Submitted To:-

Sir Farhan

Submitted By:-

Syed Usama Muzaffar

Roll No:-

29032

Section:-

Electrical Engineering
(Tech) Evening

Ex# 9.2

Solve

Question 1:-

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3)+c$$

$$\frac{3y^2}{2} = \ln(1+x^3)+3c$$

$$3y^2 = 2\ln(1+x^3)+6c$$

$$3y^2 = 2\ln(1+x^3)+c$$

Question 2:-

$$\frac{dy}{dx} + y^2\sin x = 0$$

$$\frac{dy}{dx} = -y^2\sin x$$

$$\int\frac{dy}{y^2} = -\int\sin x\,dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x)+c$$

$$-\frac{1}{y} = \cos x + c$$

Q:- 3

$$\frac{dy}{dx} = 1+x+y^2+xy^2$$

$$\frac{dy}{dx} = (1+x)+y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int\frac{dy}{1+y^2} = \int(1+x)dx$$

$$\tan^{-1}y = x+\frac{x^2}{2}+c$$

$$2\tan^{-1}y = 2x+x^2+C$$

Q:- 4

$$(xy+2x+y+2)dx+(x^2+2x)dy=0$$

$$[x(y+2)+(y+2)]dx+x(x+2)dy=0$$

$$[(y+2)(x+1)]dx+x(x+2)dy=0$$

$\div$ by $x(x+2)(y+2)$

$$\frac{x+1}{x(x+2x)}dx+\int\frac{dy}{y+2}=0$$

$$\frac{1}{2}\int\frac{2x+2}{x^2+2x}dx+\int\frac{dy}{y+2}=0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x)+\ln$$

$$y+2 = \frac{e}{\sqrt{x^2+2x}}$$

Q:- 5

$$\frac{dy}{dx} = 2x^2 x.y - x^2 y + xy - 2x - 2$$

$$= 2x^2 - 2x - 2 + y - x^2 y + xy$$

$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1)\,dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + C$$

$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-6\ln|2-y| = 2x^3 - 3x^2 - 6x + 6c$$

$$\ln|2-y| = (2x^3 - 3x^2 - 6x + 6c)\ln$$

$$-6 \qquad\qquad 2x^3 - 3x^2 - 6x + 6c$$

$$\ln|2-y| = \ln e$$

$$-6 \qquad 2x^3 - 3x^2 - 6x$$

$$|2-y| = e \qquad .e$$

$$|2-y|^{-6} = 2x^3 - 3x^2 - 6x$$

Q:- 6

$$\cos ey \, dx + \sec x \, dy = 0$$

$\div$ by $\operatorname{cosec} y \sec x$

$$\frac{1}{\sec x} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\Rightarrow \int \cos x \, dx + \int \sin y \, dy = \int 0 \, dx$$

$$\Rightarrow \sin x - \cos y = c$$

Q:- 7

$$y(1+x)dx + x(1+y)\,dy = 0$$

$\div$ by xy

$$\frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy = 0$$

$$\int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy = \int 0 \, dx$$

$$\Rightarrow \ln x + x \; \ln y + y$$

$$x + y + \ln(xy) = c$$

Q-8

$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$

$\div$ by xy

$\int \frac{\sqrt{1+x^2}}{x}\,dx + \int \frac{\sqrt{1+y^2}}{y}\,dy = \int 0\,dx$

put $\sqrt{1+x^2} = t$ put $\sqrt{1+y^2} = z$

$1+x^2 = t^2$ $1+y^2 = z^2$

$2x\,dx = 2t\,dt$ $2y\,dy = 2z\,dz$

$x\,dx = t\,dt$ $y\,dy = z\,dz$

Therefore

$= \int \frac{\sqrt{1+x^2}}{x^2}\,x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}\,y\,dy = \int 0\,dx$

$= \int \frac{t\cdot t\,dt}{t^2-1} + \int \frac{z\cdot z\,dx}{z^2-1} = c$

$\int \left(\frac{t^2-1+1}{t^2-1}\right)dt + \int \frac{z^2-1+1}{z^2-1}\,dz = c$

$\int \left(1+\frac{1}{t^2-1}\right)dt + \int \left(1+\frac{1}{z^2-1}\right)dz = c$

