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Class :

EET (2nd) Evenign

Assignment

: Math

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Exercise 9.2

Q# 1 seperable Eq

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}$$

$$\int y\,dy = \frac{1}{3}\ \frac{x^2}{1+x^3}$$

$$y^2/2 = \frac{1}{3}\ln(1+x^3)+c$$

$$3y^2/2 = 1\ \ln(1+x^3)+3c$$

$$3y^2 = 2\ln(1+x^3)+6c$$

$$3y^2 = 2\ln(1+x^3)+c$$

Q# 2

$$\frac{dy}{dx} = 2x^2+y-x^2y+xy-2x-2$$

$$= 2x^2-2x-2+y-x^2y+xy$$

$$= 2(x^2-x-1)\ y(-1+x^2-x)$$

$$\frac{dy}{dx} = (x^2-x-1)(2-y)$$

$$\int dy/2-y = \int(x^2-x-1)\,dx$$

$$-\int\frac{-dy}{+2-y} = \int(x^2-x-1)\,dx$$

$$-\ln(2-y) = x^3/3 - \frac{x^2}{2} - x + c$$

$$-\ln(2-y) = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

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$-6\ln(2-y) = 2x^3 - 3x^2 - 6x + 6c$

$\ln(2-y)^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln$

$\ln(2-y)^{-6} = \ln e^{2x} - 3x^2 - 6x + 6c$

$(2-y)^{-6} = \ln e^{2x - 3x^2 - 6x}$

$2-y = e^{2x^3} - 3x^2 - 6x \quad e^{6c}$

$(2-y)^{-6} = e \cdot e^{2x^3 - 3x^2 - 6x}$

Q#3.

$\frac{dy}{dx} + y^2 \sin x = 0$

$\frac{dy}{dx} = -y^2 \cdot \sin x$

$\int dy/dx = \int \sin x\, dx$

$y^{-1}/-1 = -(-\cos x) + c$

$-1/y = \cos x + c$

Q#4 $dy/dx = 1 + x + y^2 + xy^2$

$dy/dx = (1+x) + y^2(1+x)$

$\frac{dy}{dx} = (1+x)(1+y^2)$

$\int dy/y^2 = \int (1+x)\, dx$

$\tan^{-1} y = x + x^2/2 + c$

$2 \tan^{-1} y = 2x + x^2 + c$

Q#5 $(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$

$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$

$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$

$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$

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$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2)}\,dx + \frac{1}{y+2}\,dy = 0$$

$$\int \frac{x+1}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$$

$$y+2 = \frac{c}{\sqrt{x^2+2x}}$$

Q#6

$$\cos ec\,y\,dx + \sec x\,dy = 0$$

$\div$ $\cos ec\,y$ $\sec x$

$$\frac{1}{\sec x}\,dx + \frac{dy}{\cos ec\,y} = 0$$

$$\int \cos x\,dx + \int \sin y\,dy = \int 0\,dx$$

$$\sin x - \cos y = c$$

Q#7 $y(1+x)\,dx + x(1+y)\,dy = 0$

$\div$ by xy

$$\left(\frac{1+x}{x}\right)dx + \left(\frac{1+y}{y}\right)dy = 0$$

$$\int\left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0\,dx$$

$$\ln x + x + \ln y = c$$

$$x + y + \ln(xy) = c$$

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$Q\#8$ (4)

$(e^x+1)\,y\,dy = (y+1)e^x\,dx$

$\div$ by $(e^x+1)(y+1)$

$\int \frac{y}{y+1}\,dy = \int \frac{e^x}{e^x+1}\,dx$

$\int \left(\frac{y+1-1}{y+1}\right)dy = \int \frac{e^x}{e^x+1}$

$y - \ln(y+1) = \ln(e^x+1) + \ln c$

$y = \ln(y+1) + \ln(e^x+1) + \ln c$

$y = \ln\{(y+1)(e^x+1)c\}$

$e^y = c(y+1)(e^x+1)$

Execise 9.3

Homogeies D.Ft en (H.D.E)

$Q\#1$

$(x-y)\,dx + (x+y)\,dy = 0$

$(x+y)\,dy \quad dy = -(x-y)\,dx$

$\frac{dy}{dx} = \frac{y-x}{x+1}$ H.D.E — (i)

Put $y = vx$ — (ii)

