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APPLIED TECHNOLOGY & MATHAMATICS-2

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GCUF

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Ex9.2

① $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

Sol:- ~~$y\,dy$~~ $= \frac{x^2}{1+x^3}dx$

$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}dx$

$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$

$\frac{3y}{2} = \ln(1+x^3)+c$

$3y^2 = 2\ln(1+x^3)+6c$

$3y^2 = 2\ln(1+x^3)+c$

② $\frac{dy}{dx} + y^2\sin x = 0$

$\frac{dy}{dx} + y^2\sin x = 0$

$\frac{dy}{dx} = -y^2\sin x$

$\int \frac{dy}{y^2} = -\int \sin x\,dx$

$\frac{y^{-1}}{-1} = -(-\cos x)+c$

$-\frac{1}{y} = \cos x + c$

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Q#③ $\frac{dy}{dx} = 1+x+y^2+xy^2$

$\frac{dy}{dx} = (1+x)+y^2(1+x)$

$\frac{dy}{dx} = (1+x)(1+y^2)$

$\int \frac{dy}{\cancel{dx}\ 1+y^2} = \int (1+x)\,dx$

$\tan^{-1}y = x+\frac{x^2}{2}+C$

$2\tan^{-1}y = 2x+x^2+C$

④ $(xy+2x+y+2)dx+(x^2+2x)\,dy = 0$

$[x(y+2)+(y+2)]dx+x(x+2)\,dy = 0$

$[(y+2)(x+1)dx+x(x+2)\,dy = 0$

$\div$ by $x(x+2)(y+2)$

$\frac{x+1}{x(x+2)}dx+\frac{1}{y+2}dy = 0$

$\int \frac{x+1}{x^2+2x}dx+\int \frac{dy}{y+2} = 0$

$\frac{1}{2}\int \frac{2x+2}{x^2+2x}dx+\int \frac{dy}{y+2} = 0$

$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x)+\ln c$

$y+2 = \frac{c}{\sqrt{x^2+2x}}$

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⑤ $\operatorname{Cosec} y \, dx + \operatorname{Sec} x \, dy = 0$

$\div$ by Cosec secx

$\Rightarrow \frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$

$= \int \cos x \, dx + \int \sin dy = \int 0 \, dx$

$= \sin x - \cos y = C$ general sol

⑥ $y(1+x)dx + x(1+y)dy = 0$

$\div$ by xy

$= \frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy = 0$

$\Rightarrow \int\left(\frac{1}{x} + 1\right) dx + \int\left(\frac{1}{y} + 1\right) dy = \int 0 \, dx$

$\Rightarrow \ln x + x + \ln y + y = C$

$x + y + \ln(x+y) = C$

⑦ $(e^x+1) y \, dy = (y+1) e^x dx$

$\div$ by $(e^x+1)(y+1)$

$= \int \frac{y \, dy}{y+1} = \int \frac{e^x \, dx}{e^x+1}$

$= \int\left(\frac{y+1-1}{y+1}\right) dy = \int \frac{e^x}{e^x+1} dx$

$= \int\left(1 - \frac{1}{y+1}\right) dy = \int \frac{e^x}{e^x+1} dx$ →

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$$\Rightarrow y - \ln(y+1) = \ln(e^x+1) + \ln c$$
$$\Rightarrow y = \ln(y+1) + \ln(e^x+1) + \ln c$$
$$\Rightarrow y = \ln[(y+1)(e^x+1)c]$$

$$\Rightarrow e^y = c(y+1)(e^x+1)$$

⑱ $xe^{x^2+y}dx = ydy$

$$xe^{x^2}e^y dx = ydy$$
$$xe^{x^2}dx = ye^{-y}dy$$

$$\frac{1}{2}\int e^{x^2}(2x)dx = \int ye^{-y}dy$$

$$\because \begin{cases} e^{x^2} = t \quad e^{x^2}2xdx = dt \\ \int e^{x^2}2xdx = \int dt = t\ e^{x^2} \end{cases}$$

