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Ex. 9.2

Slove

(i) $\dfrac{dy}{dx} = \dfrac{x^2}{y(1+x^3)}$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$
$$\int y\,dy = \frac{1}{3}\int\frac{3x^2}{1+x^3}\,dx$$
$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$$
$$\frac{3y^2}{2} = \ln(1+x^3)+3c$$
$$3y^2 = 2\ln(1+x^3)+6c$$
$$3y^2 = 2\ln(1+x^3)+C \quad \text{Ans.}$$

(ii) $\dfrac{dy}{dx} + y^2\sin x = 0$

$$\frac{dy}{dx} = -y^2\sin x$$
$$\int\frac{dy}{y^2} = -\int\sin x\,dx$$
$$\frac{y^{-1}}{-1} = -(-\cos x)+C$$
$$-\frac{1}{y} = \cos x + C \quad \text{Ans.}$$

(iii) $\dfrac{dy}{dx} = 1+x+y^2+xy^2$

$$\frac{dy}{dx} = (1+x)+y^2(1+x)$$
$$\frac{dy}{dx} = (1+x)(1+y^2)$$

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$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1} y = 2x + x^2 + c' \quad \text{Ans:}$$

x——x——x——x——x

(iv) $$(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$$

$$[x(y+2) + (y+2)]\,dx + x(x+2)\,dy = 0$$

$$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$$

$$[(y+2)(x+1)]\,dx + x(x+2)\,dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2)}\,dx + \frac{1}{y+2}\,dy = 0$$

$$\int \frac{x+1}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$$

$$y+2 = \frac{c}{\sqrt{x^2+2x}}$$

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(v) $\frac{dy}{dx} = 2x^2 + y - x^2y + xy - 2x - 2$

$= 2x^2 - 2x - 2 + y^2 - x^2y + xy$

$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$

$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$

$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$

$-\int \frac{-dy}{2-y} = \int (x^2 - x - 1)\,dx$

$-\ln(2-y) = \frac{x^3}{3} - \frac{x^2}{2} + x + C$

$-\ln(2-y) = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$

$-6\ln(2-y) = 2x^3 - 3x^2 - 6x + 6c$

$\ln(2-y)^{-6} = (2x^3 - 3x^2 - 6x + 6c)\ln e$

$\ln(2-y)^{-6} = \ln e^{2x^3 - 3x^2 - 6x + 6c}$

$(2-y)^{-6} = e^{2x^3 - 3x^2 - 6x} \cdot e^{6c}$

$(2-y)^{-6} = c_1 e^{2x^3 - 3x^2 - 6x}$ Ans.

x ——— x ——— x

(vi) $\operatorname{Cosec} y\,dx + \operatorname{Sec} x\,dy = 0$

$\div$ by $\operatorname{Cosec} y \operatorname{Sec} x$

$\Rightarrow \frac{1}{\operatorname{Sec} x}\,dx + \frac{dy}{\operatorname{Cosec} y} = 0$

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$$\int \cos x \, dx + \int \sin y \, dy = \int 0 \, dx$$

$$\sin x - \cos y = C \quad \text{Ans.}$$

x——x——x——x

(vii) $y(1+x)dx + x(1+y)dy = 0$

$\div$ by xy

$$\left(\frac{1+x}{x}\right)dx + \left(\frac{1+y}{y}\right)dy = 0$$

$$\int\left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0 \, dx$$

$$\ln x + x + \ln y + y = C$$

$$x + y + \ln(xy) = C \quad \text{Ans.}$$

x——x——x——x

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(viii) $y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x}\,dx + \int \frac{\sqrt{1+y^2}}{y}\,dy = \int 0\,dx$$

Put $\sqrt{1+x^2} = t$ Put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $y\,dy = z\,dz$

Theorems

$$\int \frac{\sqrt{1+x^2}}{x^2}\,x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}\,y\,dy = \int 0\,dx$$

$$= \int \left(\frac{t \cdot t\,dt}{t^2-1}\right)dt + \int \frac{z \cdot z\,dz}{z^2-1} = C$$

$$= \int \left(\frac{t^2-1+1}{t^2-1}\right)dt + \int \frac{z^2-1+1}{z^2-1}\,dz = C$$

$$= t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = C$$

$$\Rightarrow \sqrt{1+x^2} + \frac{1}{2}\ln\left[\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right] + \sqrt{1+y^2} + \frac{1}{2}\ln\left[\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right]$$

$= C$ Ans.