$dy/dx = v + x\frac{dv}{dx}$ — (iii)

$v + x\frac{dv}{dx} = \frac{vx-x}{x+vx}$

$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(v+1)} - v$

$x\frac{dv}{dx} = \frac{\cancel{v}-1-\cancel{v}-v^2}{1+v}$

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$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1} v = -\ln x + c$$

$$\ln(v^2+1)^{1/2} + \tan^{-1} v + \ln x = c$$

$$\ln \sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}(y/x) + \ln x = c$$

$$\ln\sqrt{y^2+x^2} \quad \ln\sqrt{x^2} + \tan^{-1}(y/x) + \ln x = c$$

$$\ln\sqrt{y^2+x^2} + \tan^{-1}(y/x) = \text{Ans}$$

Q#2

$$(y^2+2xy)\,dx + x^2\,dy = 0$$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = -\frac{(y^2+2xy)}{x^2} \quad \text{—— H.D.F (i)}$$

$$\text{Put } y = vx \quad \text{(ii)}$$

$$\frac{dy}{dx} = v + \frac{dv}{dx} \quad \text{—— (iii)}$$

using (i) (ii) (iii)

$$v + x\frac{dv}{dx} = \frac{-(v^2x^2+2xvx)}{x^2}$$

$$x\frac{dv}{dx} = \frac{-x^2(v^2+2)}{x^2} - v$$

$$x\frac{dv}{dx} = -(v^2+3v)$$

$$\int \frac{-dv}{v^2+3v} = -\int \frac{dx}{x}$$

$$\int \frac{1}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{3}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

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$$\frac{1}{3}\int \frac{3}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{v+3-v}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \left(\frac{1}{v} - \frac{1}{v+3}\right) dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(v+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln c$$

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{\frac{1}{3}} = c\,(v+3)^{1/3}$$

$$x(y/x)^{1/3} = c\,(y/x+3)^{1/3}$$

$$x \cdot \frac{y^{1/3}}{\cancel{x^{1/3}}}\ \cancel{x^{1/3}} = c\,(y+3x)^{1/3}$$

$$xy^{1/3} = c(y+3x)^{1/3}$$

$$x^3y = c(y+3x) \text{ Ans}$$

Q#3 $(x^2-3y^2)\,dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2-3y^2)\,dx$$

$$dy/dx = \frac{3y^2 - x^2}{2xy} \quad \text{— H.D.E — (i)}$$

put $y = vx$ — (ii)

$$dy/dx = v + x\,dv/dx$$

$$v + x\frac{dv}{dx} = \frac{3v^2\cancel{x^2} - \cancel{x^2}}{2v\cancel{x^2}} - v$$

$$x\,dv/dx = \frac{3v^2 - 1 - 2v^2}{2v}$$

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$$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln(v^2-1) = \ln cx$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$$

$$\frac{y^2-x^2}{x^2} = cx \qquad y^2-x^2 = (cx)x^2$$

Q#4 $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{— (i)}$$

Put $y = vx$ — (ii)

$dy/dx = v + x\,dv/dx$ — (iii)

$$v + x\,dv/dx = x^2 + 3v^2x^2 / 2xvx$$

$$x\frac{dv}{dx} = \frac{x^2(1+3v^2)}{x^2 \cdot 2v} - v$$

$$x\,dv/dx = 1+3v^2-2v^2 / 2v$$

$$x\,dv/dx = 1+v^2 / 2v$$

$$\int 2v / 1+v^2 = \int dx/x$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln x + \ln c$$

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$\ln(1+y^2/x^2) = cx$

$x^2+y^2/x^2 = cx$

$x^2+y^2 = (cx)x^2$

Q#5 $(x^2-xy+y^2)\,dx - x^2dy = 0$

$(x^2-x)+y^2)\,dx = x^2dy$

$dy/dx = x^2+xy+y^2/x^2$ — H.D.E — (i)

put $y = vx$ (ii)

$\frac{dy}{dx} = v+x\,dv/dx$ — (iii)