so $\frac{1}{2}e^{x^2} = ye^{-y} - \int 1.\frac{e^{-y}}{-1}dy$

$$= -ye^{-y} + \int e^{-y}dy$$

$$= -ye^{-y} + \frac{e^{-y}}{-1} + c$$

$$\frac{1}{2}e^{x^2} = -ye^{-y} - e^{-y} + c$$

$$e^{x^2} = -2e^{-y}(y+1) + 2c$$
$$e^{x^2} = -2e^{-y}(y+1) + c'$$

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Ex # 9.3

① $(x-y)dx+(x+y)dy=0$

$$(x+y)\,dy=-(x-y)\,dx$$

$$\frac{dy}{dx}=\frac{y-x}{y+x} \text{ —— } ①$$

Put $y=vx$ —— ⑪

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \text{ —— } ⑪⑪$$

using ⑪ ⑪⑪ in ①

$$v+x\frac{dv}{dx}=\frac{vx-x}{x+vx}$$

$$x\frac{dv}{dx}=\frac{x(v-1)}{x(1+v)}-v$$

$$=\frac{v-1-v-v^2}{1+v}$$

$$x\frac{dv}{dx}=-\frac{(v^2+1)}{1+v}$$

$$\int\frac{v+1}{v^2+1}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v\,dv}{v^2+1}+\int\frac{dv}{v^2+1}=-\int\frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1)+\tan^{-1}v=-\ln x+c$$

$$\ln(v^2+1)^{\frac{1}{2}}+\tan^{-1}v+\ln x=c$$

$$\ln\sqrt{\frac{y^2}{x^2}+1}+\tan^{-1}\left(\frac{y}{x}\right)+\ln x=c$$

$$\ln\sqrt{y^2+x^2}-\ln\sqrt{x^2}+\tan^{-1}\left(\frac{y}{x}\right)+\ln x=c$$

$$\ln\sqrt{y^2+x^2}+\tan^{-1}\left(\frac{y}{x}\right)=c$$

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② $(x^2-3y^2)dx + 2xy\,dy = 0$

$2xy\,dy = -(x^2-3y^2)dx$

$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy}$ —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dy}{dx}$ —— (iii)

using (ii) (iii) in (i)

$v + x\frac{dv}{dx} = \frac{3v^2x^2-x^2}{2xvx}$

$x\frac{dv}{dx} = \frac{(3v^2-1)x^2}{2vx^2} - v$

$x\frac{dv}{dx} = \frac{v^2-1}{2v}$

$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$

$\ln(v^2-1) = \ln x + \ln c$

$\ln\left(\frac{y^2}{x^2}-1\right) = \ln cx$

$\frac{y^2-x^2}{x^2} = \ln cx$

$y^2-x^2 = (cx)x^2$

③ $(x^2+xy+y^2)\,dx - x^2\,dy = 0$

$(x^2+xy+y^2)\,dx = x^2\,dy$

$\frac{dy}{dx} = x^2+xy+y^2$ HDE ①

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

using (ii) (iii) in ①

$v + x\frac{dv}{dx} = x^2 + x(vx) + v^2x^2$

$x\frac{dv}{dx} = \frac{(1+v+v^2)\cancel{x^2}}{\cancel{x^2}} - v$

$\int\frac{dv}{1+v^2} = \int\frac{dx}{x}$

$\tan^{-1}v = \ln x + c$

$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$

④ $(x^2+3xy+y^2)\,dx - x^2\,dy = 0$

$(x^2+3xy+y^2)\,dx = x^2\,dy$

$\frac{dy}{dx} = x^2+3xy+y^2$ —— ①

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

using (iii) (ii) in ①

$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v+v^2)}{\cancel{x^2}} - v$

$x\frac{dv}{dx} = 1+2v+v^2$

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$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\frac{-1}{(v+1)} = \ln x + C$$

$$\frac{-1}{\left(\frac{y}{x}+1\right)} = \ln x + C$$

$$\frac{-1}{\frac{y+x}{x}} = \ln x + C$$

$$-\frac{x}{(x+y)} = \ln x + C$$

⑤ $x\sin\left(\frac{y}{x}\right)dy = \left(y\sin\frac{y}{x} - x\right)dx$

$$\frac{dy}{dx} = \frac{y\sin\frac{y}{x} - x}{x\sin\left(\frac{y}{x}\right)} \quad \text{———} ①$$

