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Subject. Mathematics.

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Question No. 1

Solve Differential eq with Separable Variable equation.

i. $$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^2}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + C$$

$$\frac{3y^2}{2} = \ln(1+x^3) + 3C$$

$$3y^2 = 2\ln(1+x^3) + 6C$$

$$3y^2 = 2\ln(1+x^3) + C$$

ii. $$\frac{dy}{dx} + y^2\sin x = 0$$

$$\frac{dy}{dx} = -y^2\sin x$$

$$\int \frac{dy}{y^2} = -\int \sin x\,dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + C$$

$$-\frac{1}{y} = \cos x + C$$

iii) $$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx}\ (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

iv) $$(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$$

$$[x(y+2) + (y+2)]dx + x(x+2)dy = 0$$

$$[(y+2)(x+1)]dx + x(x+2)dy = 0$$

divided by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2)}\,dx + \frac{1}{y+2}\,dy = 0$$

$$\int \frac{x+1}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$\tan^{-1} y = x + \frac{x^2}{2} + C$

$2\tan^{-1} y = 2x + x^2 + C$

v. $\frac{dy}{dx} = 2x^2 + y - x^2 y + xy - 2x - 2$

$= 2x^2 - 2x - 2 + y - x^2 y + xy$

$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$

$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$

$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$

$-\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$

$-\ln(2-y) = \frac{x^3}{3} - \frac{x^2}{2} - x + C$

$-\ln(2-y) = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$

$-6\ln(2-y) = 2x^3 - 3x^2 - 6x + 6c$

$\ln(2-y)^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln e$

$\ln(2-y)^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6c}$

$(2-y)^{-6} = e_1 e^{2x^3 - 3x^2 - 6x}$

$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$

$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$

$y + 2 = \frac{c}{\sqrt{x^2+2x}}$

vi. $\operatorname{cosec} y\,dx + \sec x\,dy = 0$

÷ by cosec y sec x

$\Rightarrow \frac{1}{\sec x}\,dx + \frac{dy}{\operatorname{cosec} y} = 0$

$= \int \cos x\,dx + \int \sin y\,dy = \int 0\,dx$

$= \sin x - \cos y = C.$

viii) $y(1+x)\,dx + x(1+y)\,dy = 0$

÷ by xy.

$\frac{(1+x)}{x}\,dx + \frac{1+y}{y}\,dy = 0$

$\int\left(\frac{1}{x} + 1\right)dx + \int\left(\frac{1}{y} + 1\right)dy = \int 0\,dx$

$\ln x + x + \ln y + y = C$

$x + y + \ln(xy) = C$

viii. $y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$$\int \frac{\sqrt{1+x^2}}{x}\,dx + \int \frac{\sqrt{1+y^2}}{y}\,dy = \int 0\,dx$$

Put $\sqrt{1+x^2} = t$ Put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $y\,dy = z\,dz$

therefor

$$\int \sqrt{\frac{1+x^2}{x^2}}\,dx + \int \sqrt{\frac{1+y^2}{y^2}}\,y\,dy = \int 0\,dx$$

$$\int \frac{t.t\,dt}{t^2-1} + \int \frac{z.z\,dz}{z^2-1} = c$$

$$\int \frac{t^2-1+1}{t^2-1}\,dt + \int \frac{z^2-1+1}{z^2-1}\,dz = c$$

$$\int \left(1 + \frac{1}{t^2-1}\right)dt + \int \left(1 + \frac{1}{z^2+1}\right)dz = c$$

$$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + \sqrt{1+y^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right) = c$$

Question No 2

Solve the Following Homogeneous diffr eq.

i. $(x-y)dx+(x+y)dy=0$

$(x+y)dy = -(x-y)dx$

$\frac{dy}{dx} = \frac{y-x}{x+y}$ —— H. DE (1)

Put $y = Vx$ —— (ii)

$\frac{dy}{dx} = V + x\frac{dv}{dx}$ —— (iii)