$v+x\,dv/dx = \frac{x^2+x(vx)+v^2x^2}{x^2}$

$x\,dv/dx = \frac{(1+v+v^2)\cancel{x^2}-v}{\cancel{x^2}}$

$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$

$\tan^{-1}v = \ln x + c$

$\tan^{-1}(v/x) = \ln x + c$

Q#6 $x\sin(y/x)\,dy = (y\sin y/x - x)\,dx$

$dy/dx = \frac{y\sin y/x - x}{x(\sin)(y/x}$ H.D.E — (i)

put $y = vx$ — (ii)

$dy/dx = v + x\,dy/dx$

$v+x\frac{dv}{dx} = \frac{vx\sin vx/x - x}{x\sin(vx/x)}$

$x\,dv/dx = \frac{x(v\sin v-1)}{x\sin v} - v$

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(a)

$x\, dv/dx = \frac{\cancel{v\sin v - v\sin v}}{\sin v}$

$\int \sin v\, dv = \int -dx/x$

$-\cos v = -\ln x + c$

$\cos v = \ln x - c$

$\cos y/x = \ln - c.$

Q #7 $dy/dx = \frac{x+y}{x}$ — (i) $y(1) = 1$

put $y = vx$ — (ii)

$dy/dx = v + x\, dv/dx$ — (iii)

$v + x\, dv/dx = x + vx/x$

$x\, dv/dx = \frac{\cancel{x}(1+v)}{\cancel{x}} - v$

$x \frac{dv}{dx} = 1$

$\int dv = \int dx/x$

$v = \ln x + c$

$y/x = \ln x + c$

$y(1) = 1$

$1/1 = \ln + c$

$\boxed{1 = 0 + c}$

so $y/x = \ln x + 1$

$y = x \ln x + x$

$= x\,[\ln x + 1)$ Ans

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Exercise # 9.4

$M(x)\,dx + N(xy)\,dy = 0$

is said to be exact diff eq if it is total diff fcn

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Q No 1

Solve $(3x^2 + 4xy)\,dx\,(2x^2 + 2y)\,dy = 0$

$M = 3x^2 + 4xy$, $N = 2x^2 + 2y =$

$\frac{\partial M}{\partial y}$ $0 + 4x$ $\frac{\partial N}{\partial x}$ $4x + 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So giving Diff is Exact.

Now

$\int M dx + \int (\text{tan of } N \text{ fonform } x)\,dy = 0$

$\int (3x^2 + 4xy)\,dx + \int 2y\,dy = c$

$\frac{3x^3}{3} + \frac{4x^2 y}{2} + \frac{2y}{2} = c$

$x^3 + 2x^2 y + y^2 = c$

Q # 2 $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)$

$M = 2xy + y - \tan y$. $N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y$ $\frac{\partial N}{\partial x} = 2x - \tan^2 y + 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So given different Exact.

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$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int (2xy + y - \tan y)\,dx + \int \sec^2 y\,dy = c$

$\frac{2x^2y}{2} + xy - x\tan y + \tan y = c$

$x^2y + xy - x\tan y + \tan y = c$

Q#3

$dy/dx = \frac{-(ax+hy)}{hx+by}$

$(hx+by)\,dy = -(ax+hy)\,dx$

$(ax+hy)\,dx + (hx+by)\,dy = 0$

$M = ax+hy \quad . \quad N = hx+by$

$\frac{\partial M}{\partial y} = 0+h \qquad \frac{\partial N}{\partial x}$

$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int (ax+hy)\,dx + \int by\,dy = c$

$\frac{ax^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c$

Q#5

$(1+\ln x)\,dx + (1 + \frac{x}{y})\,dy = 0$

$M = 1 + \ln xy \qquad N = 1 + x/y$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy} x \qquad \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int (1+\ln x)\,dx + \int 1\cdot dy = c$

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$\int dx + \int \frac{1}{x} \ln xy \, dy + \int dy = c$

$x + [\ln xy \,(x) - \int \frac{1}{y} \cdot y \cdot x \cdot dx] + y = c$

$x + x\ln y - \int dx + y = c$

$x + x \ln xy - x + y = c$

$x \ln xy + y = c$ Ans

Q No6

$(6xy + 2y^2 - 5)\,dx + (3x^2 + 4xy - 6) = 0$

$M = 6xy + 2y^2 - 5$ $\qquad N = 3x^2 + 4xy - 6$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int (\text{term of } N \text{ free } x)\,dy = c$