Put $y = vx$ ——— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ ——— (iii)

using (ii) (iii) in ①

$$v + x\frac{dv}{dx} = \frac{vx\sin\frac{vx}{x} - x}{x\sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{x^2(v\sin v - 1)}{x\sin v} - v$$

$$x\frac{dv}{dx} = \frac{v\sin v - 1 - v\sin v}{\sin v}$$

$$\int \sin v\, dv = \int -\frac{dx}{x}$$

$-\cos v = -\ln x + c$

$\cos v = \ln x - c$

$\cos \frac{y}{x} = \ln x - c$

⑥ $\frac{dy}{dx} = \frac{x+y}{x}$ —— $\textcircled{i}$ $y(1)=1$

Put $y = vx$ —— $\textcircled{ii}$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— $\textcircled{iii}$

Using $\textcircled{ii}$ $\textcircled{iii}$ in $\textcircled{i}$

$v + x\frac{dv}{dx} = \frac{x+vx}{x}$

$x\frac{dv}{dx} = \frac{x(1+v)}{x} - v$

$x\frac{dv}{dx} = \frac{\cancel{x}(1+v)}{\cancel{x}} - v$

$x\frac{dv}{dx} = 1$

$\int dv = \int \frac{dx}{x}$

$v = \ln x + c$

$\frac{y}{x} = \ln x + c$ $\quad \because y(1) = 1$

$\frac{1}{1} = \ln 1 + c$

$\boxed{1 = 0 + c}$

so $\frac{y}{x} = \ln x + 1$

$y = x\ln x + x = x[\ln x + 1]$ Ans

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⑦ $(y+\sqrt{x^2+y^2})dx - x\,dy = 0$

$(y+\sqrt{x^2+y^2})dx \underline{\ \ } x\,dy$

$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ ———— ①

Put $y = vx$ ———— ⓘⓘ

$\frac{dy}{dx} = v+x\frac{dv}{dx}$ ———— ⓘⓘⓘ

using ⓘⓘ ⓘⓘⓘ in ①

$v+x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x}$

$= x(v+\sqrt{1+v^2})$

$x\frac{dv}{dx} = v+\sqrt{1+v^2}\,\underline{\ x\ }\,v$

$\int\frac{dv}{\sqrt{1+v^2}} = \int\frac{dx}{x}$

$\sinh^{-1} v = \ln x + c$

$\sinh^{-1}(\frac{y}{x}) = \ln x + c$

$\therefore \sinh^{-1}(\frac{y}{x}) = \ln x$

$\Rightarrow \ln(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}) = \ln x$

Puting

$\frac{y+\sqrt{x^2+y^2}}{x} = x$

$y \pm \sqrt{x^2+y^2} = x^2$

$y = x^2 - \sqrt{x^2+y^2}$

$\therefore y(1) = 0$

$\therefore \sinh^{-1}(\frac{0}{1}) = \ln 1 + c$

$\boxed{0 = c}$

$\therefore \sin^{-1}x = \ln(x+\sqrt{x^2+1})$

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⑧ $(x^2+3y^2)dx-2xy\,dy=0$

$(x^2+3y^2)dx=2xy\,dy$

$\dfrac{x^2+3y^2}{2xy}=\dfrac{dy}{dx}$ ——— (i)

Put $y=vx$ ——— (ii)

$\dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$ ——— (iii)

using (ii) (iii) in (i)

$v+x\dfrac{dv}{dx}=\dfrac{x^2+3v^2x^2}{2xvx}$

$x\dfrac{dv}{dx}=\dfrac{\cancel{x^2}(1+3v^2)}{\cancel{x^2}2v}-v$

$x\dfrac{dv}{dx}=\dfrac{1+3v^2-2v^2}{2v}$

$x\dfrac{dv}{dx}=\dfrac{1+v^2}{2v}$

$\int\dfrac{2v}{1+v^2}dv=\int\dfrac{dx}{x}$

$\ln(1+v^2)=\ln x+\ln c$

$\ln(1+v^2)=\ln x+\ln c$

$\left(1+\dfrac{y^2}{x^2}\right)=cx$

$\dfrac{x^2+y^2}{x^2}=cx$

$x^2+y^2=(cx)x^2$

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Ex#9.4

① $(3x^2+4xy)dx+(2x^2+2y)dy=0$

$M = 3x^2+4xy$, $N = 2x^2+2y$

$\frac{2M}{2y} = 0+4x$ $\frac{2N}{2x} = 4x+0$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so given diff Ev is Exact

