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Section

EET

Course Title:

Math's

- GCUF -

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MATH'S

Ex #9.2

Question: 1

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

Sol.

$$y\,dy = \frac{x^2}{1+x^2}\,dx$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+C$$

$$\frac{3y^2}{2} = \ln(1+x^3)+3C$$

$$3y^2 = 2\ln(1+x^3)+6C$$

$$3y^2 = 2\ln(1+x^3)+C$$

Sum: 2

$$\frac{dy}{dx} + y^2\sin x = 0$$

Sol.

$$\frac{dy}{dx} = y^2\sin x$$

$$\int \frac{dy}{dx} = -\int \sin x\,dx$$

$$\frac{y^{-1}}{-1} = (-\cos x)+c \Rightarrow \frac{-1}{y} = \cos x + C$$

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Sum : 3

$$\frac{dy}{dx} = (1+x+y^2+xy^2)$$

Sol

$$\frac{dy}{dx} = (1+x)+y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y} = \int (1+x)\,dx$$

$$\tan^{-1}y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1}y = 2x + x^3 + c$$

Sum : 4

$$(xy+2x+y+2)dx + (x^2+2x)dy = 0$$

$$[x(y+2)+(y+2)]dx + x(x+2)\,dy = 0$$

Sol

$$[(y+2)(x+1)]dx + x(x+2)\,dy = 0$$

$(\div)$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2x)}\,dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$$

$$y+2 = \frac{c}{\sqrt{x^2+2x}}$$

Pg # 3rd

Sum : 6

Sol

$$\frac{dy}{dx} = 2x^2 x \cdot y - x^2 y + xy - 2x \cdot 2$$

$$= 2x^2 - 2x - 2 + y - x^2 y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-6\ln|2-y| = 2x^3 - 3x^2 - 6x + 6c$$

$$\ln|2-y|^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln$$

$$\ln|2-y| = \ln c^{-6}\, 2x^3 - 3x^2 - 6x$$

$$|2-y| = e \cdot e$$

$$|2-y|^{-6} = 2x^3 - 3x^2 - 6x$$

Sum : 6

$$\operatorname{cosec} y \, dx + \sec x \, dy = 0$$

$\div$ by $\operatorname{cosec} y \sec x$

Sol

$$\frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\Rightarrow \int \cos x \, dx + \int \sin y \, dy = \int 0 \, dx$$

$$\Rightarrow \sin x - \cos y = c$$

Sum : 7

$$y(1-x) dx + x(1+y) dy = 0$$

Sol

$\div$ by xy

$$\frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy = 0$$

$$\int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy \int 0 \, dx$$

$$\Rightarrow \ln x + x \ln y + y$$

$$x + y + \ln(xy) = c$$

Sum : 8

$$y\sqrt{1+x^2} dx + x\sqrt{1+y^2} dy = 0$$

$\div$ by xy

Sol

$$\int \frac{\sqrt{1+x^2}}{x} dx + \int \frac{\sqrt{1+y^2}}{y} dy = \int 0 \, dx$$

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Put $\sqrt{1+x^2} = t$ Put $\sqrt{1+y^2} = z$ Pg. 5th

$1+x^2 = t^2$ $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $y\,dy = z\,dz$

Therefor

$= \int \frac{\sqrt{1+x^2}}{x^2} x\,dx + \int \sqrt{1+y^2}\, y\,dy = \int 0\,dx$

$= \int \frac{t \cdot t\,dt}{t^2-1} + \int \frac{z \cdot z\,dz}{z^2-1} = c$

$\int \frac{t^2-1+1}{t^2-1} dt + \int \frac{z \cdot z\,dz}{z^2-1} dz = c$

$\int \left(1+\frac{1}{t^2-1}\right) + \int \frac{z^2-1+1}{z^2-1} dz = c$

$t + \frac{1}{2} \ln\left(\frac{t-1}{t+1}\right) + \int \left(1+\frac{1}{z^2-1}\right) dz = c$