$\int (6xy + 2y^2 - 5)\,dx + \int -6 \cdot dy = c$

$\frac{6x^2 y}{2} + 2xy^2 - 5x - 6y = c$

$3x^2 y + 2xy^2 - 5x - 6y = c.$

Q#7 $(y\sec^2 x + \sec x \tan x)\,dx + (\tan x + 2y)\,dy = 0$

$M = y\sec^2 x + \sec x \tan x, \quad N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x \qquad \frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M\,dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int (y\sec^2 x + \sec x \tan x)\,dx + \int 2y\,dy = c$

$y \tan x + \sec x + y^2 = c$

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$\frac{1}{y}(xy^2+y)\,dx - \frac{x}{y^2}\,dy = 0$

$(x + \frac{1}{y})\,dx - \frac{x}{y^2}\,dy = 0$ — (1)

Now

$M = x + \frac{1}{y}$ $\qquad N = -1/y^2$

$M_y = -\frac{1}{y^2}$ $\qquad N_x = -1/y^2$

$M_y = N_x$ ∴ Exact Diff eq

$\int M\,dx + \int$ terms of N free from x $dy = c$

$\int (x + \frac{1}{y})\,dx + 0 = c$

$\frac{x^2}{2} + \frac{x}{y} = c$

Q#3 $dy + (y - \sin x)\,dx = 0$ — (i)

$M = y/x - \sin x/x$ $\qquad N = 1$

$M_y = 1/x - 0$ $\qquad N_x = 0$

$M_y \neq N_x$ (i) is Non exact Diff eq

$\frac{M_y - N_x}{N} = \frac{1/x - 0}{1} = \frac{1}{x}$

$e^{\int 1/x\,dx} = e^{\ln x} = (x)$

Multiply both side of (i) by I.F

$x\,dy + \cancel{x}(y - \sin x)\,dx = 0$ — (ii)

$M = y - \sin x$ $\qquad N = x$

$M_y = 1$ $\qquad N_x = 1$

$M_y = N_x$ (ii) is Exact Eq

$\int M\,dx + \int$ terms of N free from x $dy = c$

$\int (y - \sin x)\,dx = c$

$xy + \cos x = c$

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Q#4 (15)

$x\,dy - y\,dx = (x^2+y^2)\,dx$ — (1)

$x^2+y^2+y\,dx - x\,dy = 0$

$M_y = 2y+1$ $N_x = -1$

$M_y \neq N_x$ Hence (1) non exact

$\frac{N_x - M_y}{N} = \frac{-1-2y-1}{-x}$

$\frac{M_y - N_x}{N} = \frac{2y+1+1}{-x}$

$x\,dy - y\,dx = (x^2+y^2)\,dx$

$\int \frac{x\,dy - y\,dx}{x^2+y^2} = \int dx$

$\tan^{-1}(y/x) = x+c$

$(y/x) = \tan(x+c)$

$y = x\tan(x+c)$ ~~the~~ Ans

Q#5 $(y-xy^2)\,dx + (x+x^2y^2)\,dy = 0$

$M = y-xy^2$ $N = x+x^2y$

$M_y = 1-2xy$ $N_x = 1+2xy^2$

$\frac{M_y - N_x}{N} = \frac{1-2xy-1-2xy^2}{x+x^2y^2} = \frac{-2xy(x+1)}{x(1-xy)}$

$y\,dx - xy^2\,dx + x\,dy + x^2y^2\,dy = 0$

$y\,dx + x\,dy - xy^2\,dx + x^2y\,dy = 0$

$x \div$ by $x = y\,x\,dx + x\,dy - x^2y^2\left(\frac{dx}{x}\right) + x^2y^2\,d$

by x^2y^2 in both sides

$\frac{y\,dx + x\,dy}{x^2y^2} - \frac{x^2y^2\left(\frac{dx}{x} - dy\right)}{x^2y^2} = 0$

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$\frac{-1}{xy} - \ln(x) + y = 0$

Q#6

$(3xy + y^2)dx + (x^2 + xy)dy = 0$ —— (1)

$M = 3xy + y^2$ $N = x^2 + xy$

$My = 3x + 2y$ $Nx = 2x + y$

$My \neq Nx$

$\frac{Nx - My}{M} = \frac{2x + y - 3x + 2y}{3xy + y^2} = \frac{-x + 3y}{y(3x+y)}$