Now $\int M dx + \int (\text{tan of } N \text{ free from } x)\, dy = C$

$\because \int (3x^2+4xy)dx + \int 2y dy = C$

$3\frac{x^3}{3} + \frac{4x^2y}{2} + \frac{2y^2}{2} = C$

$\cancel{3}x^3 + 2x^2y + y^2 = C$

② $(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$

$M = 2xy+y-\tan y$, $N = x^2-x\tan^2 y+\sec^2 y$

$\frac{2M}{2y} = 2x+1-\sec^2 y$, $\frac{2N}{2x} = 2x-\tan^2 y+0$

$= 2x-\tan^2 y$

$\therefore \frac{2M}{2y} = \frac{2N}{2x}$ · so given diff eq is Exact

$\int M dx + \int (\text{tan of } N \text{ free from } x)\, dy = 0$

$\int (2xy+y-\tan y)dx + \int \sec^2 y\, dy = C$

$\frac{2x^2y}{2} + xy - x\tan y + \tan y = C$

$x^2y + xy - x\tan y + \tan y = C$

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③ $\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy = 0$

$M = \frac{x+y}{y-1}$ $\quad N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$

$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}$ $\quad N = -\frac{1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$

$= \frac{y-1-x-y}{(y-1)^2}$ $\quad \frac{\partial N}{\partial x} = \frac{-(2x+2)}{2(y-1)^2}$

$\quad = \frac{-x-1}{(y-1)^2}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ∴ given diff eq is Exact

$\int M dx + \int$ (tan of free from x) $dy = c$

$\int \frac{x+y}{y-1} dx + \int -\frac{1}{2(y-1)^2} = c$

$\left(\frac{1}{y-1}\right)\int (x+y)dx + \left(-\frac{1}{2}\right)\int (y-1)^{-2} dy = c$

$\left(\frac{1}{y-1}\right)\left(\frac{x^2}{2}+xy\right) + \left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right) = c$

$\frac{x^2+2xy}{2y-1} + \frac{1}{2(y-1)} = c$

$x^2+2xy+1 = c'(y-1)$ Ans

④ $\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$

$(hx+by)dy = -(ax+hy)dx$

$(ax+hy)dx + (hx+by)dy = 0$

$M = ax+hy$ $\quad N = hx+by$

$\frac{\partial M}{\partial y} = 0+h$ $\quad \frac{\partial N}{\partial x} = h$

∴ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hance Exact diff Eq

$\int M dx + \int$ (term of N free form x) $dy = c$

$\int (ax+hy)dx + \int by\, dy = c$

$a\frac{x^2}{2} + hxy + b\frac{y^2}{2} = c \Rightarrow ax^2+2hxy+by^2 = c'$ Ans

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⑤ $(1+\ln xy)dx + (1+\frac{x}{y})dy = 0$

$M = 1+\ln xy$ $\quad N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy} x$ & $\frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hance Exact diff eq

$\int M dx + \int (\text{term of } N \text{ free form } x) dy = c$

$\int (1+\ln xy) dx + \int 1 \, dy = c$

$\int dx + \int 1 \cdot \ln xy \, dx + \int dy = c$

$x + [\ln xy \cdot (x) - \int \frac{1}{xy} \cdot y \cdot x \, dx] + y = c$

$x + x\ln xy - \int dx + y = c$

$x + x\ln xy - x + y = c$

$x \ln xy + y = c$

⑥ $(5xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6)$

$M = 6xy + 2y^2 - 5$, $\quad N = 3x^2 + 4xy - 6$

$\frac{\partial M}{\partial y} = 6x + 4y$ $\quad \frac{\partial N}{\partial x} = 6x + 4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact diff eq