$V + x\frac{dv}{dx} = \frac{Vx-x}{x+Vx}$

$x\frac{dv}{dx} = \frac{x(v-1)}{x(1+v)} - V$

$= \frac{v-1-v-v^2}{1+v}$

$x\frac{dv}{dx} = \frac{-(v^2+1)}{1+v}$

$\int\frac{v+1}{v^2+1}dv = -\int\frac{dx}{x}$

$\frac{1}{2}\int\frac{2v\,dv}{v^2+1} + \int\frac{dv}{v^2+1} = -\int\frac{dx}{x}$

$\Rightarrow \frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + c$

$\ln(v^2+1)^{1/2} + \tan^{-1}v + \ln x = c$

$\Rightarrow \ln\sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$

$\ln\sqrt{y^2+x^2} - \cancel{\ln\sqrt{x^2}} + \tan^{-1}\left(\frac{y}{x}\right) \cancel{+\ln x} = c$

$\ln\sqrt{y^2+x^2} + \tan^{-1}\left(\frac{y}{x}\right) = c$

ii) $(y^2+2xy)\,dx + x^2\,dy = 0$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = \frac{-(y^2+2xy)}{x^2} \quad \text{— (i)}$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (iii)}$$

$$v + x\frac{dv}{dx} = \frac{-(v^2x^2+2xvx)}{x^2}$$

$$x\frac{dv}{dx} = \frac{-x^2v^2+2v}{x^2} - v$$

$$x\frac{dv}{dx} = -(v^2+3v)$$

$$\int\frac{dv}{v^2+3v} = -\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{3}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{v+3-v}{v(v+3)}\,dv$$

$$\frac{1}{3}\int\left(\frac{1}{v}-\frac{1}{v+3}\right)dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(x+3) = -\ln x + \ln c$$

$$\Rightarrow \ln\left(\frac{v^{1/3}}{(v+3)^{1/3}}\right) = \ln\frac{c}{x}$$

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x\,v^{\frac{1}{3}} = c\,(v+3)^{\frac{1}{3}}$$

$$x\left(\frac{y}{x}\right)^{\frac{1}{3}} = c\left(\frac{y}{x}+3\left(\frac{1}{3}\right)\right)$$

$$x\,\frac{y^{\frac{1}{3}}}{x^{1/3}}\cdot x^{1/3} = c\,(y+3x)^{1/3}$$

$$x\,y^{1/3} = c\,(y+3x)^{\frac{1}{3}}$$

$$x^3 y = c'(y+3x)$$

Q iii) $(x^2-3y^2)dx+2xy\,dy=0$

$2xy\,dy = -(x^2-3y^2)dx$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{--- (i)}$$

Put $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (iii)}$$

$$v + x\frac{dv}{dx} = \frac{3v^2x^2-x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{(3v^2-1)x^2}{2vx^2} - v$$

$$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int\frac{2v}{v^2-1}dv = \int\frac{dx}{x}$$

$\ln(v^2-1) = \ln x + \ln c$

$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$

$$\frac{y^2-x^2}{x^2} = cx$$

$y^2 - x^2 = (cx)x^2$

iv) $(x^2+3y^2)dx-2xy\,dy=0$

$(x^2+3y^2)\,dx = 2xy\,dy$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{--- (i)}$$

Put $y = vx$ --- (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (iii)}$$

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v^2)}{\cancel{x^2}2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int\frac{2v}{1+v^2}dv = \int\frac{dx}{x}$$

$\ln(1+v^2) = \ln x + \ln c$

$\ln(1+v^2) = \ln cx$

$$\left(1+\frac{y^2}{x^2}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$x^2+y^2 = (cx)x^2$

v. $(x^2+xy+y^2)dx - x^2dy = 0$

$(x^2+xy+y^2)dx = x^2dy$

$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

$v + x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \frac{(1+v+v^2)x^2}{x^2} - v$