$\frac{My - Nx}{N} = \frac{3x + 2y - 2x - y}{x^2 + xy} = \frac{x + y}{x(x+y)} = \frac{1}{x}$

$I.F = e^{\int \frac{1}{x} dx} = e^{\ln x} = \boxed{x}$

$(3x^2y + xy^2)dx + (x^3 + x^2y)dy = 0$

$M = 3x^2y + xy^2$ $N = x^3 + x^2y$

$My = Nx$

$\int M dx + \int (\text{term not from } x) dy = c$

$\int (3x^2y + xy^2) dx + \cdot Nil = c$

$\frac{3x^3y}{3} + \frac{x^2y^2}{2} = c$

$x^3y + \frac{x^2y^2}{2} = c$ Ans

Q#7

$\because dy/dx = e^{2x} + y - 1$

$dy = (e^{2x} + y - 1)\,dx$

$(e^{2x} + y - 1)\,dx - dy = 0$ —— (i)

$M = e^{2x} + y - 1$, $N = -1$

$M_y \neq N_x$

$\frac{M_x - M_y}{N} = \frac{0 - 1}{e^{2x} + y - 1}$

$\frac{M_y - M_x}{N} = \frac{1 - 0}{1} = -1 = -x^0$

$I.F = e^{\int -1\,dx} = \boxed{e^{-x}}$

Multiply by both Side

$e^{-x}(e^{2x} + y - 1)\,dx - e^{-x}\,dy = 0$

$(e^{x} + e^{-x}y - e^{-x})\,dx - e^{-x}\,dy = $ —— (ii)

$M = e^{x} + e^{-x} - y - e^{x}$ $N = -e^{-x}$

$M_y = e^{-x}$ $N_x = e^{-x}$

$M_y = N_x$ is Exact Diff eq

$\int M\,dx + \int$ term of N (for m x) $dy = c$

$\int (e^{x} + e^{-x}y - e^{-x})\,dx + N(ii) = c$

$e^{x}\ e^{-x}y + e^{-x} = c$

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Q8 $(y\cos x + 2xe^y)dx + (\sin x + x^2 e^y - 1)$

$M = y\cos x + 2xe^y$, $N = \sin x + x^2e^y - 1$

$\frac{\partial M}{\partial y} = \cos x + 2xe^y$, $\frac{\partial N}{\partial x} = \cos x + 2xe^y$

$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial x}$

$\int M dx + \int (\text{term of } N \text{ free } x)dy = c$

$\int (y\cos x + 2xe^y)dx + \int -1\, dy = c$

$y\sin x + \frac{\cancel{2}x^2e^y}{\cancel{2}} - y = c$

$y \sin x + x^2 e^y - y = c$

Exercise 2.5

Q#1 Solving by following IF

i) $(xy^2 + y)dx - xdy = 0$ —(i)

$M = xy^2 + y$ $N = -x$

$M_y = 2xy + 1$ $N_x = -1$

$M_y \neq N_x$ Non exact

$\frac{M_y - N_x}{N} = \frac{2xy + 1 + 1}{-x}$

$\frac{N_x - M_y}{M} = \frac{1 - 2xy - 1}{xy^2 + y}$

$= \frac{-2(1+xy)}{y(x+1)} = \frac{-2}{y}$

$\int \frac{2}{y} dy = 2\ln y = \ln y^{-2}$

Multiply both side of eq (i) by IF

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Q#8

$(4x+3y^2)\,dx + 2x\,dy = 0$ — ①

$M = 4x+3y^2$ $\qquad N = 2xy$

$M_y = 0+6y$ $\qquad N_x = 2y$

$M_y \neq N_x$ Non Exact Diff eq

$\dfrac{N_x - M_y}{M} = \dfrac{2y-6y}{4x+3y^2}$

$\dfrac{M_y - N_x}{N} = \dfrac{6y-2y}{2xy} = \dfrac{4y}{2xy} = \dfrac{2}{x}$

Multiply both side ① by I.F

$M_y = 6yx^2$ $\qquad N_x = 6x^2y$

$M_y = N_x$ Exact Diff eq

$\int M\,dx + \int (\text{term } N \text{ of } (\text{const}))\,dy = c$

$\int (4x^3 + 3y^2 x^2)\,dx + N'\,l = c$

$\dfrac{4x^4}{4} + 3y^2\dfrac{x^3}{3} = c$

$x^4 + y^2x^3 = c$ Ans

Exercise #9.6

Q#1

$dy/dx + \dfrac{(2x+1)}{x}y = e^{-2x}$ LDE is y

$I.F = e^{\int p\,dx} = e^{\int \frac{(2x+1)}{x}dx} = e^{\int (\frac{2+1}{x})dx}$