$\int M dx + \int (\text{term of } N \text{ free form } x) dy = c$

$\int (6xy + 2y^2 - 5) dx + \int -6 dy = c$

$6x^2y + 2xy^2 - 5x - 6y = c$

$3x^2y + 2xy^2 - 5x - 6y = c$

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⑦ $(y\sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$

$M = y\sec^2 x + \sec x \tan x$, $N = \tan x + 2y$

$\frac{\partial M}{\partial y} = \sec^2 x$ $\frac{\partial N}{\partial x} = \sec^2 x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact diff eq

$\int M dx + \int (\text{term of N o free form } x) dy = \cancel{0}C$

$\int (y\sec^2 x + \sec \tan x) dx + \int 2y\, dy = C$

$y x \tan x + \sec x + y^2 = C$

⑧ $(y\cos x + 2xe^y)dx + (\sin x + x^2 e^y - 1)dy = 0$

$M = y\cos x + 2xe^y$, $N = \sin x + x^2 e^y - 1$

$\frac{\partial N}{\partial x} = \cos x + 2xe^y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence Exact diff eq

$\int M dx + \int (\text{term of N Free form } x) dy = C$

$\int (y\cos x + 2xe^y) dx \int -1\, dy = C$

$y\sin x + \frac{2x^2}{2} e^y - y = C$

$y\sin x + x^2 e^y - y = C$

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Ex # 9.5

Solve by finding an I.F

① $(xy^2+y)\,dx - x\,dy = 0$ —— ①

$M = xy^2+y \qquad N = -x$

$M_y = 2xy+1 \qquad N_x = -1$

$\therefore M_y \neq N_x \quad \therefore$ Non Exact

$\frac{M_y - N_x}{N} = \frac{2xy+1+1}{-x}$ Not for of x axis

$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$

$= \frac{-2(1+xy)}{y(xy+1)} = \frac{-2}{y}$

$\therefore I.F = e^{-\int \frac{2}{y} dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$

Multiply by both sides of eq ① by $I.F = \frac{1}{y^2}$

$\frac{1}{y^2}(xy^2+y)\,dx - \frac{x}{y^2}\,dy = 0$

$(x + \frac{1}{y})\,dx - \frac{x}{y^2}\,dy = 0$ —— ①

Now $M = x + \frac{1}{y} \qquad N = -\frac{x}{y^2}$

$M_y = -\frac{1}{y^2} \qquad N_x = \frac{-1}{y^2}$

$\therefore M_y = N_x \therefore$ Exact Diff eq

so $\int M\,dx + \int(\text{terms of } N \text{ free from } x)\,dy = c$

$\int (x + \frac{1}{y})\,dx + \text{Nil} = c$

$\frac{x^2}{2} + \frac{x}{y} = c$

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② $(x^2+x-y)\,dx + x\,dy = 0$

$M = x^2+x-y \quad N = x$

$M_y = -1 \quad N_x = 1$

$M_y \neq N_x$ ∴ Non Exact diff eq

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$ fun of x alone

$\therefore I.F = e^{\int \frac{-2}{x} dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

Multiply both sides of eq ① by $I.F = \frac{1}{x^2}$

$\frac{1}{x^2}(x^2+x-y)\,dx + \frac{x}{x^2}\,dy = 0$

$\left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + \frac{1}{x}\,dy = 0$

Now $M = 1+\frac{1}{x}-\frac{y}{x^2} \quad N = \frac{1}{x}$

$M_y = \frac{-1}{x^2} \quad N_x = \frac{-1}{x^2}$

$M_y = N_x$ ∴ ① is Exact diff Eq

∴ $\int M dx + \int (\text{term of } N \text{ free from } x)\,dy = c$

$\int\left(1+\frac{1}{x}-\frac{y}{x^2}\right)dx + Nil = C$

$x + \ln x + \frac{y}{x} = C$

③ $dy + \left(\frac{y-\sin x}{x}\right)dx = 0$

$M = \frac{y-\sin x}{x} \quad N = 1$

$M_y = \frac{1}{x} - 0 \quad N_x = 0$

$M_y \neq N_x$ ∴ ③ is Non Exact diff eq

Now $\frac{M_y - N_x}{N} = \frac{\frac{1}{x} - 0}{1} = \frac{1}{x}$ [illegible]