$\int\frac{dv}{1+v^2} = \int\frac{dx}{x}$

$\tan^{-1}v = \ln x + C$

$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + C$

vi) $(x^2+3xy+y^2)dx - x^2dy = 0$

$(x^2+3xy+y^2)dx = x^2dy$

$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$ —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v+v^2)}{\cancel{x^2}} - v$

$x\frac{dv}{dx} = 1+2v+v^2$

$\int\frac{dv}{(1+v)^2} = \int\frac{dx}{x}$

$\frac{1}{(v+1)} = \ln x + C$

$\frac{-1}{\left(\frac{y}{x}+1\right)} = \ln x + C$

$\frac{-1}{\frac{y+x}{x}} = \ln x + C$

$\frac{-x}{(x+y)} = \ln x + C$

vii) $\frac{dy}{dx} = \frac{4y-3x}{2x-y}$ — (i)

Put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (iii)

$v + x\frac{dv}{dx} = \frac{4vx - 3x}{2x - vx}$

$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$

$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$

$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x}$ — (iv)

$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$

$2-v = A(v-1) + B(v+3)$

$A = \frac{-5}{4}, \quad B = \frac{1}{4}$

$-\frac{5}{4}\int \frac{dv}{v+3} + \frac{1}{4}\int \frac{dv}{v-1} = \int \frac{dx}{x}$

$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-1) = \ln x + \ln c$

$\ln \frac{(v-1)}{(v+3)^5} = \ln c^4 x^4$

$\frac{(y-x)x^5}{x(y+3x)^5 x^4} = c'$

$\frac{y-x}{(y+3x)^5} = c'$

viii) $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\frac{y}{x} - x\right) dx$

$\frac{dy}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\frac{y}{x}}$ — (i)

Put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (iii)

$v + x\frac{dv}{dx} = \frac{vx \sin\frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$

$x\frac{dv}{dx} = \frac{\cancel{x}(v \sin v - 1)}{\cancel{x} \sin v} - v$

$x\frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v}$

$\int \sin v\,dv = \int -\frac{dx}{x}$

$-\cos v = -\ln x + c$

$\cos v = \ln x - c$

$\cos\frac{y}{x} = \ln x - c$

Question No 3

1) $\dfrac{dy}{dx} = \dfrac{x+3y-5}{x-y-1}$

Put $\left.\begin{array}{l} x = X+h \\ y = Y+k \end{array}\right] \Rightarrow \dfrac{dy}{dx} = \dfrac{dY}{dX}$

$$\frac{dY}{dX} = \frac{X+h+3(Y+k)-5}{X+h-(Y+k)-1}$$

$$\frac{dY}{dX} = \frac{X+3Y}{X-Y} \quad \text{—— (i)}$$

Put $Y = vX$ —— (ii)

$$\frac{dY}{dX} = v + X\frac{dv}{dX} \quad \text{—— (iii)}$$

$$v + x\frac{dv}{dx} = \frac{x+3vx}{x-vx}$$

$$x\frac{dv}{dx} = \frac{x(1+3v)}{x(1-v)} - v$$

$$x\frac{dv}{dx} = \frac{1+3v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{(1+v)^2}{1-v}$$

$$\int \frac{1-v}{(1+v)^2}\,dv = \int \frac{dx}{x}$$

$$\int \frac{1\,dv}{(1+v)^2} - \int \frac{v\,dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\int (1+v)^{-2}\,dv - \int \frac{1+v-1}{(1+v)^2}\,dv = \int \frac{dx}{x}$$

$$\int (1+v)^{-2}\,dv - \int \frac{1}{1+v}\,dv + \int \frac{1}{(1+v)^2}\,dv = \int \frac{dx}{x}$$