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(ix) $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$; $|x| < 1$, $|y| < 1$

$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} y = -\sin^{-1} x + C$

$y = \sin(C - \sin^{-1} x)$ is g. sd

$\frac{dy}{dx} + \sqrt{\frac{y^2-1}{x^2-1}} = 0$; $|x| > 1$, $|y| > 1$

(xi)

$\frac{dy}{dx} = -\sqrt{\frac{y^2-1}{x^2-1}}$

$\int \frac{dy}{\sqrt{y^2-1}} = -\int \frac{dx}{\sqrt{x^2-1}}$

$\cosh^{-1} y = -\cosh^{-1} x + C$

$y = \cosh(C - \cosh^{-1} x)$ Ans.

(X) $(e^x + 1) y\, dy = (y+1) e^x\, dx$

$\div$ by $(e^x + 1)(y+1)$

$\Rightarrow \int \frac{y\,dy}{y+1} = \int \frac{e^x\,dx}{e^x+1}$

$\int \left(\frac{y+1-1}{y+1}\right) dy = \int \frac{e^x}{e^x+1}\, dx$

$$\Rightarrow \int \left(1 - \frac{1}{y+1}\right) dy = \int \frac{e^x}{e^x+1}\, dx$$

$$\Rightarrow y - \ln(y+1) = \ln(e^x+1) + \ln c$$

$$\Rightarrow y = \ln(y+1) + \ln(e^x+1) + \ln c$$

$$\Rightarrow y = \ln(y+1) + \ln(e^x+1) + \ln \cdot c$$

$$y = \ln[(y+1)(e^x+1)c]$$

$$e^y = c(y+1)(e^x+1) \text{ Ans.}$$

x———x———x———x

(xi) $$\frac{dy}{dx} = \frac{y^3+2y}{x^2+3x}$$

$$\frac{dy}{y^3+2x} = \frac{dx}{x^2+3x}$$

By Partial fraction form (i) and (ii)

$$\left[\frac{1}{dy} - \frac{y}{2(y^2+3)}\right] dy = \left[\frac{1}{3x} - \frac{1}{3(x+3)}\right] dx$$

$$\int \frac{dy}{2y} - \int \frac{(2y)\,dy}{4(y^2+2)} = \int \frac{dx}{3x} - \int \frac{dx}{3(x+3)}$$

$$\frac{1}{2}\ln y - \frac{1}{4}\ln(y^2+2)^{\frac{1}{4}} = \ln x^{1/3} - \ln(x+3)^{1/3} + \ln c^{1/3}$$

$$\ln\left[\frac{y^{1/2}}{(y^2+2)^{1/4}}\right] = \ln\left[\frac{cx}{x+3}\right]^{1/3}$$

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$$\frac{y^{1/2}}{(y^2+2)^{1/4}} = \left(\frac{cx}{x+3}\right)^{1/3} \quad \text{Ans.}$$

x——x——x——x

(xii) $(\sin x + \cos x)dy + (\cos x - \sin x)dx = 0$

$\div$ by $(\sin x + \cos x)$

$$\int dy + \int \frac{\cos x - \sin x}{(\sin x + \cos x)} dx = \int 0 \, dx$$

$$y + \ln(\sin x + \cos x) = c$$

$$y \ln e + \ln(\sin x + \cos x) = c \ln e$$

$$\ln e^y + \ln(\sin x + \cos x) = \ln e^c$$

$$\ln[e^y(\sin x + \cos x)] = \ln e^c$$

$$e^y(\sin x + \cos x) = c'$$

$$e^y = \frac{c'}{\sin x + \cos x} \quad \text{Ans.}$$

x——x——x——x

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(xiii) $(1+2y^2)dy = y\cos x\, dx$

$\div$ by y

$\dfrac{(1+2y^2)}{y}\, dy = \cos x\, dx$

$\int\left(\frac{1}{y}+2\right)dy = \int \cos x\, dx$

$\ln y + \cancel{2}\frac{y^2}{\cancel{2}} = \sin x + C$

$y(0) = 1$

$\ln 1 + 1 = 0 + C$

$1 = C$

$\ln y + y^2 = \sin x + 1$ Ans.

x —— x —— x —— x —— x

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Exercise No 9.3

Q.1) A differential eq of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is said to be homogeneous diff eq of same degree.

$$(x-y)dx + (x+y)dy = 0$$

$$(x+y)dy = -(x-y)dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{--- (1)}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dy}{dx}$$

$$v + x\frac{dy}{dx} = \frac{vx - x}{x+vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(1+v)} - v$$

$$= \frac{\cancel{v}-1-\cancel{v}-v^2}{1+v} \Rightarrow \frac{v^2+1}{1+v}$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + c$$