$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$

$\sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + \sqrt{1+y^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right)$

Ex#A. 3

Q:- 1

$$(x-y)\,dx + (x+y)\,dy = 0$$

$$(x+y)dy = -(x-y)\,dx$$

$$\frac{dy}{dx} = \frac{y-x}{x+y}$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(1+v)} - v$$

$$= \frac{\cancel{v} - 1 - \cancel{v} - v^2}{1+v}$$

$$x\frac{dy}{dx} = \frac{-(v^2+1)}{1+v}$$

$$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1) + \tan^{-1}v = -\ln x + c$$

$$\ln(v^2+1)^{\frac{1}{2}} + \tan^{-1}v + \ln x = c$$

$$\ln\sqrt{\frac{y^2}{x^2}+1} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$$

$$\ln\sqrt{y^2+x^2} - \ln\sqrt{x^2} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$$

$$\ln\sqrt{y^2+x^2} + \tan^{-1}\left(\frac{y}{x}\right) = c$$

Q.2

$$(y^2+2xy)\,dx + x^2 dy = 0$$

$$x^2 dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = \frac{-(y^2+2xy)}{x^2}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Using (ii) and (iii) in (i)

$$v + x\frac{dv}{dx} = -\left(\frac{v^2x^2+2x\,vx}{x^2}\right)$$

$$x\frac{dv}{dx} = -\cancel{x^2}\frac{(v^2+2x)}{\cancel{x^2}} - v$$

$$x\frac{dv}{dx} = -(v^2+3v)$$

$$\int\frac{dv}{v^2+3v} = -\int\frac{dx}{x}$$

$$\int\frac{1}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\int\frac{3}{v(v+3)}\,dv = -\int\frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(x+3) = -\ln x + \ln c$$

$$\ln\left[\frac{v^{\frac{1}{3}}}{(v+3)^{\frac{1}{3}}}\right] = \ln\frac{c}{x}$$

$$\frac{v^{\frac{1}{3}}}{(v+3)^{\frac{1}{3}}} = \frac{c}{x}$$

$$x\cdot v^{\frac{1}{3}} = c\,(v+3)^{\frac{1}{3}}$$

$$x\left(\frac{y}{x}\right)^{\frac{1}{3}} = c\left(\frac{y}{x}+3\right)^{\frac{1}{3}}$$

$$x\,\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\,x^{\frac{1}{3}} = c\,(y+3x)^{\frac{1}{3}}$$

$$xy^{\frac{1}{3}} = c\,(y+3x)^{\frac{1}{3}}$$

$$x^3y = c(y+3x)$$

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Q. 3

$(x^2 - 3y^2)\,dx + 2xy\,dy = 0$

$2xy\,dy = -(x^2 - 3y^2)\,dx$

$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$ —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

using (ii) (iii) in (i)

$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2x\,vx}$

$x\frac{dv}{dx} = \frac{(3v^2 - 1)\cancel{x^2}}{2v\cancel{x^2}} - v$

$x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$\int \frac{2v}{v^2 - 1}\,dv = \int \frac{dx}{x}$

$\ln(v^2 - 1) = \ln x + \ln c$

$\ln\left(\frac{y^2}{x^2} - 1\right) = \ln cx$

$\frac{y^2 - x^2}{x^2} = cx$

$y^2 - x^2 = (cx)\,x^2$

Q.4

$(x^2+3y^2)\,dx - 2xy\,dy = 0$

$(x^2+3y^2)\,dx = 2xy\,dy$

$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx}$ —— (i)

put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$ —— (iii)

using (ii) (iii) in (i)