$$\frac{-1}{1+v} - \ln(1+v) - \frac{1}{1+v} = \ln x + \ln c$$

$$\frac{-2}{1+v} = \ln(1+v) + \ln x + \ln c$$

$$\frac{-2}{1+v} = \ln c\,x(1+v)$$

$$\frac{-2}{1+\frac{y}{x}} = \ln c\,x\left(1+\frac{y}{x}\right)$$

$$\frac{-2x}{x+y} = \ln c(x+y)$$

$$\frac{-2(x-2)}{x-2+y-1} = \ln c\,(x-2+y-1)$$

$$\frac{-2x+4}{x+y-3} = \ln c'\,(x+y-3) \quad \text{Ans}$$

ii) $\dfrac{dy}{dx} = -\dfrac{4x+3y+15}{2x+y+7}$

Put $\left.\begin{array}{l} x = X+h \\ y = Y+k \end{array}\right] \Rightarrow \dfrac{dy}{dx} = \dfrac{dY}{dX}$

$$\frac{dY}{dX} = \frac{-(4X+4h+3Y+3k+15)}{2X+2h+Y+k+7}$$

$$\frac{dY}{dX} = \frac{-4X-3Y}{2X+Y} \quad \text{—— (i)}$$

Put $Y = VX$ —— (ii)

$$\frac{dY}{dX} = V + X\frac{dV}{dX} \quad \text{—— (iii)}$$

$$V + X\frac{dV}{dX} = \frac{-4X-3VX}{2X+VX}$$

$$X\frac{dV}{dX} = \frac{-X(4+3V)}{X(2+V)} - V$$

$$= \frac{-4-3V-2V-V^2}{2+V}$$

$$X\frac{dV}{dX} = \frac{-(V^2+5V+4)}{2+V}$$

$$\int\frac{(V+2)\,dV}{V^2+5V+4} = -\int\frac{dX}{X}$$

$$\frac{1}{3}\int\frac{dV}{(V+1)} + \frac{2}{3}\int\frac{dV}{V+4} = -\int\frac{dX}{X}$$

$$\frac{1}{3}\ln(V+1) + \frac{2}{3}\ln(V+4) = -\ln X + \ln c$$

$$\frac{1}{3}\ln\left[(V+1)(V+4)^2\right] = \ln\left(\frac{c}{X}\right)$$

$$\ln\left(\left(\frac{Y}{X}+1\right)\left(\frac{Y}{X}+4\right)^2\right)^{1/3} = \ln\left(\frac{c}{X}\right)^3$$

$$\frac{(Y+X)}{X}\,\frac{(Y+4X)^2}{X^2} = \frac{C^3}{X^3}$$

$$(Y+X)(Y+4X)^2\,\frac{X^3}{X^3} = C$$

$$(y+1+x+3)(y+1+4x+12) = c^2$$

$$(x+y+4)(4x+y+13) = c$$

iii) $(3y-7x-3)dx + (7y-3x-7)dy = 0$

$(7y-3x-7)dy = -(3y-7x-3)dx$

$$\frac{dy}{dx} = \frac{-3y+7x+3}{7y-3x-7}$$

Put $\left.\begin{array}{l} x = X+h \\ y = Y+k \end{array}\right] \Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$

$$\frac{dY}{dX} = \frac{-3(Y+k)+7(X+h)+3}{7(Y+k)-3(X+h)-7}$$

$$\frac{dY}{dX} = \frac{-3Y+7X}{7Y-3X} \quad \text{—— (i)}$$

Put $Y = vX$ —— (ii)

$$\frac{dY}{dX} = v + X\frac{dv}{dX} \quad \text{—— (iii)}$$

$$v + x\frac{dv}{dx} = \frac{-3vx+7x}{7vx-3x}$$

$$x\frac{dv}{dx} = \frac{x(-3v+7)}{x(7v-3)} - v$$

$$x\frac{dv}{dx} = \frac{-3v+7-7v^2+3v}{7v-3}$$

$$\int \frac{7v-3}{7(1-v^2)}dv = \int \frac{dx}{x}$$

$$\frac{2}{7}\int\frac{dv}{1-v} - \frac{5}{7}\int\frac{dv}{1+v} = \int\frac{dx}{x}$$

