$$\int \sin v dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + C$$

$$\cos v = \ln x - C$$

$$\cos \frac{y}{x} = \ln x - C \quad \text{Ans}$$

←—————→

$$Q4) (x^3 + y^2 \sqrt{x^2 + y^2}) dx - xy \sqrt{x^2 + y^2} dy = 0$$

$$x^3 + y^2 \sqrt{x^2 + y^2} dx = xy \sqrt{x^2 + y^2} dy$$

$$\frac{dy}{dx} = \frac{x^3 + y^2 \sqrt{x^2 + y^2}}{xy \sqrt{x^2 + y^2}} \quad (i)$$

$$\text{Put } y = vx \quad \text{--- (ii)}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (iii)}$$

Using (ii) & (iii) in (i)

$$\begin{aligned} v + x \frac{dv}{dx} &= \frac{x^3 + v^2 x^2 \sqrt{x^2 + v^2 x^2}}{x vx \sqrt{x^2 + v^2 x^2}} \\ &= \frac{x^2 (1 + v^2 \sqrt{1 + v^2}) - v}{x^2 v \sqrt{1 + v^2}} \end{aligned}$$

$$x \frac{dv}{dx} = \frac{1 + v^2 \sqrt{1 + v^2} - v^2 \sqrt{1 + v^2}}{v \sqrt{1 + v^2}}$$

$$x \frac{dv}{dx} = \frac{1}{v \sqrt{1 + v^2}}$$

$$\int v \sqrt{1 + v^2} dv = \int \frac{dx}{x}$$
$$\frac{1}{2} \sqrt{1 + v^2} (2v) dv = \int \frac{dx}{x}$$

$$\frac{1}{2} (1 + v^2)^{3/2} = 3 \ln x + c$$
$$\left(1 + \frac{y^2}{x^2}\right)^{3/2} = 3 \ln x + c$$

$$\left(\frac{x^2 + y^2}{x^2}\right)^{3/2} = \ln x^3 + c'$$

$$\frac{(x^2 + y^2)^{3/2}}{x^3} = \ln x^3 + c'$$

$$(x^2 + y^2)^{3/2} = x^3 \ln x^3 + c' x^3 \text{ Ans}$$



$$\text{Q5) } (3x^2 + 9xy + 5y^2) dx - (6x^2 + 4xy) dy = 0$$

$$(6x^2 + 4xy) dy = (3x^2 + 9xy + 5y^2) dx$$

$$\frac{dy}{dx} = \frac{3x^2 + 9xy + 5y^2}{6x^2 + 4xy}$$

$$\text{Put } y = vx \quad \text{--- ii}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- iii}$$

$$v + x \frac{dv}{dx} = \frac{3x^2 + 9x \cdot vx + 5v^2x^2}{6x^2 + 4x \cdot vx}$$

$$x \frac{dv}{dx} = \frac{x^2 (3 + 9v + 5v^2)}{x^2 (6 + 4v)} - v$$

$$x \frac{dv}{dx} = \frac{3 + 9v + 5v^2 - 6v - 4v^2}{6 + 4v}$$

$$x \frac{dv}{dx} = \frac{v^2 + 3v + 3}{6 + 4v}$$

$$\int \frac{(4v+6) dv}{v^2+3v+3} = \int \frac{dx}{x}$$

$$2 \int \frac{(2v+3) dv}{v^2+3v+3} = \int \frac{dx}{x}$$

$$2 \cdot \ln(v^2+3v+3) = \ln x + \ln C$$

$$\ln(v^2+3v+3)^2 = \ln Cx$$

$$(v^2+3v+3)^2 = Cx$$

$$\left(\frac{y^2}{x^2} + \frac{3y}{x} + 3\right)^2 = Cx$$

$$\frac{(y^2 + 3yx + 3x^2)^2}{x^2} = Cx$$

$$\frac{y^2 + 3yx + 3x^2}{x^2} = \sqrt{Cx}$$

$$y^2 + 3yx + 3x^2 = x^2 \sqrt{Cx}$$

$$\therefore y(1) = -6$$

$$\frac{12}{4} = \sqrt{2C}$$

$$(3)^2 = 2C \Rightarrow C = 9 \text{ Ans}$$

←—————→

$$\text{Q6) } \frac{dy}{dx} = - \frac{(4x + 3y + 15)}{2x + y + 7}$$

$$\text{Put } \left. \begin{array}{l} x = X + h \\ y = Y + k \end{array} \right\} \Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = - \frac{(4X + 4h + 3Y + 3k + 15)}{2X + 2h + Y + k + 7}$$