$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$

$x\frac{dv}{dx} = \frac{\cancel{x^2}(1+3v^2)}{\cancel{x^2}2v} - v$

$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$

$x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$

$\ln(1+v^2) = \ln x + \ln c$

$\ln(1+v^2) = \ln cx$

$\left(1+\frac{y^2}{x^2}\right) = cx$

$\frac{x^2+y^2}{x^2} = cx$

$x^2+y^2 = (cx)\,x^2$

Q.5

$$(x^2+xy+y^2)\,dx - x^2 dy = 0$$

$$(x^2+xy+y^2)\,dx = x^2 dy$$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2} \quad \text{—— (i)}$$

put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

using (ii) (iii) in (i)

$$v + x\frac{dy}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \frac{(1+v+v^2)\,x^2}{x^2} - v$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{v}$$

$$\tan^{-1} v = \ln x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$$

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Q.6

$$(x^2+3xy+y^2)\,dx - x^2dy=0$$

$$(x^2+3xy+y^2)\,dx = x^2dy$$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \quad \text{(i)}$$

put $y = vx$ — (ii)

$$\frac{dy}{dx} = v+x\frac{dv}{dx} \quad \text{(iii)}$$

Using (ii)(iii) in (i)

$$v+x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = \cancel{x^2}\left(\frac{1+3v+v^2}{\cancel{x^2}}\right)-v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$\int\frac{dv}{(1+v)^2} = \int\frac{dx}{x}$$

$$\frac{1}{(v+1)} = \ln x + c$$

$$\left(\frac{-1}{\frac{y}{x}+1}\right) = \ln x + c$$

$$\frac{-1}{\frac{y+x}{x}} = \ln x + c$$

$$\frac{-x}{(x+y)} = \ln x + c$$

Q.7

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y} \quad \text{—— (i)}$$

put $y = vx$ —— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iii)}$$

Using (ii) (iii) in (i)

$$v + x\frac{dv}{dx} = \frac{4vx-3x}{2x-vx}$$

$$x\frac{dv}{dx} = \frac{x(4v-3)}{x(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$$

$$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x} \quad \text{—— (iv)}$$

$$\int \frac{2-v}{v^2+2v-3}\,dv = \int \frac{dx}{x}$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$$2-v = A(v-1) + B(v+3)$$

put $v+3=0 \Rightarrow +5 = -4A \Rightarrow \boxed{A = \frac{-5}{4}}$

put $v-1=0 \Rightarrow 1 = 4B = \boxed{B = \frac{1}{4}}$

$$\frac{2-v}{(v+3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v-1)}$$

$$-\frac{5}{4}\int\frac{dv}{v+3} + \frac{1}{4}\int\frac{dv}{v-1} \int\frac{dx}{x}$$

$$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-1) = \ln x + \ln c$$

$$-\ln(v+3)^5 + \ln(v-1) - 4\ln cx$$

$$\ln\frac{(v-1)}{(v+3)^5} = \ln c^4x^4$$

Anti_log

$$\frac{\left(\frac{y}{x}-1\right)}{\left(\frac{y}{x}+3\right)} = cx^4$$

$$\frac{(y-x)\,x^5}{x(y+3x)x^4} = c'$$

$$\frac{y-x}{(y+3x)^5} = c'$$

Q#8

$$x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\frac{y}{x} - x\right) dx$$

$$\frac{dy}{dx} = \frac{y \sin\frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \quad \text{——— (i)}$$

put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——— (iii)}$$

using (ii)(iii) in (i)

$$v + \frac{x\,dv}{dx} = \frac{vx \sin\frac{vx}{x} - x}{x \sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{v \sin v - 1 - v \sin v}{\sin v}$$

$$\int \sin v\, dv = \int -\frac{dx}{x}$$

$$-\cos v = -\ln x + c$$

$$\cos v = \ln x - c$$

$$\cos\frac{y}{x} = \ln x - c$$

Ex # 9.4

Q#1

$$(3x^2+4xy)\,dx+(2x^2+2y)\,dy=0$$

$M = 3x^2+4xy, \quad N = 2x^2+2y$

$\frac{\partial M}{\partial y} = 0+4x \qquad \frac{\partial N}{\partial x} = 4x+0$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Now $\int M dx + \int (\text{terms of } N \text{ free from } x)\, dy = c$

$$\int (3x^2+4xy)\,dx + \int 2y\,dy = c$$

$$3\frac{x^3}{3} + \frac{4x^2y}{2} + \frac{2y^2}{2} = c$$

$$x^3+2x^2y+y^2=c$$

Q#2

$$(2xy+y\tan y)\,dx+(x^2-x\tan y+\sec^2 y)\,dy=0$$

$M = 2xy+y-\tan y, \quad N = x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial y} = 2x+1-\sec^2 y, \quad \frac{\partial N}{\partial x} = 2x-\tan^2 y+0$