$I.f = e^{\int \frac{1}{x} dx} = e^{\ln x} = \boxed{x}$

Multiplying both sides of eq ③ by I.f = x

$x\,dy + x\left(\frac{y-\sin x}{x}\right)dx = 0$

$M = y - \sin x \quad N = x$

$M_y = 1 \quad N_x = 1$

$M_y = N_x$ ∴ ③ is Exact diff eq

$\int M dx + \int (\text{term of } N \text{ free from } x)$

$dy = C$

$\int (y - \sin x)\,dx = C$

$xy + \cos x = C$

∵ $M_y = N_x$ ∴ ④ is Exact diff eq

$\int M dx + \int$ term of N free

from x) dy = C

$\int\left(y + \frac{2}{y^2}\right)dx + \int 2y\,dy = C$

$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = C$

$xy + \frac{2x}{y^2} + y^2 = C$

Q④ $Y(2xy+e^x)dx - e^x dy = 0$

$(2xy^2+e^x y)dx - e^x dy = 0$ —— ①

$M = 2xy^2+e^x y$ $N = -e^x$

$M_y = 4xy+e^x$ $N_x = -e^x$

$M_y \neq N_x$ ∴ ① is Non Exact diff eq

$\frac{M_y - N_x}{N} = \frac{4xy+e^x+e^x}{-e^x}$ Not fox of x alone

$\frac{N_x - M_y}{M} = \frac{-e^x-4xy-e^x}{2xy^2+ye^x} = \frac{-2e^x-4xy}{y(2xy+e^x)}$

$= \frac{-2(e^x+2xy)}{y(2xy+e^x)} = \frac{-2}{y}$

I.F $= e^{\int \frac{-2}{y}dy} = e^{-2\ln y} = e^{(\ln y)^{-2}} = y^{-2} = \boxed{\frac{1}{y^2}}$

Multiply both sides of ① by I.F $= \frac{1}{y^2}$

$\frac{1}{y^2}(2xy^2+e^x y)dx - \frac{1}{y^2}e^x dy = 0$

$\left(2x+\frac{e^x}{y}\right)dx - \frac{e^x}{y^2}dy = 0$ — ①

$M = \left(2x+\frac{e^x}{y}\right)dx$ $N = -\frac{e^x}{y^2}$

$M_y = 0+\left(\frac{e^x}{y^2}\right)$ $N_x = \frac{-e^x}{y^2}$

$M_y = N_x$ ∴ is Exact diff eq

$\int M dx + \int(\text{term of } N \text{ Free from } x)dy = 0$

$\int\left(2x+\frac{e^x}{y}\right)dx + Nil = C$

$x^2+\frac{e^x}{y} = C$

Q⑤ $(x^2+y^2+2x)dx + 2y\,dy = 0$ – ①

$M = x^2+y^2+2x$ $N = 2y$

$M_y = 2y$ $N_x = 0$

$M_y \neq N_x$ ∴ ① is Non Exact diff eq

$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2x}$ Not fox of .

$\frac{M_y - N_x}{N} = \frac{2y-0}{2y} = 1 = x^0$ ~~fox~~ fx of x only

I.F $= e^{\int 1\,dx} = e^x$

Date:

Multibly both sides of eq ① by I.F $= \boxed{e^{x}}$

$e^{x}(x^2+y^2+2x)dx + e^{x}(2y)\,dy = 0$ —— (1.1)

$M = e^{x}(x^2+y^2+2x) \qquad N = e^{x}2y$

$M_y = e^{x}2y \qquad N_x = e^{x}2y$

$M_y = N_x$ ∴ is Exact diff eq

$\therefore \int M dx + \int (\text{terms of } N \text{ free from } x)\,dy = c$

$\int e^{x}(x^2+y^2+2x)\,dx + Nil = c$

$\int e^{x}x^2\,dx + \int e^{x}y^2\,dx + \int e^{x}2x\,dx = c$

$x^2 e^{x}\,dx + \int e^{x}y^2\,dx + \int e^{x}2x\,dx = c$

$x^2 e^{x} - \int 2\cancel{x}e^{x}\,dx + e^{x}y^2 + \int e^{x}\cancel{2x}\,dx = c$