$$\frac{dY}{dX} = - \frac{4X - 3Y}{2X + Y} \quad \text{--- (i)}$$

$$\text{Put } Y = VX \quad \text{--- (ii)}$$

$$\frac{dY}{dX} = V + X \frac{dV}{dX} \quad \text{--- (iii)}$$

Using (ii) & (iii) in (i)

$$V + X \frac{dV}{dX} = - \frac{4X - 3VX}{2X + VX}$$

$$x \frac{dv}{dx} = - \frac{x(4+3v)}{x(2+v)} - v$$

$$= - \frac{4+3v-2v-v^2}{2+v}$$

$$x \frac{dv}{dx} = - \frac{(v^2+5v+4)}{2+v}$$

$$\int \frac{(v+2) dv}{v^2+5v+4} = - \int \frac{dx}{x}$$

$$\frac{1}{3} \int \frac{dv}{v+1} + \frac{2}{3} \int \frac{dv}{v+4} = - \int \frac{dx}{x}$$

$$\frac{1}{3} \ln(v+1) + \frac{2}{3} \ln(v+4) = - \ln x + \ln c$$

$$\frac{1}{2} \ln[(v+1)(v+4)^2] = \ln\left(\frac{c}{x}\right)$$

$$\ln\left[\left(\frac{y}{x}+1\right)\left(\frac{y}{x}+4\right)^2\right]^{1/3} = \ln\left(\frac{c}{x}\right)$$

$$\frac{(y+x)(y+4x)^2}{x^3} = \frac{c^3}{x^3}$$

$$(y+x)(y+4x)^2 \frac{x^3}{x^3} = c'$$

$$\cancel{(y+1)} \cancel{(x+3)} (y+1+x+3)(y+1+4x+12) = c'$$

$$(x+y+4)(4x+y+13) = c' \text{ Ans}$$

$$\text{or) } \frac{dy}{dx} = \frac{y-x+1}{y-x+5} \quad \text{--- (i)}$$

$$\text{Put } y-x = z \quad \text{(ii)}$$

$$\frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{dz}{dx} \quad \text{--- (iii)}$$

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

$$= \frac{z+1-2-5}{z+5}$$

$$\frac{dz}{dx} = \frac{-4}{z+5}$$

$$\int (z+5) dz = -\int dx$$

$$\frac{z^2}{2} + 5z = -4x + C$$

$$\frac{z^2 + 10z}{2} = -4x + C$$

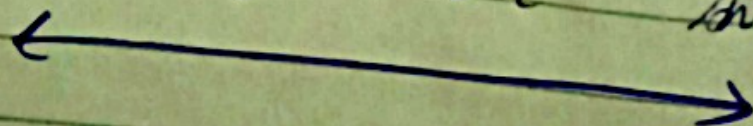
$$z^2 + 10z = -8x + 2C$$

$$(y-x)^2 + 10(y-x) = -8x + C'$$

$$(y-x)^2 + 10(y-x) + 8x = C'$$

$$(y-x)^2 + 10y - 10x + 8x = C'$$

$$(y-x)^2 + 10y - 2x = C' \quad \text{Ans}$$



Ex 11 9.4

$$Q1) (2xy + y \tan y) dx + (x^2 - x \tan y + \sec^2 y) dy = 0$$

$$M = 2xy + y \tan y, \quad N = x^2 - x \tan y + \sec^2 y$$

$$\frac{2M}{2y} = 2x + 1 - \sec^2 y, \quad \frac{2N}{2x} = 2x - \tan y + \sec^2 y$$