$= 2x-\tan^2 y$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N \text{ free from } x)\, dy = c$

$$\int (2xy+y-\tan y\, dx + \int \sec^2 y\, dy = c$$

$$\frac{2x^2y}{2} + xy - x\tan y + \tan y = c$$

$$x^2y+xy-x\tan y+\tan y = c$$

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Q# 3

$$\left(\frac{x+y}{y-1}\right)dx.\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2 dy=0$$

$$M=\frac{x+y}{y-1} \qquad N=-\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$$

$$\frac{\partial M}{\partial y}=\frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2} \qquad N=-\frac{1}{2}\frac{(x^2+2x+1)}{(y-1)^2}$$

$$=\frac{y-1-x-y}{(y-1)^2}=\frac{-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \quad \because \text{given } d\ldots$$

$\int M dx+\int$ (terms of N force from x) $dy=c$

$$\int\left(\frac{x+y}{y-1}\right)dx+\int-\frac{1}{2}\frac{dy}{(y-1)^2}=c$$

$$\left(\frac{1}{y-1}\right)\int(x+y)dx+-\frac{1}{2}\int(y-1)dy=c$$

$$\frac{1}{(y-1)}\left(\frac{x^2}{2}+xy\right)+\left(-\frac{1}{2}\right)\left(\frac{-1}{y-1}\right)=c$$

$$\frac{x^2+2xy}{2(y-1)}+\frac{1}{2(y-1)}=c$$

$$x^2+2xy+1=c'(y-1)$$

Q#4

$$\frac{dy}{dx}=\frac{-(ax+hy)}{hx+by}$$

$$(hx+by)dy=-(ax+hy)dx$$

$$(ax+hy)dx+(hx+by)dy=0$$

$$M=ax+hy \qquad N=hx+by$$

$$\frac{\partial M}{\partial y}=0+h \qquad \frac{\partial N}{\partial x}=h$$

$$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$\int M dx+\int$ (terms of N ...

$$\int(ax+hy)dx+\int by\, dy=c$$

$$a\frac{x^2}{2}+hxy+\frac{by^2}{2}=c \implies ax^2+2hxy+by^2=c$$

Q# 5:- $(1+\ln xy)\,dx + \left(1+\frac{x}{y}\right)dy = 0$

$M = 1+\ln xy \quad N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{xy}\cdot x \quad \frac{\partial N}{\partial x} = 0+\frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N \text{ free from } x) dy = c$

$\int (1+\ln xy)\,dx + \int 1 \cdot dy = c$

$\int dx + \int 1 \cdot \ln xy\, dx + \int dy = c$

$x + \left[\ln xy \cdot (x) - \int \frac{1}{xy}\cdot y \cdot x\, dx\right] + y = c$

$x + x\ln xy - \int dx + y = c$

$\cancel{x} + x\ln xy - \cancel{x} + y = c$

$x \ln xy + y = c$

Q#6

$(4x+3y^2)\,dx + 2xy\,dy = 0 \quad \text{—(1)}$

$M = 4x+3y^2 \qquad N = 2xy$

$M_y = 0+6y \qquad N_x = 2y$

$M_y \neq N_x$

$\frac{N_x - M_y}{M} = \frac{2y-6y}{4x+3y^2}$

$\frac{M_y - N_x}{N} = \frac{6y-2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x}$

Multiplying both side

$(4x^3 + 3y^2x^2)\,dx \ (2x^3y)\,dy = 0$

$M_y = 6yx^2 \quad N_x = 6x^2y$

$My = Nx$

$\int M dx + \int$ ...