$+ z + \frac{1}{2} \ln\left(\frac{z-1}{z+1}\right) = c$

$\sqrt{1+x^2} + \frac{1}{2} \ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + \sqrt{1+y^2} + \frac{1}{2} \ln \frac{(\sqrt{1+y^2}-1)}{(\sqrt{1+y^2}+1)}$

Pg # 6th

Ex # 9.3

Sum: 1

$(x-y)dx + (x+y)dy = 0$

Sol

$$(x+y)dy = -(x-y)dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{— (1)}$$

Put $y = vx$ — (2)

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \quad \text{— (3)}$$

$$v + \frac{x\,dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{x(v-1)}{x(1+v)} - v$$

$$= \frac{v - 1 - \cancel{v} - v^2}{1+v}$$

$$x\frac{dy}{dx} = -\frac{(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = \ln x + c$$

$$\ln(v^2+1)^{\frac{1}{2}} + \tan^{-1}v + \ln x = c$$

$$\ln\sqrt{y^2+x^2} - \ln\sqrt{x^2} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$$

$$\ln\sqrt{y^2+x^3}+\tan^{-1}\left(\frac{y}{x}\right)=c$$

Sum: 2

$$(y^2+2xy)dx+x^2dy=0$$

Sol:

$$x^2dy=-(y^2+2xy)dx$$

$$\frac{dy}{dx}=\frac{-(y^2+2xy)}{x^2}$$

Put $y=vx$

$$\frac{dy}{dx}=v+\frac{xdv}{dx}$$

Using (ii)/(iii) in (i)

$$v+\frac{xdv}{dx}=-\left(\frac{v^2x^2+2xvx}{x^2}\right)$$

$$\frac{xdv}{dx}=-\frac{x^2(v^2+2x)}{x^2}-v$$

$$\frac{xdv}{dx}=-(v^2+3v)$$

$$\int\frac{dv}{v^2+3v}=\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}dv=-\int\frac{dx}{x}$$

$$\frac{1}{3}\ln v-\frac{1}{3}\ln(x+3)=\ln x+\ln c$$

$$\ln\left[\frac{v^{\frac{1}{3}}}{(v+3)^{\frac{1}{3}}}\right]=\ln\frac{c}{x}$$

$$\frac{v^{\frac{1}{3}}}{(v+3)^{\frac{1}{3}}}=\frac{c}{x}$$

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Pg 8th

$$3 \cdot v^{\frac{1}{3}} = c(v+3)^{\frac{1}{3}}$$

$$x\left(\frac{y}{x}\right)^{\frac{1}{3}} = c\left(\frac{y}{x}+3\right)^{\frac{1}{3}}$$

$$x\,\frac{y^{\frac{1}{3}}}{\cancel{x^{1/3}}}\,\cancel{x^{1/3}} = c(y+3x)^{\frac{1}{3}}$$

$$xy^{\frac{1}{3}} = c(y+3x)^{\frac{1}{3}}$$

$$x^2 y = c(y+3x)$$

Sum : 3

$$(x^2-3y^2)dx + 2xydx = 0$$

sol

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \quad \text{--- (1)}$$

Put $y = vx$ --- (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{--- (3)}$$

Using (3)(2) in (1)

$$v + x\frac{dv}{dx} = \frac{(3v^2-1)x^2}{2vx^2} - v$$

$$v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$$

$$\frac{y^2-x^2}{x^2} = cx$$

$$y^2 - x^2 = (cx)x^2$$

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Pg 9th

Sum : 4

$$(x^2+3y^2)dx-2xy\,dy=0$$

Sol

$$(x^2+3y^2)dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{— (i)}$$

Put $y = vx$ — (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (3)}$$

Using (iii)/(ii) & (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{x^2(1+3x^2)}{x^2 2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{11v^2}{2v}$$

$$\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln cx$$

$$\left(1+\frac{y^2}{x^2}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$x^2+y^2 = (cx)x^2$$

Sum: 5

$$(x^2+xy+y^2)dx - x^2dy = 0$$

Sol

$$(x^2+xy+y^2) = x^2dy$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{— (i)}$$

Put $= vx$ — (2)