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$\ln(v^2+1)^{1/2} + \tan^{-1}v + \ln x = c$

$\ln\sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$

$\ln\sqrt{y^2+x^2} - \ln\sqrt{x^2} - \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$

$\ln\sqrt{y^2+x^2} + \tan^{-1}\left(\frac{y}{x}\right) = c$ Ans.

x ——— x ——— x ——— x

Q.(ii) $(y^2+2xy)dx + x^2dy = 0$

$x^2dy = -(y^2+2xy)\,dx$

Put $y = vx$

$\frac{dy}{dx} = -\frac{(y^2+2xy)}{x^2}$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = -\frac{(v^2 x^2 + 2x\,vx)}{x^2}$

$x\frac{dv}{dx} = -\frac{\cancel{x^2}(v^2+2v)}{\cancel{x^2}} - v$

$x\frac{dv}{dx} = -(v^2+3v)$

$\int\frac{dv}{v^2+3v} = -\int\frac{dx}{x}$

$\int\frac{1}{v(v+3)}\,dv = -\int\frac{dx}{x}$

$\frac{1}{3}\int\frac{3}{v(v+3)}\,dv = -\int\frac{dx}{x}$

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$$\frac{1}{3}\int \frac{y+3-y}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int\left(\frac{1}{v} - \frac{1}{v+3}\right)dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(v+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$

Solving

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{1/3} = c(v+3)^{1/3}$$

$$x \cdot \left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x \cdot \frac{y^{1/3}}{x^{1/3}} = c(y-3x)^{1/3}$$

$$x y^{1/3} = c(y+3x)^{1/3}$$

$$x^3 y = c'(y+3x)$$ Ans.

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(iii) $(x^2-3y^2)dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2-3y^2)dx$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \xrightarrow{H.DE} (i)$$

Put $y = vx$ — (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (iii)}$$

using (ii) (iii) is (i)

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = \frac{3v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$$

$$\frac{y^2-x^2}{x^2} = cx$$

$$y^2 - x^2 = (cx)x^2 \quad \text{Ans.}$$

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(iv) $(x^2+xy+y^2)dx - x^2 dy = 0$

$(x^2+xy+y^2)dx = x^2 dy$

$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ — (i)

Put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (iii)

using (ii) (iii) and (i)

$v + x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \frac{(1+v+v^2)\cancel{x^2}}{\cancel{x^2}} - v$

$\int\frac{dv}{1+v^2} = \int\frac{dx}{x}$ solving using

$\tan^{-1} v = \ln x + c$

$\tan\left(\frac{y}{x}\right) = \ln x + c$ Ans.

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(iv) $(x^2+3y^2)dx - 2xy \quad dy = 0$

$$(x^2+3y^2)dx = 2xy\, dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{—(i)}$$

$$\text{Put } y = vx \quad \text{—(ii)}$$

$$\frac{dy}{dx} = v + x\frac{dy}{dx} \quad \text{—(iii)}$$

using (ii)(iii) is (i)

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{x^2(1+3v^2)-v}{x^2 2x}$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int\frac{2v}{1+v^2}dv = \int\frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\ln(1+v^2) = \ln cx$$

$$\left(\frac{1+y^2}{x}\right) = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

$$x^2+3y^2 = (x)x^2 \quad \text{Ans}$$

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(vi) $(x^2+3xy+y^2)dx - x^2dy = 0$

$(x^2+3xy+y^2)dx = x^2dy$

$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$ —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

using (ii) (iii) and (i)

$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \frac{x^2(1+3v+v^2)}{x^2} - v$

$x\frac{dv}{dx} = (1+2v+v^2)$

$\int\frac{dv}{(1+v)^2} = \int\frac{dx}{dx}$

$-\frac{1}{v+1} = \ln x + c$

$-\frac{1}{\frac{y}{x}+1} = \ln x + c$

$-\frac{1}{\frac{y+x}{x}} = \ln x + c$

$-\frac{x}{x+y} = \ln x + c$ Ans.