$(x^2+y^2)e^{x} = c$

Q⑥ $(y^2+xy)\,dx - x^2\,dy = 0$

$\frac{dy}{dx} = \frac{y^2+xy}{x^2}$ HD eq — ①

put $y = vx$ — (11)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (12)

$v + \frac{x\,dv}{dx} = \frac{v^2x^2+xvx}{x^2}$

$v + \frac{x\,dv}{dx} = \frac{\cancel{x^2}(v^2+v)}{\cancel{x^2}}$

$\frac{x\,dv}{dx} = v^2 + v - v$

$\frac{dv}{dv^2} = \frac{dx}{x}$

$\int v^{-2}\,dv = \int \frac{dx}{x}$

$\frac{-1}{v} = \ln x + c$

$0 = \ln x + \frac{1}{v} + c$

$= \ln x + \frac{x}{y} + c$

$= y = vx$

$\frac{y}{x} = v$

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⑦ $(y-xy^2)dx+(x^2+x^2y^2dy=0$

$M\cancel{x}=y-xy^2$ $\quad N=x+x^2y^2$

$M_y=1-2xy$ $\quad N_x=1+2xy^2$

$\frac{M_y-N_x}{N}=\frac{1-2xy-1-2xy^2}{x+xy^2}=\frac{-2x(y+y^2)}{x(1+xy^2)}$ not fx of

$\frac{N_x-M_y}{M}=\frac{1-2xy-1+2xy}{y-xy^2}=\frac{2xy(x+1)}{y(1-xy)}$

Rearranging $ydx-xy^2dx+xdy+x^2y^2dy=0$

$ydx+xdy-xy^2dx+x^2y^2dy=0$

x ÷ by x $\quad ydx+xdy-x^2y^2(\frac{dx}{x})+x^2y^2dy=0$

$ydx+xdy-x^2y^2(\frac{dx}{x}-dy)=0$

÷ by x^2y^2 both sides $\quad \frac{ydx+xdy}{x^2y^2}-\frac{x^2y^2}{x^2y^2}(\frac{dx}{x}-dy)=0$

$d(-\frac{1}{xy})-\frac{dx}{x}+dy=0$

$-\frac{1}{xy}-\ln(x)+y=c$

⑧ $xdy-ydx=(x^2+y^2)dx$ — ①

$(x^2+y^2+y)dx-xdy=0$

$M_y=2y+1$ $\quad N_x=-1$

$\therefore M_y \pm N_x$ Hence ① is Non Exact

$\frac{N_x-M_y}{M}=\frac{-1-2y-1}{x^2+y^2+y}$ Not x or y

$\frac{M_y-N_x}{N}=\frac{2y+1+1}{-x}$

From ① $xdy-ydx=(x^2+y)dx$

$\int\frac{xdy-ydx}{x^2+y^2}=\int dx$

$\tan^{-1}(\frac{y}{x})=x+c$

$(\frac{y}{x})=\tan(x+c)$

$y=x\tan(x+c)$ Ans

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Ex #9.6

① $\frac{dy}{dx} + \frac{(2x+1)}{x}y = e^{-2x}$

$I.F = e^{\int P dx} = e^{\int \frac{2x+1}{x} dx} = e^{\int (2+\frac{1}{x}) dx}$

$= e^{2x+\ln x} = e^{2x} e^{\ln x} = e^{2x} x$

$\therefore$ Sol is given by $\int d(y \times I.F) = \int Q \times I.F dx + c$

$= \int d(y e^{2x} x) = \int e^{-2x} . e^{2x} x \, dx + C$

$= y e^{2x} x = \int x dx + C$

$= x y e^{2x} = \frac{x^2}{2} + C$

② $\frac{dy}{dx} + \frac{3}{x}y = 6x^2$

$I.F = e^{\int P dx} = e^{\int \frac{3}{x} dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