$$-\frac{2}{7}\ln(1-v) - \frac{5}{7}\ln(1+v) = \ln x + \ln c$$

$$\ln\left[\left(\frac{x-y}{x}\right)^{-2}\left(\frac{x+y}{x}\right)^{-5}\right] = \ln c^7 x^7$$

$$\frac{X^2}{(X-Y)^2}\,\frac{X^5}{(X+Y)^5} = c^7 X^7$$

$$\frac{\cancel{x^7}}{\cancel{x^7}\,c^7} = (x-y)^2(x+y)^5$$

$$c' = (x-y)^2(x+y)^5$$

$$c' = (x-(y-1))^2(x+y-1)^5$$

$$c' = (x-y+1)^2(x+y-1)^5$$

iv. $\frac{dy}{dx} = \frac{x-2y+5}{2x+y-1}$

$\frac{dy}{dx} = \frac{x-2y}{2x+1}$ — (i)

Put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (iii)

$v + x\frac{dv}{dx} = \frac{x-2vx}{2x+vx}$

$x\frac{dv}{dx} = \frac{x(1-2v)}{x(2+v)} - v$

$x\frac{dv}{dx} = \frac{1-2v-2v-v^2}{2+v}$

$\frac{2+v}{1-4v-v^2}dv = \frac{dx}{x}$

$\int \frac{2+v}{v^2+4v-1}dv = -\int\frac{dx}{x}$

$\frac{1}{2}\left[\ln(v^2+4v-1)\right] = -\ln x + \ln c$

$\ln\sqrt{v^2+4y-1} = \ln\frac{c}{x}$

$\sqrt{\frac{y^2}{x^2} + \frac{4y}{x} - 1} = \frac{c}{x}$

$\frac{y^2+4xy-x^2}{x^2} = \frac{c^2}{x^2}$

$y^2+4xy-x^2 = c^2$

$\left(y-\frac{11}{5}\right)^2 + 4\left(x+\frac{3}{5}\right)\left(y-\frac{11}{5}\right)$

$- \left(x+\frac{3}{5}\right)^2 = c^2$

v. $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$ — (i)

Put $3x-4y = z$ — (ii)

$3 - 4\frac{dy}{dx} = \frac{dz}{dx}$ — (iii)

$\frac{1}{4}\left(3-\frac{dz}{dx}\right) = \frac{z-2}{z-3}$

$3 - \frac{dz}{dx} = \frac{4z-8}{z-3}$

$3 - \frac{(4z-8)}{z-3} = \frac{dz}{dx}$

$\frac{3z-9-4z+8}{z-3} = \frac{dz}{dx}$

$-\frac{(1+z)}{z-3} = \frac{dz}{dx}$

$-dx = \frac{(z-3)}{1+z}dz$

$\frac{z-3-1+1}{1+z} = -dx$

$\int\frac{(z+1)-4}{1+z}dz = -\int dx$

$\int\left(1 - \frac{4}{1+z}\right)dz = -\int dx$

$z - 4\ln(1+z) = -x + c$

$(3x-4y) - 4\ln(1+3x-4y) = -x + c$

$4x-4y - 4\ln(1+3x-4y) = c$

$x - y - \ln(1+3x-4y) = \frac{c}{4}$

$x - y - \ln(1+3x-4y) = c'$

vi) $\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$ — (i)

Put $y - x = z$ — (ii)

$\frac{dy}{dx} - 1 = \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = 1 + \frac{dz}{dx}$ — (iii)

$1 + \frac{dz}{dx} = \frac{z+1}{z+5} \Rightarrow \frac{dz}{dx} = \frac{z+1}{z+5} - 1$.

$= \frac{z+1-z-5}{z+5}$

$\frac{dz}{dx} = \frac{-4}{z+5} \Rightarrow \int (z+5)\,dz = -4\int dx$

$\frac{z^2}{2} + 5z = -4x + c \Rightarrow \frac{z^2+10z}{2} = -4x + c$

$z^2 + 10z = -8x + 2c$

$(y-x)^2 + 10(y-x) = -8x + c'$

$(y-x)^2 + 10(y-x) + 8x = c'$

$(y-x)^2 + 10y - 10x + 8x = c'$

$(y-x)^2 + 10y - 2x = c'$

Question No 04.