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(vii) $x \sin\left(\frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$

$$\frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{— (i)}$$

Put $y = Vx$ — (ii)

$$\frac{dy}{dx} = V + x\frac{dV}{dx} \quad \text{— (iii)}$$

using (ii) (iii) and (i)

$$V + \frac{x\,dv}{dx} = \frac{Vx \sin \frac{Vx}{x} - x}{x \sin\left(\frac{Vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{x(V\sin V - 1) - x}{x \sin v}$$

$$x\frac{dv}{dx} = \frac{V\sin V - 1 - V\sin V}{\sin V}$$

$$\int \sin dv = \int -\frac{dx}{x}$$

$$-\cos V = -\ln x + C$$

$$\cos V = \ln x \div C$$

$$\cos \frac{y}{x} = \ln x - C \quad \text{Ans.}$$

(viii) $\frac{dy}{dx} = \frac{4y - 3x}{2x - y}$ HDE — (i)

Put $y = Vx$ — (ii)

$$\frac{dy}{dx} = V + x\frac{dV}{dx} \quad \text{— (iii)}$$

using (ii) (iii) and (i)

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$$v + x\frac{dv}{dx} = \frac{4vx - 3x}{2x - vx}$$

$$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v - 3 - 2v + v^2}{2v}$$

$$\int \frac{2-v}{v^2+3v-3}\,dv = \int \frac{dx}{x} \quad \text{— (iv)}$$

By Parti function.

$$-\frac{5}{4}\int \frac{dv}{v+3} + \frac{1}{4} \pm \int \frac{dx}{x}$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{-5}{(v+3)} + \frac{1}{4(v-1)}$$

Put $v+3=0 \Rightarrow +5 = -4A \Rightarrow A = -\frac{5}{4}$

Put $v-1=0 \Rightarrow 1 = 4B \Rightarrow B \Rightarrow \frac{1}{4}$

$$-\frac{5}{4}\int \frac{dv}{v+3} + \frac{1}{4} + \int \frac{dx}{x}$$

$$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-1) = \ln x + \ln c$$

$$\ln(v+3)^5 + \ln(v-1) = 4\ln cx$$

$$\ln\left(\frac{v-1}{(v+3)^5}\right) = \ln c^4 x^4$$

$$\frac{\left(\frac{y}{x}-1\right)}{\left(\frac{y}{x}+3\right)^5} = c^4 x^4$$

$$\frac{(y-x)x^5}{x(y+3x)^5\cancel{x}} = c'$$

(ix)

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$$\frac{(y-x)^{x^4}}{(y+3x)^5} = c' \text{ Ans.}$$

x———x———x———x

(ix) $$\frac{dy}{dx} = \frac{x+y}{x} \quad \text{—(i)} \qquad y(1)=1$$

Put $y = vx$ —(ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—(iii)}$$

using (ii) (iii) is (i)

$$v + x\frac{dv}{dx} = \frac{x+vx}{x}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(1+v)}{\cancel{x}} - v$$

$$x\frac{dv}{dx} = 1$$

$$\int\frac{dv}{dx} = \int\frac{dx}{x}$$

$$v = \ln x + c$$

$$\frac{y}{x} = \ln x + c$$

$$y(1) = 1$$

$$\frac{1}{1} = \ln 1 + c$$

$$1 = 0 + c$$

So $\frac{y}{x} = \ln x + 1$

$$y = x\ln x + x = x(\ln x + 1) \text{ Ans}$$

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Exact Diff Eq

A diff eq of the form

$M(x,y)\,dx + N(x,y)\,dy = 0$

is Side to be an Exact diffa eq

is it a eqvation in total diffa Eqation

$[d(f(x),)] = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

Condition for an Exact Diffg Eq

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

To solve

(i) Integration M.w.r.t is keeping

M, N, have I at a same Gradian Partem distribution

$M = \frac{\partial f}{\partial x}, \quad N = \frac{\partial f}{\partial y}$

$\frac{\partial M}{\partial y} = \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial N}{\partial x} = \frac{\partial^2 f}{\partial x \partial y}$

Ex. (9.4)

(1) Solve $(3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$

$M = 3x^2 + 4xy, \quad N = 2x^2 + 2y$

$\frac{\partial M}{\partial y} = 0 + 4x \qquad \frac{\partial N}{\partial x} = 4x + 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So Given Diffg its is Exact

Now $\int M\,dx + \int$ (tan is of N face from x) $dy = C$

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$$\int (3x^2+4xy)\,dx + \int 2y\,dy = c$$

$$3\frac{x^3}{3} + \frac{4x^2y}{2} + \frac{2y^2}{2} = c$$

$$x^3 + 2x^2y + y^2 = c \text{ Ans.}$$

x——x——x——x——x

(ii) $(2xy + y^2\tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

$M = 2xy + y - \tan,\quad N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y,\quad \frac{\partial N}{\partial x} = 2x - \tan^2 y + 0$

$= 2x - \tan^2 y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ So given Eq is Exact

$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int (2xy + y - \tan y)\,dx + \int \sec^2 y\,dy = c$