Using (2)(3) in (i)

$$v + \frac{x\,dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{(1+v+v^2)\,x^2 - v}{\cancel{x^2}}$$

$$\int \frac{dv}{dx} = \int \frac{dx}{v}$$

$$\tan^{-1}v = \ln x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$$

Sum: 6

$$(x^2+3xy+y^2)dx - x^2dy = 0$$

Sol

$$(x^2+3xy+y^2)dx - x^2dy$$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \quad (i)$$

Put $= vx$ — (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad (3)$$

Using (3) and (2) in (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3x\,vx+v^2x^2}{x^2}$$

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$$x\frac{dv}{dx} = \frac{x^2(1+3v+v^2)}{x^2} - v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\left(\frac{1}{v+1}\right) = \ln x + c$$

$$\left(\frac{1}{\frac{y}{x}+1}\right) = \ln x + c$$

$$\frac{-1/y+x}{x} = \ln x + c$$

$$\frac{-x}{(x+y)} = \ln x + c$$

Sum: 7

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y}$$

Sol #

Put $y = vx$ — (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (3)}$$

Using (3) (2) in (1)

$$v + x\frac{dv}{dx} = \frac{4vx-3x}{2x-vx}$$

$$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$$

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$$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x} \quad \text{---(iv)}$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$2 - v = A(v-1) + B(v+3)$ = Put $v+3=0$ so $A = -\frac{5}{4}$

$$\frac{2-v}{(v-3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v+1)}$$

$$-\frac{5}{4}\int \frac{dv}{v+3} + \frac{1}{4}\int \frac{dv}{v-1} \quad \int \frac{dx}{x}$$

$$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-1) = \ln x + \ln c$$

$$-\ln(v+3)^5 + \ln(v-1) = 4\ln cx$$

$$\ln \frac{(v-1)}{(v+3)^5} = \ln c^4 x^4$$

$$\frac{\left(\frac{y}{x}-1\right)}{\left(\frac{y}{x}+3\right)^2} = c^4 x^4$$

$$\frac{(y-x)\,x^5}{(y+3x)x^4} = c'$$

$$\frac{y+x}{(y+3x)^5} = c'$$

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Sum: 8

$$x \sin(y/x)\, dy = (y \sin \frac{y}{x} - x)\, dx$$

sol

$$\frac{dy}{dx} = \frac{y \sin y/x - x}{x \sin(y/x)} \quad \text{—(1)}$$

Put $y = vx$ —(2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—(3)}$$

Using (2)(3) in (1)

$$v + x\frac{dv}{dx} = \frac{vx \sin \frac{vx}{x} - x}{x \sin(\frac{vx}{x})}$$

$$x\frac{dv}{dx} = \frac{x(v\sin v - 1)}{x \sin v} - v$$

$$x\frac{dv}{dx} = \frac{v \sin v - 1 - v\sin v}{\sin v}$$

$$\int \sin v\, dv = \int -\frac{dx}{x}$$

$$-\cos v = \ln x + c$$

$$\cos v = \ln x - c$$

$$\cos y/x = \ln x - c$$

Ex #9.4

Sum: 1

$$(3x^2+4xy)dx+(2x^2+2y)dy=0$$

Sol:

$M=3x^2+4xy$, $N=2x^2+2y$

$$\frac{\partial M}{\partial y}=0+4x \quad \frac{\partial N}{\partial x}=4x+0$$

$$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

Now $\int M dx+\int$ (turmes of N from) $dy=c$

$$\int(3x^2+4xy)dx+\int 2y\,dy=c$$

$$\frac{3x^3}{3}+\frac{4x^2y}{2}+\frac{2y^2}{\cancel{2}}=c$$

$$x^3+2x^2y+y^2=c$$

Sum: 02

$$(2xy+y\tan y)dx+(x^2-x\tan y+\sec^2 y)dy=0$$

Sol:

$M=2xy+y-\tan y$, $N=x^2-x\tan^2 y+\sec^2 y$

$$\frac{\partial M}{\partial y}=2x+1-\sec^2 y \ , \ \frac{\partial N}{\partial x}=2x-\tan^2 y+0$$

$$=2x-\tan^2 y$$

$$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$\int M dx+\int$ (terms of N froce from) $dy=c$

$$\int(2xy+y-\tan dx+\int\sec^2 y\,dy=c$$

$$\frac{2x^2y}{2}+xy-x\tan y+\tan y=c$$

$$x^2y+xy-x\tan y+\tan y=c$$

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Sum : 03

$$\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)dy = 0$$

Sol

$$M = \frac{x+y}{y-1} \qquad N = \frac{-1}{2}\left(\frac{x+1}{y-1}\right)^2$$

$$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1) - (x-y)(1)}{(y-1)^2} \qquad N = \frac{-1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$$

$$= \frac{y-1-x-y}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \because \text{Givend---}$$

$$\int M dx \int (\text{Term of } N \text{ free form } x)\, dy = c$$

$$\int \left(\frac{x+y}{y-1}\right)dx + \int -\frac{1}{2}\frac{dy}{(y-1)^2} = c$$

$$\int (x+y)\left(\frac{1}{y-1}\right)dx - \frac{1}{2}\int (y-1)^{-2}dy = c$$

$$\frac{1}{(y-1)}\left(\frac{x^2}{2} + xy\right| + \left(\frac{-1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2 + 2xy + 1 = c'(y-1)$$

Sum: 4

$$\frac{dy}{dx} = -\frac{(ax+by)}{ax+by}$$

sol

$$(hx+by)dy = -(ax+hy)dx$$

$$(ax+hy)dx + (hx+by)dy = 0$$

$$M = ax+hy \qquad N = hx+dy$$

$$\frac{\partial M}{ay} = 0+h \qquad \frac{\partial N}{\partial x} = h$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int Mdx + \int (\text{terms of } N)$$

$$\int (ax+hy)\,dx + \int by\,dy = C$$

$$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = C$$

$$ax^3 + 2hxy + by^2 = C$$

Sum: 05

sol

$$(1+\ln xy)dx + \left(1+\frac{x}{y}\right)dy = 0$$

$$M = 1+\ln xy \qquad N = 1+\frac{x}{y}$$

$$\frac{\partial M}{\partial y} = 0+\frac{1}{xy}\cdot x \qquad \frac{\partial N}{\partial x} = 0+\frac{1}{y}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int Mdx + \int \text{terms of } N \text{ form } x)dy = C$$

$$\int (1+\ln xy)dx + \int 1\cdot dy = C$$

$$x + \int \ln xy\,(x) - \int \frac{1}{xy}\cdot y\cdot x\,dx] + y = C$$

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$$x + x\ln xy - \int dx + y = C$$

$$x + x\ln xy - x + y = C$$

$$x\ln xy + y = C$$

Sum: 06

$$\frac{y\,dx + x\,dy}{1-x^2y^2} + x\,dx = 0$$

sol

$$\frac{y\,dx}{1-x^2y^2} + \frac{x\,dy}{1-x^2y^2} + x\,dy = 0$$

$$\left(\frac{x+y}{1-x^2y^2}\right)dx + \frac{x\,dy}{1-x^2y^2} = 0$$

$$M = \frac{x+y}{1-x^2y^2}$$

$$\frac{\partial M}{\partial y} = 0 + \frac{(1-x^2y^2)\cdot 1 - y(-2x^2y)}{(1-x^2y^2)^2}$$

$$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{1-x^2y^2}$$

$$N = \frac{x}{1-x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2) - x(-2xy^2)}{(1-x^2y^2)^2}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int (\text{terms of } N \text{ --- } x)\,dy = c$$

$$\int M\,dx + \int \frac{y\,dx}{1-x^2y^2} = c$$

$$\frac{x^2}{2}+\int \frac{y/y^2}{\frac{\frac{1}{y^2}-x^2y^2}{y^2}}\,dx = C$$

$$\frac{x^2}{2}+\frac{1}{y}\int \frac{dx}{(\frac{1}{y})^2-x^2} = C$$

$$\frac{x^2}{2}+\frac{1}{y}\left[\frac{1}{2}\left(\frac{1}{y}\right)\ln\left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right] = C$$

$$\frac{x^2}{2}+\frac{1}{2}\ln\left|\frac{x+y}{1-xy}\right| = C$$

$$x^2+\ln\left|\frac{1+xy}{1-xy}\right| = C$$

Sum: 0]