$\therefore$ Sol is given by $\int d(y \times I.F) = \int Q \times I.F dx + C$

$= \int d(y x^3) = \int 6 x^2 . x^3 dx + C$

$= y x^3 = \int 6 x^5 dx + C$

$= y x^3 = 6 \frac{x^6}{6} + c$

$= x^3 y = x^6 + C$

③ $\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3x^2}{\ln x}$

$I.F = e^{\int P dx} = e^{\int \frac{1}{x \ln x} dx} = e^{\int \frac{\frac{du}{x}}{\ln x}}$

$I.F = e^{\ln(\ln x)} = \boxed{\ln x}$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F dx + c$

$\Rightarrow \int d(y \ln x) = \int \frac{3x^2}{\ln x} \ln x \, dx + C$

$\Rightarrow y \ln x = 3 \frac{x^3}{3} + C$

$y = \frac{x^3 + c}{\ln x}$

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Q#④ $\frac{dy}{dx}+3y=3x^2e^{-3x}$

$I.F = e^{\int P dx} = e^{\int 3 dx} = \boxed{e^{3x}}$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + C$

$\Rightarrow \int d(ye^{3x}) = \int 3x^2 e^{-3x} e^{3x}\, dx + C$

$ye^{3x} = x^3 + C$

$y = e^{-3x}(x^2+C)$

Q⑤ $(x^2+1)\frac{dy}{dx}+2xy=4x^2$

$\frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{4x^2}{x^2+1}$

$I.F = e^{\int\left(\frac{2x}{x^2+1}\right)dx} = e^{\ln(x^2+1)} = \boxed{x^2+1}$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + C$

$\Rightarrow \int d(y(x^2+1)) = \int \frac{4x^2}{(x^2+1)}(x^2+1)\, dx + C$

$y(x^2+1) = \frac{4x^3}{3} + C$

$3y(x^2+1) = 4x^3 + C$

Q⑥ $x\frac{dy}{dx}+2y=\sin x$

$\frac{dy}{dx}+\frac{2}{x}y=\frac{\sin x}{x}$

$I.F = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\ln x} = e^{\ln x^2} = \boxed{x^2}$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F\, dx + C$

$\Rightarrow \int d(yx^2) = \int \frac{\sin x}{x} x^2\, dx + C$

$yx^2 = x(-\cos x) - \int 1(-\cos x)\, dx + C$

$y = \frac{1}{x^2}(-x\cos x + \sin x + C)$

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Q 7 $\frac{dy}{dx} = \frac{1}{e^y - x}$

$\frac{dx}{dy} - e^{y} - x$

$\frac{dx}{dy} + x = e^{y}$

$I.F = e^{\int 1.dy} = \boxed{e^{y}}$

Sol is given by $\int d(x \times I.F) = \int Q \times I.F \, dy + C$

$\Rightarrow \int d(xe^{y}) = \int e^{y} e^{y} dy + C$

$\Rightarrow xe^{y} = \int e^{2y} dy + C$

$x = \frac{1}{e^{y}}\left(\frac{e^{2y}}{2} + C\right)$

$x = \frac{e}{2} + Ce^{-y}$

Q 8 $(x+1)\frac{dy}{dx} - ny = e^{x}(x+1)^{n+1}$

$\frac{dy}{dx} - \frac{n}{x+1}y = e^{x}(x+1)^{n}$

$I.F = e^{\int P dx} = e^{\int \frac{n}{x+1} dx} = e^{-n \ln(x+1)} = e^{\ln(x+1)^{-n}}$

$I.F = (x+1)^{-n} = \boxed{\frac{1}{(x+1)^{n}}}$

Sol is given by $\int d(y \times I.F) = \int Q \times I.F \, dx + C$

$\Rightarrow \int d\left(y \frac{1}{(x+1)^{n}}\right) = \int e^{x}(x+1)^{n} \frac{1}{(x+1)} dx + C$

$\frac{y}{(x+1)^{n}} = e^{x} + C$

$y = (e^{x} + C)(x+1)^{n}$