$\frac{2x^2y}{2} + xy - x\tan y + \tan y = c$

$x^2y + xy - x\tan y + \tan y = c$ Ans

x——x——x——x——x

(iii) $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$M = \frac{x+y}{y-1}\qquad N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$

$N = -\frac{1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1) - (x+y)(1)}{(y-1)^2},\quad \frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$

$$= \frac{y-1-x-y}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ given diff eq is Exact

$$\int M dx + \int (\text{term of } N \text{ free from } x)\, dy = c$$

$$\int \left(\frac{x+y}{y-1}\right) dx + \int -\frac{1}{2(y-1)^2} dy = c$$

$$\left(\frac{1}{y-1}\right) \int (x+y)\, dx + \left(-\frac{1}{2}\right) \int (y-1)^{-2} dy = c$$

$$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2} + xy\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2 + 2xy + 1 = c'(y-1) \quad \text{Ans.}$$

x——x——x——x

(iv) $$\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$$

$$(hx+by)\,dy = -(ax+hy)\,dx$$

$$(ax+hy)\,dx + (hx+by)\,dy = 0$$

$$M = ax+hy \qquad N = hx+by$$

$$\frac{\partial M}{\partial y} = 0 + h \qquad \frac{\partial N}{\partial x} = h$$

(v)

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$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{is Exact Dig}$$

$$\int M\,dx + \int (\text{trm of } N \text{ free from } x)\,dy = c$$

$$\int (ax + hy)\,dx + \int by\,dy = c$$

$$a\frac{x^2}{2} + hxy + \frac{by^2}{2} = c$$

$$ax^2 + 2hxy + by^2 = c'' \quad \text{Ans.}$$

(V)

$$(1 + \ln x)\,dx + \left(1 + \frac{x}{y}\right)dy = 0$$

$$M = 1 + \ln xy \qquad N = 1 + \frac{x}{y}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Hence Exact Dig Eq}$$

$$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$$

$$\int (1 + \ln xy)\,dx + \int 1.\,dy = c$$

$$\int dx + \int 1.\ln xy\,dx + \int dy = c$$

$$x + \left[\ln xy.\,(x) - \int \frac{1}{xy}.\,y.x\,dx\right] + y = c$$

$$x + x\ln xy - \int dx + y = c$$

$$\cancel{x} + x\ln xy - \cancel{x} + y = c$$

$$x\ln xy + y = c \quad \text{Ans.}$$

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(vi) $(6xy+2y^2-5)\,dx+(3x^2+4xy-6)\,dy=0$

$M=6xy+2y^2-5$, $N=3x^2+4xy-6$

$\frac{\partial M}{\partial y}=6x+4y$ $\frac{\partial N}{\partial x}=6x+4y$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Have Exact Diff. eqt

$\int M\,dx+\int(\text{terms of } N \text{ free from } x)\,dy=c$

$\int(6xy+2y-5)\,dx+\int -6\,dy=c$

$\frac{6x^2y}{2}+2xy^2-5x-6y=c$

$3x^2y+2xy^2-5x-6y=c$ Ans.

x — x — x — x — x

(vii) $(y\sec^2 x+\sec\tan x)\,dx+(\tan x+2y)\,dy=0$

$M=y\sec^2 x+\sec x\tan x$, $N=\tan x+2y$

$\frac{\partial M}{\partial y}=\sec^2 x$ $\frac{\partial N}{\partial x}=\sec^2 x$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Have Exact Diffeqt

$\int M\,dx+\int(\text{terms of } N \text{ free from } x)\,dy=C$

$\int(y\sec^2 x+\sec x\tan x)\,dx+\int 2y\,dy=c$

$y\tan x+\sec x+y^2=C$ Ans.

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(viii) $(y\cos x + 2xy)\,dx + (\sin x + x^2 e^{y-1})\,dy = 0$

$M = y\sec^2 x + \sec x \tan x$, $N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x \qquad \frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hase Exact D.E.q

$\int M\,dx + \int (\text{terms of } N \text{ free from } x)\,dy = C$

$\int (y\sec^2 x + \sec x \tan x)\,dx + \int 2y\,dy = C$

$y\tan x + \sec x + y^2 = C$ Ans.