Sol $(6xy+2y^2-5)dx+(3x^2+4xy-6)=0$

$M = 6xy+2y^2-5$, $N = 3x^2+4xy-6$

$\frac{\partial M}{\partial y} = 6x+4y$ $\qquad \frac{\partial N}{\partial x} = 6x+4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence exactly

$\int M dx + \int(\text{terms of } N \text{ --- } x)\,dy = C$

$\int(6xy+2y^2-5)\,dx+\int -6\,dy = C$

$\frac{6x^2y}{2}+2xy^2-5x-6y = C$

$3x^2y+2xy^2-5x-6y = c$

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Q no:- 8

$$(y\sec^2 x+\sec x\tan x)dx+(\tan x+2y)dy=0$$

Sol

$$M=y\sec^2 x+\sec x\tan x \quad , \quad N=\tan x+2y$$

$$\frac{\partial M}{\partial y}=\sec^2 x \qquad \frac{\partial N}{\partial x}=\sec^2 x$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$$\int M\,dx+\int(\text{terms of } N \text{ --- } x)\,dy=C$$

$$y\tan x+\sec x+y^2=C$$

Ex#9.5

Sum : 1

$$(xy^2 + y)dx - x\,dy = 0$$

Sol

$M = xy^2 + y \qquad N = -x$

$M_y = 2xy + 1 \qquad N_x = -1$

$\because M_y \neq N_x \qquad \therefore$ Non Exact

$$\frac{M_y - N_x}{M} = \frac{2xy + 1 + 1}{-x}$$

$$\frac{N_x - M_y}{M} = \frac{-1 - 2xy - 1}{xy^2 + y}$$

$$= \frac{-2(1 + xy)}{y(xy + 1)} = \frac{-2}{y}$$

$\int \frac{2}{y} dy - 2 \log$

$\frac{1}{y}(xy^2 + y)dx - \frac{x}{y} = dy = 0$

$(x + \frac{1}{y})dx - \frac{x}{y}dy = 0$ — (2)

Now $M = x + \frac{1}{y} \qquad N = -\frac{x}{y^2}$

$M_y = \frac{1}{y^2}, N_x = \frac{1}{y^2}$

$\because M_y = N_x$

$\int M dx + \int$ terms of N --- x

$\int (x + \frac{1}{y})dx + Nil = c$

$$\frac{x^2}{2} + \frac{x}{y} = c$$

Sum: 02

$$(x^2+xy)dx + x\,dy = 0 \quad -(1)$$

Sol

$M = x^2+x-y \qquad N = x$

$M_y = -1 \qquad N_x = 1$

$M_y \neq N_x \qquad \therefore$ Non exact

$$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = -\frac{2}{x}$$

$\int -\frac{2}{x}\,dx \quad -2\ln x \quad \ln x^{-2}$

multiply both sides of eq (1)