$= e^{2x+\ln x} = e^{2x}e^{\ln x} = e^{2x}x$

Solving by $\int d(y \cdot I.f) = \int Q \cdot I.F\,dx + c$

$\int d(ye^{2x}x) = \int e^{-2x}\cdot e^{2x}\,dx + c$

$ye^{2x}x = \int x\,dx + c$

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$xye^{2x} = x^2/2 + c$

Q#2

$dy/dx + 3/x\, y = 6x^2$

$I.F = e^{\int 3/x\, dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

Solution is given by $\int d(y \times I.F) = \int (Q \times I.F\, dx + c)$

$\int d(yx^3) = \int 6x^2 \cdot x^3 dx + c$

$y x^3 = \int 6x^5 dx + c$

$y x^3 = \frac{6x^6}{6} + c$

$x^3 y = x^6 + c$

Q#3

$dy/dx + y/x \ln x = \frac{3x^2}{\ln x}$ (LDE)

$I.F = e^{\int P dx} = e^{\int \frac{1}{x \ln x} dx} = e^{\int \frac{dx/x}{\ln x}}$

$I.F = e^{\ln(\ln x)} = \ln x$

$\int d(y \ln x) = \int \frac{3x^2}{\ln x} \ln x\, dx + c$

$y \ln x = \frac{3x^3}{3} + c$

$y = \frac{x^3 + c}{\ln x}$

Q#4

$dy/dx + 3y = 3x^2 e^{-3x}$ (LDE)

$I.F = e^{\int P dx} = e^{3x}$

Solve by $\int d(y \times I.F) = \int Q\, I.F\, dx + c$

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$\int d(Ye^{3x}) = \int 3x^2 \cancel{e^{-3x}} e^{3x} dx + c$

$Ye^{3x} = x^3 + c$

$Y = e^{-3}(x^3 + c)$

Q # 6

$(x+1) \, dy/dx - ny = e^x (x+1)^{n+1}$

$dy/dx - \frac{n}{n+1} Y = e^x (x+1)^n$ (LDE)

I.F $e^{\int P dx} = e^{-\int \frac{n}{x+1} dx} = e^{-n \ln(x+1)} \quad e^{\ln(x+1)}$

$I.F = (x+1)^{-n} = \frac{1}{(x+1)^n}$

$\int d(Y \times I.F) = \int Q \times I.F \, dx + c$

$\int d(Y \cdot \frac{1}{x+1}) = \int e^x \cancel{(x+1)^n} \frac{1}{\cancel{(x+1)^n}} dx$

$Y \cdot (x+1)^{-n} = e^{x} + c$

$Y = (e^x + c)(x+1)^n$ Ans

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$(x^2+1) \, dy/dx + 2xy = 4x^2$

$dy/dx + \left(\frac{2x}{x^2+1}\right) Y = \frac{4x^2}{x^2+1}$ (LDE)

$\int \frac{2x}{x^2+1} dx = e^{\ln(x^2+1)}$

Sol is given by $\int d(Y \times I.F) = \int Q \, I.F \, dx + c$

$\int d[Y(x^2+1)] = \int \frac{4x^2}{\cancel{(x^2+1)}} \cancel{(x^2+1)} dx + c$

$Y(x^2+1) = \frac{4x^3}{3} + c$

$3Y(x^2+1) = 4x^3 + c$ Ans

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2#8

$dy/dx = \frac{1}{e^{y}-x}$

$dx/dy = e^{y} - x$

$\frac{dx}{dy} + x = e^{y}$ LLDE

$I.F = e^{\int dy} = e^{y}$

Sol ie is given by $\int d(x \cdot IF) = \int Q \times IF\, dy + c$

$\int d(xe^{y}) = \int e^{y} e^{y} dy + c$

$xe^{y} = \int e^{2y} dy + c$

$x = e^{y}/2 + ce^{-y}$

$x = e^{y}/2 + ce^{-y}$ Ans