Ex 9.5

Slove by finding an I.f

(1) $(xy^2 + y)\,dx - x\,dy = 0$

$M = xy^2 + y \qquad N = -x$

$M_y = 2xy + 1 \qquad N_x = -1$

$M_y \neq N_x \quad \therefore$ Non Exact

$\frac{M_y - N_x}{N} = \frac{2xy + 1 + 1}{-x}$ Nothin x other

$\frac{N_x - M_y}{M} = \frac{-1 - 2xy - 1}{xy^2 + x}$

$= \frac{-2(1 + xy)}{y(xy + 1)} = \frac{-2}{y}$

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$-\int \frac{2}{y} dy \quad -2\log y \quad \log y^{-2} \quad = y^{-2} = \frac{1}{y^2}$

I.f $= e \quad = e \quad = e$

Multiply both side of eq(i) by I.f $= \frac{1}{y^2}$

$\frac{1}{y^2}(xy^2+y)dx - \frac{x}{y^2}dy = 0$

$\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0$

$\left(x+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0$ — (ii)

Now $M = x+\frac{1}{y}$ $\quad N = -\frac{1}{y^2}$

$M_y = N_x$ ∴ Exact using Eq

$\int M dx + \int$(terms of N free from x) $dy = 0$

$\int \left(x+\frac{1}{y}\right)dx + Nil = C$

$\frac{x^2}{2} + \frac{x}{y} = C$ Ans.

(ii) $(x^2+x-y)dx + x\,dy = 0$ — (i)

$M = x^2+x-y$ $\quad N = x$

$M_y = -1$ $\quad N_x = 1$

$M_y \neq N_x$ ∴ Non Exact Diff Eq

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$ fct of x alone

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$I.f = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

Multiply both Side of eq (i) by I.f $= \frac{1}{x^2}$

$$\frac{1}{x^2}(x^2+x-y)\,dx + \frac{x}{x^2}\,dy = 0$$

$$\left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + \frac{1}{x}\,dy = 0 \quad \text{—(ii)}$$

Now $M = 1+\frac{1}{x}-\frac{y}{x^2}$ $\quad N = \frac{1}{x}$

$M_y = -\frac{1}{x^2}$ $\quad N_x = -\frac{1}{x^2}$

$M_y = \frac{-1}{x^2}$ $\quad N_x = \frac{-1}{x^2}$

$M_y = N_x$ $\quad \therefore$ Exact Diff Eq

$\int M dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int \left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + \text{Nil} = c$

$x + \ln x + \frac{y}{x} = c$ Ans.

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$$dy + \left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{— (i)}$$

(حل)

$$M = \frac{y - \sin x}{x} \qquad N = 1$$

$$M_y = \frac{1}{x} - 0 \qquad N_x = 0$$

$M_y \neq N_x \therefore$ (i) is Non Exact Diff eq

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$ mg

$$I.f = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Multpling both Sides g of (i) by I.f. x

$$x\,dy + x\left(\frac{y - \sin x}{x}\right) dx = 0 \quad \text{—(ii)}$$

$$M = y - \sin x \qquad N = x$$

$$M_y = 1 \qquad N_x = 1$$

$M_y = N_x \therefore$ (ii) is Exact Digget

$$\int M\,dx + \int (\text{tames of } N \text{ free form})\,dy = c$$

$$\int (y - \sin x)\,dx = C$$

$$xy + \cos x = C \quad \text{Ans.}$$

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(iv) $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$

$M=y^4+2y \qquad N=xy^3+2y^4-4x$

$M_y=4y^3-2 \qquad N_x=y^3-4$

$M_y \neq N_x \quad \therefore$ (1) is Non Exact Diff

$\dfrac{N_x-M_y}{M}=\dfrac{y^3-4-4y^3-2}{y^4+2y}=\dfrac{-3y^3-6}{y(y^3+2)}=-\dfrac{3}{y}$

$I.f=e^{\int-\frac{3}{y}dy}=e^{-3\ln y}=e^{\ln y^{-3}}=y^{-3}=\boxed{\dfrac{1}{y^3}}$