$$\frac{1}{x^2}(x^2+x-y)dx + \frac{x}{x^2}dy = 0$$

$$\left(1+\frac{1}{x} - \frac{y}{x^2}\right)dx + \frac{1}{x}dy = 0 \quad -(2)$$

Now $M = 1+\frac{1}{x} - \frac{y}{x^2} \qquad N = \frac{1}{x}$

$M_y = \frac{1}{x^2} \qquad N_x = \frac{1}{x^2}$

$M_y = N_x$

So $\int M\,dx\ \int$ (terms of N form ... x) $dy = C$

$$\int\left(1+\frac{1}{x} - \frac{y}{x^2}\right)dx + N = C$$

$$x + \ln x + \frac{y}{x} = C$$

Sum:03

$$y\,dx + (2xy - e^{-2y})\,dy = 0$$

Sol

$M = y$ $\qquad N = 2xy - e^{-2y}$

$M_y = 1$ $\qquad N_x = 2y$

$M_y \neq N_x$ is non excat Differs

$$\frac{M_y - N_x}{N} = \frac{1-2y}{2xy - e^{-2y}}$$

$$\frac{N_x - M_y}{M} = \frac{2y-1}{y} = 2 - \frac{1}{y}$$

if $\int(2 - \frac{1}{y})\,dy = e^{2y - \ln y}$

$= e^{2y + \ln y^{-1}} = e^{2y} e^{\ln(\frac{1}{y})}$

$= e^{2y} \cdot \frac{1}{y}$

Multiply (i) by if $= e^{2y} \cdot \frac{1}{y}$

$$e^{2y}\frac{1}{y}\,y\,dx + e^{2y} \cdot \frac{1}{y}(2xy - e^{-2y})\,dy = 0$$

$$e^{2y}\,dx + (e^{2y} \cdot \frac{1}{y}(2xy - e^{-2y}))\,dy = 0$$

$M = e^{2y}$ $\qquad N = e^{2x}\,2x - \frac{1}{y}$

$M_y = 2e^{2y}$ $\qquad N_x = 2e^{2y} - 0$

$\therefore \int M\,dx + \int(\text{term of } N \text{ free from } x)\,dy = c$

$\Rightarrow \int e^{2y}\,dx + \int -\frac{1}{y}\,dy = c$

$\Rightarrow xe^{2y} - \ln y = c$

Sum: 04

$$dy + \left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{—(i)}$$

Sol →

$M = \frac{y - \sin x}{x}$ $\qquad N = 1$

$M_y = \frac{1}{x} - 0$ $\qquad N_x = 0$

$M_y \neq N_x \therefore$ (i) is NON - - -

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$

multiply in both side.

$x\,dy + x\left(\frac{y - \sin x}{x}\right) dx = 0$

$M_y = 1$ $\qquad N = x$

$M = y - \sin x$ $\qquad N_x = 1$

$M_y = N_x$ $\qquad \therefore$ (ii) is exact

$\int (y - \sin x)\, dx = c$

$xy + \cos x = c$

$\therefore M_y = N_x \quad \therefore$ (ii) is exact

$\int M\,dx + \int (\text{term's of} - - - x)\, dy = c$

$\int \left(y + \frac{2}{y^2}\right) dx + \int 2y\, dy = c$

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$

$xy + \frac{2x}{y} + y = c$

Sum: 05

$$(x^2+y^2+2x)dx + 2y\,dy = c \quad \text{---(i)}$$

sol

$M = x^2+y^2+2x \qquad N = 2y$

$M_y = 2y \qquad N_x = 0$

$M_y \neq N_x \qquad \therefore$ (i) is Non...

$$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2y}$$

$$\frac{M_y - N_x}{N} = \frac{2y-0}{2y}$$

multiply both side

$e^x(x^2+y^2+2x) \qquad N = e^x 2y$

$M_y = e^x 2y \qquad N_x = e^x 2y$

$M_y = N_x \qquad \therefore$ (i) is

$\int e^x(x^2+y^2+2x)dx + Nil = c$

$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x \cdot dx = c$

$x^2 e^x - \cancel{\int 2x e^x dx} + e^x y^2 + \cancel{\int e^x 2x\, dx} = c$

$$\underline{(x^2+y^2)e^x = c}$$

Som: 6

$(4x+3y^2)dx + 2xy\,dy = 0$ — (i)

sol

$M = 4x + 3y^2$ $\quad N = 2xy$

$M_y = 0 + 6y$ $\quad N_x = 2y$

$M_y \neq N_x$

$\frac{N_x - M_y}{M} = \frac{2y - 6y}{4x + 3y}$

$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$

multiply both sides

$(4x^3 + 3y^2x^2)dx + (2x^3y)dy = 0$ — (ii)