$$2y = 2x - \tan y$$

$$\therefore \frac{2M}{2y} = \frac{2N}{2x}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (2xy + y \tan y) dx + \int \sec^2 y dy = C$$

$$\frac{2x^2y}{2} + xy - x \tan y + \tan y = C$$

$$x^2 + xy - x \tan y + \tan y = C$$



$$Q2) \left(\frac{x+y}{y-1} \right) dx - \frac{1}{2} \left(\frac{x+1}{y-1} \right)^2 dy = 0$$

$$M = \frac{x+y}{y-1}, \quad N = -\frac{1}{2} \left(\frac{x+1}{y-1} \right)^2$$

$$N = -\frac{1}{2} \frac{(x^2 + 2x + 1)}{(y-1)^2}$$

$$\frac{2M}{2y} = \frac{(y-1)(0+1) - (x+y)(1)}{(y-1)^2}, \quad \frac{2N}{2x} = \frac{-(2x+1)}{2(y-1)^2}$$

$$= \frac{y-1-x-y}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$\int M dx + \int$ (terms of x from N) $dy = C$

$$\int \frac{(x+y)}{y-1} dx + \int -\frac{1}{2(y-1)^2} dy = C$$

$$\left(\frac{1}{y-1}\right) \int (x+y) dx + \left(-\frac{1}{2}\right) \int (y-1)^{-2} dy = C$$

$$\left(\frac{1}{y-1}\right) \left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right) \left(\frac{-1}{y-1}\right) = C$$

$$\frac{x^2 + 2xy}{2(y-1)} + \frac{1}{2(y-1)} = C$$

$$x^2 + 2xy + 1 = C(y-1) \text{ Ans}$$



Q3) $(6xy + 2y^2 - 5) dx + (3x^2 + 4xy - 6) dy = 0$

$$M = 6xy + 2y^2 - 5, \quad N = 3x^2 + 4xy - 6$$

$$\frac{\partial M}{\partial y} = 6x + 4y \quad \frac{\partial N}{\partial x} = 6x + 4y$$

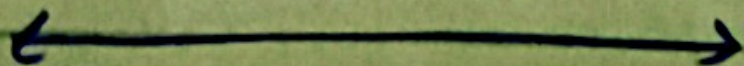
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\int M dx + \int$ (terms of N from N) $dy = C$

$$\int (6xy + 2y^2 - 5) dx + \int -6 dy = C$$

$$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = C$$

$$3x^2y + 2xy^2 - 5x - 6y = C \text{ Ans}$$



$$24) (y \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy = 0$$

$$M = y \sec^2 x + \sec x \tan x, \quad N = \tan x + 2y$$

$$\frac{\partial M}{\partial y} = \sec^2 x \quad \frac{\partial N}{\partial x} = \sec^2 x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int \frac{\partial M}{\partial x} dx = \int \frac{\partial N}{\partial y} dy$$

$$\int M dx + \int (\text{terms of } N \text{ from } x) dy = C$$

$$\int (y \sec^2 x + \sec x \tan x) dx + \int 2y dy = 0$$

$$y \tan x + \sec x + y^2 = C \quad \text{Ans}$$

$$5) (y \cos x + 2x e^y) dx + (\sin x + x^2 e^y - 1) dy = 0$$

$$M = y \cos x + 2x e^y, \quad N = \sin x + x^2 e^y - 1$$

$$\frac{\partial M}{\partial y} = \cos x + 2x e^y \quad \frac{\partial N}{\partial x} = \cos x + 2x e^y$$