$\dfrac{1}{y^3}(y^4+2y)dx+\dfrac{1}{y^3}(xy^3+2y^4-4x)dy=0$

$\left(y+\dfrac{2}{y^2}\right)dx+\left(x+2y-\dfrac{4x}{y^3}\right)dy=0$

Now $M=y+\dfrac{2}{y^2} \qquad N=x+2y-\dfrac{4x}{y^3}$

$M_y=1-\dfrac{4}{y^3} \qquad N_x=1+0-\dfrac{4}{y^3}$

$M_y=N_x \quad \therefore$ is Exact Diff

$\int\left(y+\dfrac{2}{y^2}\right)dx+\int 2y\,dy=c$

$xy+\dfrac{2x}{y^2}+\dfrac{2y^2}{2}=c$

$xy+\dfrac{2x}{y^2}+y^2=c$ Ans

(v) $(4x+3y^2)\,dx + 2xy\,dy = 0$

$M = 4x+3y^2 \qquad N = 2xy$

$M_y = 0+6y \qquad N_x = 2y$

$M_y \neq N_x \quad \therefore$ Non Exact Disgtin

$\dfrac{N_x - M_y}{M} = \dfrac{2y-6y}{4x+3y^2}$ not of Exact Disgtion

$\dfrac{M_y - N_x}{N} = \dfrac{6y-2y}{2xy} = \dfrac{4y}{2xy} = \dfrac{2}{x}$

$I.f = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

Mulpling both Sides (i) by $I.f = x^2$

$(4x^3 + 3y^2x^2)\,dx + (2x^3y)\,dy = 0$

$M_y = 6yx^2 \qquad N_x = 6x^2y$

$M_y = N_x \quad \therefore$ Exact Disgt E q

$\int M\,dx + \int (\text{tams of } N \text{ free from } x)\,dy = C$

$\int (4x^3 + 3y^2x^2)\,dx + Nil = C$

$\cancel{4}\dfrac{x^4}{\cancel{4}} + \cancel{3}y^2\dfrac{x^3}{\cancel{3}} = C$

$x^4 + y^2x^3 = C$ Ans

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(vi) $(x^2+y^2)\,dx - 2xy\,dy = 0$ --- (i)

$M = x^2+y^2 \qquad N = -2xy$

$M_y = 2y \qquad N_x = -2y$

$M_y \neq N_x$ ∴ is Non Exact Digation

$\frac{N_x - M_y}{M} = \frac{-2y-2y}{x^2+y^2}$ not ful on x

$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = \frac{-2}{x}$ for fonty

$I.f = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

Muliply both Saids of eq (i) by

$I.f = \frac{1}{x^2}$

$\frac{1}{x^2}(x^2+y^2)\,dx - \frac{1}{x^2}(2xy)\,dy = 0$

$(1+y^2)\,dx - \frac{2y}{x}\,dy = 0$

$M = 1+\frac{y^2}{x^2} \qquad N = -\frac{2y}{x}$

$M_y = \frac{2y}{x^2} \qquad N_x = \frac{2y}{x^2}$

$M_y = N_x$ ∴ is Exact Digahet

$\int M\,dx + \int(\text{terms of } N \text{ free from } x)\,dy = c$

$\int \frac{(1+y^2)}{x^2}\,dx + Nil = c$

$x - \frac{y^2}{x} = c$ Ans

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(vii) $\frac{dy}{dx} = e^{2x} + y - 1$

$dy = (e^{2x} + y - 1)dx$

$(e^{2x} + y - 1)dx - dy = 0$

$M = e^{2x} + y - 1 \qquad N = -1$

$M_y = 1 \qquad N_x = 0$

$M_y \neq N_x \therefore$ is Non Exact Diff eqst

$\frac{M_x - N_y}{M} = \frac{0-1}{e^{2x}+y-1}$ Not y alone

$\frac{M_y - N_x}{N} = \frac{1-0}{-1} = -1 = x^0$ for fn x alone

$I.f = e^{\int -1\, dx} = e^{-x}$

Multipling both sides of eq (1) by e^{-x}

$e^{-x}(e^{2x} + y - 1)dx - e^{-x}dy = 0$

$(e^{x} + e^{-x}y - e^{-x})dx - e^{-x}dy = 0$

$M = e^{x} + e^{-x}y - e^{-x} \qquad N = e^{-x}$

$M_y = e^{-x} \qquad N_x = +e^{-x}$

$M_y = N_x \therefore (1)$ is Exact Diff eqn

So $\int M\,dx + \int$ (terms of N free from x) $dy = c$

$\int (e^{x} + e^{-x}y - e^{-x})dx + Nil = c$

$e^{x} - e^{-x}y + e^{-x} = c$ Ans.

(viii) $(y^2 + xy)\,dx - x^2\,dy = 0$

$\frac{dy}{dx} = \frac{y^2 + xy}{x^2}$ HDE — (i)

Put $y = vx$ — (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{v^2x^2 + xvx}{x^2}$

$v + x\frac{dv}{dx} = \frac{\cancel{x^2}(v^2 + v)}{\cancel{x^2}}$

$x\frac{dv}{dx} = v^2 + \cancel{v} - \cancel{v}$

$\frac{dv}{v^2} = \frac{dx}{x}$

$\int v^{2}\,dv = \int \frac{dx}{x}$

$-\frac{1}{v} = \ln x + C$

$0 = \ln x + \frac{1}{v} + C$

$\ln x + \frac{x}{y} + C$

$y = vx$

$\frac{y}{x} = v$ Ans.