$M_y = 6yx^2$ $\quad N_x = 6x^2y$

$M_y = N_x$

$\therefore \int M dx + \int (\text{term} \text{ --- } x)\,dy = c$

$\int (4x^3 + 3y^2x^2)\,dy + \text{Nil} = c$

$4\frac{x^4}{4} + 3y^2\frac{x^3}{3} = c$

$x^4 + y^2x^3 = c$

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Sum: 07

$$(x^2+y^2)dx - 2xy\,dy = 0 \quad \text{---- (i)}$$

Sol

$M = x^2+y^2$ $\qquad$ $N = -2xy$

$M_y = 2y$ $\qquad$ $N_x = -2y$

$M_y \neq N_x$

$$\frac{N_x - M_y}{M} = \frac{-2y-2y}{x^2+y^2}$$

$$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = -\frac{2}{x}$$

multiply both sides

$$\frac{1}{x^2}(x^2+y^2)dx - \frac{1}{x^2}(2xy)dy = 0$$

$$\left(1+\frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$$

$M = 1+\frac{y^2}{x^2}$ $\qquad$ $N = -\frac{2y}{x}$

$M_y = \frac{2y}{x^2}$ $\qquad$ $N_x = +\frac{2y}{x^2}$

$M_y = N_x$

$\because \int M dx + \int (\text{term of } \ldots x)dy = c$

$$\int \left(\frac{1+y^2}{x^2}\right) dx + \text{Nil} = c$$

$$x - \frac{y^2}{x} = c$$

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Sum: 08

$$\frac{dy}{dx} = e^{2x} + y - 1$$

Sol

$$dy = (e^{2x} + y -)dx$$

$$(e^{2x} + y - 1)\, dx - dy = 0 \quad \text{—(i)}$$

$M = e^{2x} + y - 1$ $\qquad N = -1$

$M_y = 1$ $\qquad N_x = 0$

$M_y \neq N_x$ $\qquad \because$ (1) is non Exact

$$\frac{N_x - M_y}{M} = \frac{0 - 1}{e^{2x} + y - 1}$$

$$\frac{M_y - N_x}{N} = \frac{1 - 0}{-1} = -1 = -x^0$$

Multiply both sides

$$(e^{-x} + y - 1)\, dx - e^{-x}\, dy = 0$$

$M = e^{x} + e^{-x} y - e^{x}$ $\quad N = -e^{x}$

$M_y = e^{-x}$ $\qquad N_x = +e^{x^{-x}}$

$M_y = N_x$ $\quad$ (ii) is exact

So

$$\int M\,dx + \int (\text{term from } --- x)\, dy = c$$

$$\int e^{x} + e^{-x}y - e^{-x})\, dx + Nil = c$$

$$\underline{e^{x} - e^{-}y^{x} + e^{-x} = C}$$

Ex # 9.6

Pg #28th

Question no:- 1

$$\frac{dy}{dx} + \frac{(2x+1)}{x}y = e^{-2x}$$

Sol

$\int P dx \quad \int \frac{2x+1}{x} dx \quad \int \left(2+\frac{1}{x}\right) dx$

$I.f = e = e = e$

$2x + \ln x \qquad 2x \ln x \; 2x$

$= e$

$\because$ Solving by $\int d(y \times I.f)$

$\int d(ye^{2x}x) = \int e^{-2x} e^{2x} x\,dx + c$

$= ye^{2x}x = \int x\,dx + c$

$= xy^2 e^x = \frac{x^2}{2} + c$

Sum : 02

$$\frac{dy}{dx} + \frac{3y}{x} = 6x^2$$

Sol

$\int P dx \quad \int \frac{3}{x} dx \quad 3 \ln x$

$I.f = e = e = e = e = x^3$

Sol is given by $\int d(y \times I.f) = \int Q \times I.F\,dx + c$

$= \int d(yx^3) = \int 6x^2 \cdot x^3\,dx + c$

$= yx^3 = \int 6x^5\,dx + c$

$yx^3 = \frac{6x^6}{6} + c \Rightarrow x^3 y = x^6 + c$

Pg 29^th

Sum: 03

$$\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x^2}{\ln x}$$

$\underline{\text{Sol}}$ $\int P dx \int \frac{1}{x \ln x} dx \int \frac{\frac{dx}{x}}{\ln x}$

I.f = e = e = e.