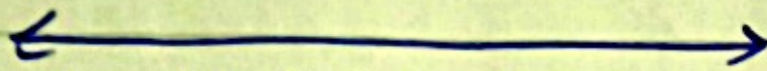
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M dx + \int (\text{terms of } N \text{ from } x) dy = C$$

$$\int (y \cos x + 2x e^y) dx - \int 1 dy = C$$

$$y \sin x + \cancel{2} x^2 e^x - y = C$$

$$y \sin x + x^2 e^x - y = C \quad \text{Ans}$$



7) Solve the internal value problem

$$(2xy - 3) dx + (x^2 + 4y) dy = 0,$$

$$M = 2xy - 3 \quad N = x^2 + 4y$$

$$\frac{\partial M}{\partial y} = 2x \quad \frac{\partial N}{\partial x} = 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (2xy - 3) dx + \int 4y dy = C$$

$$2 \frac{x^2 y}{2} - 3x + \frac{4y^2}{2} = C$$

$$x^2 y - 3x + 2y^2 = C$$

$$\therefore y(1) = 2$$

$$\therefore 2 - 3 + 8 = C$$

$$\underline{7 = C}$$

$$\text{Hence } x^2 y - 3x + 2y^2 = 7$$

$$8) (2x \cos y + 3x^2 y) dx + (x^3 - x^2 \sin y - y) dy$$

$$M = 2x \cos y + 3x^2 y \quad N = x^3 - x^2 \sin y - y$$

$$\frac{\partial M}{\partial y} = 2x(-\sin y) + 3x^2 \quad \frac{\partial N}{\partial x} = 3x^2 - 2x \sin y = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (2x \cos y + 3x^2 y) dx + \int -y dx = C$$

$$\frac{2x^2}{2} \cos y + \frac{3x^3}{3} y - \frac{y^2}{2} = C$$

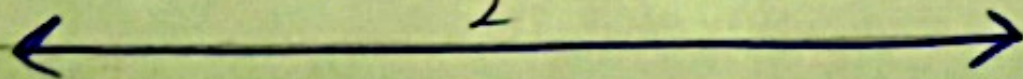
$$x^2 \cos y + x^3 y - \frac{y^2}{2} = C$$

$$\therefore y(0) = 2$$

$$0 + 0 - \frac{4}{2} = C$$

$$-2 = C$$

$$x^2 \cos y + x^3 y - \frac{y^2}{2} = -2 \quad \text{Ans}$$



Exercise: 9.5

Q.1

$$(xy^2 + y)dx - xdy = 0$$

Solution:

$$(xy^2 + y)dx - xdy = 0 \quad \text{--- (1)}$$

$$M = xy^2 + y$$

$$N = -x$$

$$M_y = 2xy + 1$$

$$N_x = -1$$

$$\therefore M_y \neq N_x$$

\therefore Not Exact

$$\frac{M_y - N_x}{N} = \frac{2xy + 1 + 1}{-x} \quad \text{Nothing of } x \text{ shows}$$

$$\frac{M_x - M_y}{M} = \frac{-1 - 2xy - 1}{xy^2 + y}$$

$$= \frac{-2(1 + xy)}{y(xy + 1)} = \frac{-2}{y}$$

$$\therefore \text{I.F} = e^{\int \frac{-2}{y} dy} = e^{-2 \ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Multiply both sides of eq (1) by IF = $\frac{1}{y^2}$

$$\frac{1}{y^2} (xy^2 + y) dx - \frac{x}{y^2} dy = 0$$

$$\left(x + \frac{1}{y}\right) dx - \frac{x}{y^2} dy = 0 \quad \text{--- (2)}$$

$$\text{Now } M = x + \frac{1}{y}$$

$$N = -\frac{x}{y^2}$$

$$M_y = -\frac{1}{y^2}$$

$$N_x = -\frac{1}{y^2}$$

$\therefore M_y = N_x \quad \therefore$ Exact Diff Eq

$$\text{So } \int M dx + \int (\text{term of } N \text{ from } x) dy = C$$

$$\int \left(x + \frac{1}{y}\right) dx + N dx = C$$

$$\frac{x^2}{2} + \frac{x}{y} = C \quad \text{Ans}$$

Q.5

$$(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0 \quad \text{--- (1)}$$

$$M = y^4 + 2y$$

$$N = xy^3 + 2y^4 - 4x$$

$$M_y = 4y^3 + 2$$

$$N_x = y^3 - 4$$

$M_y \neq N_x \quad \therefore \text{(1) is Non Exact Diff Eq}$

$$\frac{N_x - M_y}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-3y^3 - 6}{y(y^3 + 2)} = -\frac{3}{y}$$