1) $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

$M = 3x^2 + 4xy$, $N = 2x^2 + 2y$.

$\frac{dM}{dy} = 0 + 4x \qquad \frac{dN}{dx} = 4x + 0$

$\int M dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int (3x^2 + 4xy)dx + \int 2y\,dy = c$

$\frac{3x^3}{3} + \frac{4x^2 y}{2} + \frac{2y^2}{2} = c$

$x^3 + 2x^2 y + y^2 = c$

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ii. $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0$

$M = 2xy + y - \tan y$, $N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y$, $\frac{\partial N}{\partial x} = 2x - \tan^2 y + 0 = 2x - \tan^2 y$

$\int M dx + \int (\tan N)\,dy = c$

$\int (2xy + y - \tan y)dx + \int \sec^2 y\, dy = c$

$\frac{2x^2 y}{2} + xy - x\tan y + \tan y = c$

iii. $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)dy = 0$

$M = \frac{x+y}{y-1}$ $\qquad N = \frac{-1}{2}\left(\frac{x+1}{y-1}\right)^2$

$N = \frac{-1}{2}\left(\frac{x^2+2x+1}{(y-1)^2}\right) \Rightarrow \frac{\partial M}{\partial y} = \frac{(y-1)(0+1) - (x+y)(1)}{(y-1)^2}$

$\frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2} = \frac{\cancel{y} - 1 - x - \cancel{y}}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \int \frac{x+y}{y-1}dx + \int -\frac{1}{2(y-1)^2}dy = C$

$\left(\frac{1}{y-1}\right)\int (x+y)dx + \left(-\frac{1}{2}\right)\int (y-1)^{-2} dy = C$

$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2} + xy\right) + \left(\frac{-1}{2}\right)\left(\frac{-1}{y-1}\right) = c$

$\frac{x^2 - 2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$

$x^2 - 2xy + 1 = c'(y-1)$

iv. $\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$

$(hx+by)dy = -(ax+hy)dx$

$(ax+hy)dx + (hx+by)dy = 0$

$M = ax+hy \quad N = hx+b$

$\frac{dM}{dy} = 0+h \quad \frac{dN}{dx} = h$

$\int(ax+hy)dx + \int by\, dy = c$

$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c'$

vi) $\frac{y\,dx + x\,dy}{1-x^2y^2} + x\,dx = 0$

$\frac{y\,dx}{1-x^2y^2} + \frac{x\,dy}{1-x^2y^2} + x\,dx = 0$

$\left(x + \frac{y}{(1-x^2y^2)}\right)dx + \frac{x\,dy}{1-x^2y^2} = 0$

$M = x + \frac{y}{1-x^2y^2}$

$\frac{dM}{dy} = 0 + \frac{(1-x^2y^2)\cdot 1 - y(-2x^2y)}{(1-x^2y^2)^2}$

$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{(1-x^2y^2)^2}$

$N = \frac{x}{1-x^2y^2}$

$\frac{dN}{dx} = \frac{(1-x^2y^2)1 - x(-2xy^2)}{(1-x^2y^2)^2}$

$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y)^2} = \frac{1+x^2y^2}{(1-x^2y^2)^2}$