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(iv) $\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$ (LDE in 1)

$I.f = e^{\int p\,dx} = e^{\int 3\,dx} = e^{3x}$

Solving by $\int d(y \times I.F) = \int Q \times I.F\,dx + c$

$= \int d(ye^{3x}) = \int 3x^2 \cancel{e^{-3x}} \cancel{e^{3x}}\,dx + c$

$ye^{3x} = x^3 + c$

$y = e^{-3x}(x^3 + c)$ Ans.

(v) $(x+1)\frac{dy}{dx} - ny = e^x(x+1)^{n+1}$

$\frac{dy}{dx} - \frac{n}{x+1}y = e^x(x+1)^n$

$I.f = e^{\int p\,dx} = e^{-\int \frac{n}{x+1}dx} = e^{-n\ln(x+1)} = e^{\ln(x+1)^{-n}}$

$I.f = (x+1)^{-n} = \frac{1}{(x+1)^n}$

Sol is given $\int d(y \times I.f) = \int Q \times I.f\,dx + C$

$\int d\left(y \cdot \frac{1}{(x+1)^n}\right) = \int e^x \cancel{(x+1)^n} \cdot \frac{1}{\cancel{(x+1)^n}}\,dx + C$

$\frac{y}{(x+1)^n} = e^x + C$

$y = (e^x + c)(x+1)^n + c$ Ans.

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$$\int d(yx^3) = \int 6x^2 \cdot x^3 dx + C$$

$$yx^3 = \int 6x^5 dx + C$$

$$yx^3 = \frac{6x^6}{6} + C$$

$$x^3 y = x^6 + C \quad Ans.$$

(iii) $\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$ (DE = 1)

$$e^{\int P dx} = e^{\int \frac{1}{x \ln x} dx} = e^{\int \frac{dx/x}{\ln x}}$$

$$I.f = e^{\ln(\ln x)} = \ln x$$

solution by $\int d(y x \, I.f) = \int Q \times I.F \, dx + C$

$$\Rightarrow \int d(y \ln x) = \int \frac{3x^2}{\cancel{\ln x}} \cancel{\ln x} \, dx + C$$

$$y \ln x = \frac{3x^3}{3} + C$$

$$y = \frac{x^3 + C}{\ln x} \quad Ans.$$

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(vi) $x\frac{dy}{dx} + (1 + x\cot x)y = x$

$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$

$I.f = e^{\int P dx} = e^{\int(\frac{1}{x} + \cot x)} = e^{\ln x + \ln \sin x}$

$I.f = e^{\ln(x \sin x)} = x\sin x$

Solution is given by $\int d(y \times I.f) = \int Q \times I.f \, dx + c$

$\int d(y x \sin x) = \int \frac{x}{x} \sin x \, x \, dx + c$

$y x \sin x = x(-\cos x) - \int 1(-\cos x) \, dx$

$= x(-\cos x) + \int \cos x \, dx$

$y x \sin x = x\cos x + \sin x + c$

$y = -\cot x + \frac{1}{x} + \frac{c}{x}\cot x$ Ans

x——x——x——x——x

(vii) $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)dy = \frac{4x^2}{x^2+1}$ (LDE - y)

$I.f = e^{\int \left(\frac{2x}{x^2+1}\right) dx} = e^{\ln(x^2+1)} = x^2+1$

Sol is given by $\int d(y\, I.F) = \int Q \cdot I.f\, dx + C$

$\Rightarrow \int d\left(y(x^2+1)\right) = \int \frac{4x^2}{(x^2+1)} (x^2+1)\, dx + C$

$y(x^2+1) = \frac{4x^3}{3} + C$

$3y(x^2+1) = 4x^3 + C$ Ans.

(viii) $x \frac{dy}{dx} + 2y = \sin x$

$\frac{dy}{dx} + \frac{2}{x} y = \frac{\sin x}{x}$ (LDE is y)

$If = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

Sol is given by $\int d(y \times I.f) = \int Q \times I.f\, dx + C$

$\Rightarrow \int d(yx^2) = \int \frac{\sin x}{x} x^2\, dx + C$

$yx^2 = \int x \sin x\, dx + C$

$yx^2 = x(-\cos x) - \int 1(-\cos x)\, dx + C$

$y = \frac{1}{x^2}\left(-x\cos x + \sin x + C\right)$ Ans

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