$$I.F = e^{\int -\frac{3}{y} dy} = e^{-3 \log y} = e^{\log y^{-3}} = y^{-3} = \frac{1}{y^3}$$

$$\frac{1}{y^3} (y^4 + 2y) dx + \frac{1}{y^3} (xy^3 + 2y^4 - 4x) dy = 0$$

$$\left(y + \frac{2}{y^2}\right) dx + \left(x + 2y - \frac{4x}{y^3}\right) dy = 0 \quad \text{--- (2)}$$

$$\text{Now } M = y + \frac{2}{y^2} \quad N = x + 2y - \frac{4x}{y^3}$$

$$M_y = 1 - \frac{4}{y^3}$$

$$N_x = 1 + 0 - \frac{4}{y^3}$$

$\therefore M_y = N_x \quad \therefore \text{(2) is Exact Diff Eq}$

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$$

$$\int \left(y + \frac{2}{y^2}\right) dx + \int 2y dy = c$$

$$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$$

$$xy + \frac{2x}{y^2} + y^2 = c \quad \text{Ans}$$

Q.2

$$(x^2 + x - y) dx + x dy = 0 \quad \text{--- (1)}$$

Solution:

$$M = x^2 + x - y \quad N = x$$

$$M_y = -1 \quad N_x = 1$$

$M_y \neq N_x \quad \therefore$ Non Exact Diff Eq

$$\frac{M_y - N_x}{N} = \frac{-1 - 1}{x} = -\frac{2}{x}$$

$$\therefore I.F = e^{\int -\frac{2}{x} dx} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

Multiply both sides of eq (1) by I.F = $\frac{1}{x^2}$

$$\frac{1}{x^2} (x^2 + x - y) dx + \frac{x}{x^2} dy = 0$$

$$\left(1 + \frac{1}{x} - \frac{y}{x^2}\right) dx + \frac{1}{x} dy = 0 \quad \text{--- (2)}$$

$$\text{Now } M = 1 + \frac{1}{x} - \frac{y}{x^2} \quad N = \frac{1}{x}$$

$$M_y = -\frac{1}{x^2} \quad N_x = -\frac{1}{x^2}$$

$M_y = N_x \quad \therefore$ (2) is Exact Diff Eq

So

$$\int M dx + \int (\text{terms of } N \text{ from } x) dy = C$$

$$\int \left(1 + \frac{1}{x} - \frac{y}{x^2}\right) dx + \text{Nil} = C$$

$$x + \ln x + \frac{y}{x} = C$$

Q.8

$$(4x + 3y^2) dx + 2xy dy = 0 \quad \text{--- (1)}$$

$$M = 4x + 3y^2$$

$$N = 2xy$$

$$M_y = 0 + 6y$$

$$N_x = 2y$$

$M_y \neq N_x$ \therefore (1) is Non Exact Diff Eq.

$$\frac{N_x - M_y}{M} = \frac{2y - 6y}{4x + 3y^2}$$

$$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$$

$$\therefore \text{I.F} = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = e^{\ln x^2} = x^2$$

Multiply both sides of (1) by I.F. x^2

$$(4x^3 + 3y^2 x^2) dx + (2x^3 y) dy = 0 \quad \text{--- (2)}$$

$$\frac{M}{y} = 6yx^2$$

$$N_x = 6x^2 y$$

$M_y = N_x$ \therefore (2) is Exact Diff Eq.