V. $(1+\ln xy)dx + \left(1+\frac{x}{y}\right)dy = 0$

$M = 1+\ln xy \quad N = 1+\frac{x}{y}$

$\frac{dM}{dy} = 0+\frac{1}{xy} \quad \frac{dN}{dx} = 0+\frac{1}{y}$

$\int(1+\ln xy)dx + \int 1\, dy = c$

$\int dx + \int \ln xy\, dx + \int dy = c$

$x + \left[\ln xy\cdot(x) - \int \frac{1}{xy}\cdot y\cdot x\,dx\right] + y = c$

$x + x\ln xy - \int dx + y = c$

$\cancel{x} + x\ln xy - \cancel{x} + y = c$

$x\ln xy + y = c$

vii) $(6xy+2y^2-5)dx + (3x^2+4xy-6)dy$

$M = 6xy+2y^2-5, \quad N = 3x^2+4xy-6$

$\frac{dM}{dy} = 6x+4y \quad \frac{dN}{dx} = 6x+4y$

$\int(6xy+2y^2-5)dx + \int -6\,dy = c$

$\frac{6x^2y}{2} + 2xy^2 - 5x - 6y = c$

$3x^2y + 2xy^2 - 5x - 6y = c$

$\int\left(x+\frac{y}{1-x^2y^2}\right)dx + Nx = c$

$\int x\,dx + \int \frac{y\,dx}{1-x^2y^2} = c$

$\frac{x^2}{2} + \int \frac{y/y^2}{1/y^2 - x^2y^2/y^2}\,dx = c$

$\frac{x^2}{2} + \frac{1}{y}\int \frac{dx}{(1/y^2) - x^2} = c$

$\frac{x^2}{2} + \frac{1}{y}\left[\frac{1}{2(1/y)} \ln\left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right] = c$

$x^2 + \ln\left|\frac{1+xy}{1-xy}\right| = c$ Ans

viii) $(y\sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$

$M = y\sec^2 x + \sec x \tan x$, $N = \tan x + 2y$

$\frac{dM}{dy} = \sec^2 x \qquad \frac{dN}{dx} = \sec^2 x$

$\int M\,dx + \int(\tan N)dy = c$

$\int(y\sec^2 x + \sec x\tan x)dx + \int 2y\,dy = c$

$y\tan x + \sec x + y^2 = c$

Question No 5

i) $(xy^2 + y)dx - x\,dy = 0$ — (i)

$M = x^2y + y \qquad N = -x$

$M_y = 2xy + 1 \qquad N_x = -1$

$\frac{M_y - N_x}{N} = \frac{2xy+1+1}{-x}$

$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$

$= \frac{-2(\cancel{1+xy})}{y(\cancel{xy+1})} = \frac{-2}{y}$

$\frac{1}{y^2}(xy^2+y)dx - \frac{x}{y^2}dy = 0$

$\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0$ — (ii)

$\to$ Now $M = x + \frac{1}{y} \quad N = \frac{-y}{y^2}$

$M_y = \frac{-1}{y^2} \quad N_x = \frac{-1}{y^2}$

$\int\left(x+\frac{1}{y}\right)dx + N:l = C$

$\frac{x^2}{2} + \frac{2x}{y} = c$

ii) $(x^2+x-y)dx + x\,dy = 0$ — (i)

$M = x^2+x-y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$M_y \neq N_x$

$\dfrac{M_y - N_x}{N} = \dfrac{-1-1}{x} = \dfrac{-2}{x}$

$\dfrac{1}{x^2}(x^2+x-y)dx + \dfrac{x}{x^2}dy = 0$

$\left(1+\dfrac{1}{x}-\dfrac{y}{x^2}\right)dx + \dfrac{1}{x}dy = 0$ — (ii)

Now $M = 1+\dfrac{1}{x} - \dfrac{y}{x^2} \qquad N = \dfrac{1}{x}$

$M_y = \dfrac{-1}{x^2} \qquad N_x = \dfrac{-1}{x^2}$

$\int\left(1+\dfrac{1}{x}-\dfrac{y}{x^2}\right)dx + Nil = c$

$x + \ln x + \dfrac{y}{x} = c$

iii) $dy + \left(\dfrac{y-\sin x}{x}\right)dx = 0$ — (i)

$M = \dfrac{y-\sin x}{x} \qquad N = 1$

$M_y = \dfrac{1}{x} - 0 \qquad N_x = 0$

$x\,dy + x\left(\dfrac{y-\sin x}{x}\right)dx = 0$ — (ii)