$I \cdot f \; e^{1}(\ln x) = \boxed{\ln x}$

Sol is Given by $\int d(y \times I.f) = \int Q \times I.F \, dx + C$

$= \int d(y \ln x) = \int \frac{3x^2}{\ln x} \ln x \, dx + C$

$y \ln x = 3\frac{x^3}{3} + C$

$$y = \frac{x^3 + C}{\ln x}$$

Sum: 04

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$\underline{\text{Sol}}$ $I.f = e^{\int P dx} = e^{\int P dx} = \boxed{e^{3x}}$

Sol is Given by $\int d(y \times I.f) = \int Q \times I.f \, dx + C$

$= \int d(y e^{3x}) = \int 3x^2 e^{-3x} e^{3x} \, dx + C$

$y e^{3x} = x^3 + C$

$y = e^{-3x}(x^3 + C)$

Pg # 30th

Sum: 05

$$\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$$

Sol

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \sec^2 x\, y = \sec^2 x \tan x$$

$$\int P\,dx \quad \int \sec^2 x\,dx$$

$$I.f = e = e = e^{\tan x}$$

Sol is given by $\int d(y \times I.f) = \int Q \times I.f\,dx + c$

$$\int d(y\,e^{\tan x}) = \int \sec^2 x \tan x\, e^{\tan x}\,dx + c$$

$$\Rightarrow y e^{\tan x} = \int e^t\, t\, dt + c$$

$$= Te^T - \int 1 \cdot e^t\,dt + c$$

$$= Te^T - \int 1 \cdot e^t\,dT + c$$

$$= te^t - e^t + c$$

$$ye^{\tan x} = e^t(t-1) + c$$

$$y e^{\tan x} = e^{\tan x}(\tan x - 1) + c$$

$$y = (\tan x - 1) + e^{-\tan x}$$

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Sum : 06

Pg # 31st

$$x\frac{dy}{dx}+(1+x\cot x)y=x$$

sol

$$\frac{dy}{dx}+\left(\frac{1}{x}+\cot x\right)y=1$$

$\int P dx \int \left(\frac{1}{x}+\cot x\right)dx$

I.f $e=e=e$

I.f $= e^{\ln}(x \sin x)$

Sol is Given By $\int d(y \times I.f) = \int P \times I.f \, dx + c$

$= \int d(y + \sin x) = \int x \sin x \, dx + c$

$y \times \sin x = x(-\cos x) - \int 1(-\cos x)dx$

$= x(-\cos x) + \int \cos x \, dx$

$y\, x \sin x = -x\cos x + \sin x + c$

$y = -\cos x + \frac{1}{x} + \frac{c}{x} - \cos x$

Sum : 07

$$(x+1)\frac{dy}{dx} - ny = e^{x}(x+1)^{n+1}$$

sol

$$\frac{dy}{dx} - \frac{n}{(x+1)}y = e^{x}(x+1)^{n}$$

$$\int P\,dx - \int \frac{n}{x+1}\,dx - n\ln(x+1) \ln(x+1)^{-n}$$

$$I.f = e = e = e$$

$$I.f = (x+1)^{-n} = \boxed{\frac{1}{(x+1)}^{n}}$$

sol is Given by $\int d(y \times I.f) = \int Q \times I.f\,dx + c$

$$= \int d\left(y\frac{1}{(x+1)}\right)^{n} = \int e^{x}\cancel{(x+1)^{n}}\frac{1}{\cancel{(x+1)^{n}}}\,dx + c$$

$$\underline{y = (e^{x}+c)(x+1)^{n}}$$

Sum : 08

$$(x^{2}+1)\frac{dy}{dx} + 2xy = 4x^{2}$$

sol

$$\frac{dy}{dx} + \left(\frac{2x}{x^{2}+1}\right)y = \frac{4x^{2}}{x^{2}+1}$$

$$\int\left(\frac{2x}{x^{2}+1}\right)dx$$

$$I.f = e = e = \boxed{x^{2}+1}$$

sol is Given by $\int d(y \times I.f) = \int Q \times I.f\,dx + c$

$$\int d(y(x^{2}+1)) = \int \frac{4x^{2}}{\cancel{(x^{2}+1)}}\cancel{(x^{2}+1)}\,dx + c$$

$$\underline{y(x^{2}+1) = \frac{4x^{3}}{3} + c \Rightarrow 3y(x^{2}+1) = 4x^{3} + c}$$

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