$$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (4x^3 + 3y^2 x^2) dx + \text{null} = C$$

$$\frac{Mx^4}{4} + \frac{3y^2 x^3}{3} = C$$

$$x^4 + y^2 x^3 = C \quad \text{Ans}$$

Q.4

$$y(2xy + e^x) dx - e^x dy = 0$$

Solution:

$$(2xy^2 + e^x y) dx - e^x dy = 0 \quad \text{--- (1)}$$

$$M = 2xy^2 + e^x y \quad N = -e^x$$

$$M_y = 4xy + e^x \quad N_x = -e^x$$

$M_y \neq N_x$ \therefore (1) is Not Exact Diff Eq

$$\frac{M_y - N_x}{N} = \frac{4xy + e^x + e^x}{-e^x}$$

$$\frac{N_x - M_y}{M} = \frac{-e^x - 4xy - e^x}{2xy^2 + ye^x} = \frac{-2e^x - 4xy}{y(2xy + e^x)}$$

$$= \frac{-2(e^x + 2xy)}{y(2xy + e^x)} = \frac{-2}{y}$$

$$\text{I.F} = e^{\int \frac{-2}{y} dy} = e^{-2 \ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Multiply both sides of (1) by I.F = $\frac{1}{y^2}$

$$\frac{1}{y^2} (2xy^2 + e^x y) dx - \frac{1}{y^2} e^x dy = 0$$

$$(2x + \frac{e^x}{y}) dx - \frac{e^x}{y^2} dy = 0 \quad \text{--- (2)}$$

$$M = 2x + \frac{e^x}{y}$$

$$N = -\frac{e^x}{y^2}$$

$$M_y = 0 + (-\frac{e^x}{y^2})$$

$$N_x = -\frac{e^x}{y^2}$$

$M_y = N_x$ \therefore (2) is Exact Diff Eq

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (2x + \frac{e^x}{y}) dx + N \cdot y = C$$

$$x^2 + \frac{e^x}{y} = C \quad \text{Ans}$$

Q.6

$$(x^2 + y^2 + 2x) dx + 2y dy = 0 \quad \text{--- (1)}$$

$$M = x^2 + y^2 + 2x$$

$$N = 2y$$

$$M_y = 2y$$

$$N_x = 0$$

$$M_y \neq N_x$$

\therefore (1) is Non Exact Diff Eq

$$\frac{N_x - M_y}{M} = \frac{0 - 2y}{x^2 + y^2 + 2x}$$

$$\frac{M_y - N_x}{N} = \frac{2y - 0}{2y} = 1 = \lambda$$

$$I.F = e^{\int \lambda dx} = e^x$$

Multiply both sides of eq (1) by I.F = e^x

$$e^x (x^2 + y^2 + 2x) dx + e^x (2y) dy = 0 \quad \text{--- (2)}$$

$$M = e^x (x^2 + y^2 + 2x)$$

$$N = e^x 2y$$

$$M_y = e^x 2y$$

$$N_x = e^x 2y$$

$$M_y = N_x$$

\therefore (2) is Exact Diff Eq

$$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int e^x (x^2 + y^2 + 2x) dx + Nil = C$$

$$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x dx = C$$

$$x^2 e^x - \int 2x e^x dx + e^x y^2 + \int e^x 2x dx = C$$

$$(x^2 + y^2) e^x = C$$

Q.3

$$\frac{dy}{dx} + \left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{--- (1)}$$

Solution:

$$M = \left(\frac{y - \sin x}{x}\right) dx \quad N = 1$$

$$M_y = \frac{1}{x} - 0 \quad \frac{N_x}{x} = 0$$

$M_y \neq N_x \quad \therefore$ (1) is Non Exact Eq

$$\text{Now } \frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$$

$$\text{I.F} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Multiplying both sides of eq (1) by I.F = x

$$x dy + \cancel{x} \left(\frac{y - \sin x}{\cancel{x}}\right) dx = 0 \quad \text{--- (2)}$$

$$M = y - \sin x \quad N = x$$

$$M_y = 1 \quad N_x = 1$$

$M_y = N_x \quad \therefore$ (2) is Exact Diff Eq

$$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = C$$

$$\int (y - \sin x) dx = C$$

$$xy + \cos x = C \text{ Ans}$$