$M = y - \sin x \qquad N = x$

$M_y = 1 \qquad N_x = 1$

$M_y = N_x$

$\int M\,dx + \int(\text{ter of } N)\,dy = c$

$\int(y-\sin x)\,dx = c$

$xy + \cos x = c$.

iv) $y(2xy+e^x)dx - e^x\,dy = 0$

$(2xy^2+e^x y)dx - e^x\,dy = 0$ — (i)

$M = 2xy^2+e^x y \qquad N = -e^x$

$M_y = 4xy+e^x \qquad N_x = -e^x$

$\dfrac{M_y - N_x}{N} = \dfrac{4xy+e^x+e^x}{-e^x}$

$\dfrac{N_x - M_y}{M} = \dfrac{-e^x-4xy-e^x}{2xy^2+ye^x}$

$= \dfrac{-2\cancel{(e^x+2xy)}}{y\cancel{(2xy+e^x)}} = \dfrac{-2}{y}$

$\dfrac{1}{y^2}(2xy^2+e^x y)dx - \dfrac{1}{y^2}e^x dy$

$2x+\dfrac{e^x}{y}\,dx - \dfrac{e^x}{y^2}dy = 0$

$M = 2x+\dfrac{e^x}{y} \qquad N = \dfrac{-e^x}{y^2}$

$\int\left(2x+\dfrac{e^x}{y}\right)dx + Nil = c$

$x^2 + \dfrac{e^x}{y} = c$ Ans

vi) $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$ — (i)

$M = y^4+2y \qquad N = xy^3+2y^4-4x$

$M_y = 4y^3+2 \qquad N = xy^3+2y^4-4x$

$M_y \neq N_x \qquad N_x = y^3-4$

$\frac{N_x-M_y}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} = \frac{-3y^3-6}{y(y^3+2)} = \frac{-3}{y}$

$\int -\frac{3}{y}dy = -3\ln y \qquad \ln y^{-3}$

$I = e^{\int -\frac{3}{y}dy} = e^{-3\ln y} = e^{\ln y^{-3}} = y^{-3} = \frac{1}{y^3}$

$(y+\frac{2}{y^2})dx+(x-2y-\frac{4x}{y^3})dy=0$ — (ii)

Now $M = y+\frac{2}{y^2} \qquad N = x+2y-\frac{4x}{y^3}$

$M_y = 1-\frac{4}{y^3} \qquad N_x = 1+0-\frac{4}{y^3}$

$\int y+\frac{2}{y^2}\,dx+\int 2y\,dy = c$

$xy+\frac{2x}{y^2}+y^2 = c$

vii) $(x^2+y^2+2x)dx+2y\,dy=0$ — (i)

$M = x^2+y^2+2x \qquad N = 2y$

$M_y = 2y \qquad N_x = 0$

$\frac{N_x-M_y}{M} = \frac{0-2y}{x^2+y^2+2x}$

$e^x(x^2+y^2+2x)dx+e^x(2y)dy=0$ — (ii)

$\int e^x(x^2+y^2+2x)dx + Nil = c$

$\int e^x x^2\,dx+\int e^x y^2\,dx+\int e^x 2x\,dx = c$

$x^2e^x-\int 2\cancel{xe^x}\,dx+e^x y^2+\int \cancel{e^x 2x}\,dx = c$

$(x^2+y^2)e^x = c$

viii) $(x^2+y^2)dx - 2xy\,dy = 0$ — (i)

$M = x^2+y^2 \qquad N = -2xy$

$M_y = 2y \qquad N_x = -2y$

$$\frac{N_x - M_y}{M} = \frac{-2y-2y}{x^2+y^2}$$

$$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = -\frac{2}{x}$$

$$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$$

$$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0 \text{ — (ii)}$$

$M = 1+\frac{y^2}{x^2} \qquad N = -\frac{2y}{x}$

$M_y = N_x$

$$\int\left(1+\frac{y^2}{x^2}\right)dx + \text{Nil} = c$$

$$x - \frac{y^2}{x} = c \text{ Ans}$$