$\int (4x^3 + 3y^2 x^2)\, dy + N i d = c$

$$4\frac{x^4}{4} + 3y^2 \frac{x^3}{3} = c$$

$$x^4 + y^2 x^3 = c$$

Question #7

$(x^2+y^2)\,dx - 2xy\,dy = 0$

$M = x^2 + y^2$ $\quad$ $N = -2xy$

$My = 2y$ $\quad$ $Nx = -2y$

$My \neq Nx$

$$\frac{Nx - My}{M} = \frac{2y - 2y}{x^2+y^2}$$

$$\frac{My - Nx}{N} = \frac{2y + 2y}{-2xy} = \frac{4y}{-2xy} = \frac{-2}{x}$$

Multiply both side

$$\frac{1}{x^2}(x^2+y^2)\,dx - \frac{1}{x^2}(2xy)\,dy = 0$$

$$\left(1 + \frac{y^2}{x^2}\right)dx - \frac{2y}{x}\,dy = 0$$

$M = 1 + \frac{y^2}{x^2}$ $\quad$ $N = -\frac{2y}{x}$

$My = \frac{2y}{x^2}$ $\quad$ $Nx = \frac{2y}{x^2}$

$$x - \frac{y^2}{x} = c$$

Exercise # 9.6

Question # 1

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$$

$$\int P\,dx \quad \int \frac{2x+1}{x}\,dx \quad \int \left(2 + \frac{1}{x}\,dx\right)$$

$$I.F = e^{2x + \ln x} = e^{2x} \ln x = e^{2x}$$

Sol is given by $\int d'$

$$\int d(y e^{2x} x) = \int e^{-2x} e^{2x} x\,dx + C$$

$$= y e^{2x} x = \int x\,dx + C$$

$$x\,y^2 e^{x} = \frac{x^2}{2} + C$$

Question # 2

$$\frac{dy}{dx} + \frac{3}{x} y = 6x^2$$

$$\int P\,dx \quad \int \frac{3}{x}\,dx \quad 3\ln x$$

$$I.F = e = e = e = x^2$$

Solution $\int d(yx^3) = \int 6x^2 \cdot x^3\,dx + C$

$$yx^3 = \int 6x^5\,dx + C$$

$$yx^3 = \frac{6x^6}{6} + C$$

$$x^3 y = x^6 + C$$

Question #3

$$\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x^2}{\ln x}$$

$$\int P\,dx \quad \int \frac{1}{x\ln}\,dx \quad \int \frac{\frac{dx}{x}}{\ln x}$$

$$I.F = e = e = e$$

$$\int d(y\ln x) = \int \frac{3x^2}{\ln x} \ln x \, dx + c$$

$$y\ln x = 3\frac{x^3}{3} + c$$

$$y = \frac{x^3 + c}{\ln x}$$

Question #4

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$$I.F = e^{\int P dx} = e^{\int 3 dx}$$

$$\int d(ye^{3x}) = \int 3x^2 e^{-3} e^{3x} dx + c$$

$$ye^{3x} = x^3 + c$$

$$y = e^{-3x}(x^3 + c)$$

Question #5

$$\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$$

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^2 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \sec^2 x y = \sec^2 x \tan x$$

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$$\int d(y\, e^{\tan x}) = \int \sec^2 x \tan x\, e^{\tan x}\, dx + c$$

$$y\, e^{\tan x} = \int e^t t\, dt + c$$

$$= te^t - \int 1\, e^t\, dt + c$$

$$= te^t - e^t + c$$

$$y e^{\tan x} = e^t (t-1) + c$$

$$y e^{\tan x} = e^{\tan x} (\tan x - 1) + c$$

$$y = (\tan x - 1) + C e^{-\tan x}$$

Question # 7

$$(x+1)\frac{dy}{dx} - xy = e^x (x+1)^{x+1}$$

$$\frac{dy}{dx} - \frac{x}{x+1}(y) = e^x (x+1)^x$$

$$I.F \quad e = e = e$$

$$I.F = (x+1)^{-x} = \frac{1}{(x+1)^x}$$

$$\int d\left(y \frac{1}{(x+1)^x}\right) = \int e^x (x+1)^x \frac{1}{(x+1)^x}\, dx + c$$

$$\frac{y}{(x+1)^x} = e^x + c$$

$$y = (e^x + c)(x+1)^x$$

Question # 8

$$(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right) y = \frac{4x^2}{x^2+1}$$

$$\int \left(\frac{2x}{x^2+1}\right) dx$$

$$\int d(y(x^2+1)) = \int \left(\frac{4x^2}{\cancel{x^2+1}}\right)(\cancel{x^2+1})\, dx + c$$

$$y(x^2+1) = \frac{4x^3}{3} + c$$

$$3y(x^2+1) = 4x^3 + c$$