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THE INTERACTIONS OF MATHEMATICS EDUCATION WITH CULTURE

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ABSTRACT

Many developments in education are fuelled by perceived problems in society, and mathematics education is no different. There is an increasing concern in many countries, including the UK, about the education of children whose family culture does not resonate with that of the school. In some countries pressure has mounted to reflect in the school curriculum the multi-cultural nature of their societies, and there has been widespread recognition of the need to re-evaluate the total school experience in the face of the educational failure of many children from ethnic minority communities. In countries like Papua New Guinea, Mozambique and Iran, there is a concern to re-examine the 'colonial' or 'western' educational experience, and to try to create instead an education which is in tune with the 'home' culture of the society. The same concern emerges in other debates about the formal education of Aborigines, of Amerindians, of the Laps and of Eskimos. In all of these cases, a culture-conflict situation is recognised and curricula are being re-examined. One particular version of this problem relates to the school Mathematics curriculum and its relationship with the home culture of the child.

1. Introduction

Up to five or so years ago, the conventional wisdom was that mathematics was 'culture-free' knowledge. After all, the argument went, "a negative times a negative gives a positive" wherever you are, and triangles the world over have angles which add up to 180 degrees. This view though, confuses the 'universality of truth' of mathematical ideas with the cultural basis of that knowledge. The ideas are decontextualised and abstracted in such a way that 'obviously' they can apply everywhere. Religious ideas are no different and yet we can also recognise different religions in different cultures. Can we recognise the possible existence of different mathematics?

Recently, research evidence from anthropological and cross-cultural studies has emerged which demonstrates convincingly that the mathematics which we know is a culture-bound phenomenon, and that other cultures have created ideas which are clearly 'other mathematics'. One can cite the work of Zaslavsky (1973), who has shown in her book *Africa Counts*, the range of mathematical ideas existing in indigenous African cultures. Van Sertima's *Blacks in Science* (1986) is another source as is Gerdes (1985). On other continents, the research of Lancy (1983), Lean (1986) and Bishop (1979) in Papua New Guinea, Harris (1980) and Lewis (1976) in Aboriginal Australia, and Pinxten (1983) and Closs (1986) with the Amerindians, has also shown us evidence which points to the fact that mathematics is a pan-human activity.

Mathematics curricula, though, have been slow to change, due partly to the fundamental misconception described above. Now there is an urgent need to find ways to 'culturalise' the Mathematics curriculum. This then is the problem-context which motivated the analysis reported in this paper. The principal task was how to conceptualise adequately Mathematics as a cultural phenomenon. This work is reported in more detail in Bishop (1988).

2. *Mathematics as a cultural phenomenon*

The most productive starting point was provided by White (1959) in his book *The evolution of culture* in which he argues, as others have done, that "the functions of culture are to relate man to his environment on the one hand, and to relate man to man, on the other" (p.8). White, though, went further, and divided the components of culture into four categories:

-ideological—composed of beliefs, dependent on symbols, philosophies;
-sociological—the customs, institutions, rules and patterns of interpersonal behaviour;
-sentimental—attitudes, feelings concerning people, behaviour;
-technological—manufacture and use of tools and implements.

Moreover whilst showing that these four components are interrelated he argues strongly that "the technological factor is the basic one; all others are dependent on it. Furthermore, the technological factor determines, in a general way at least, the form and content of the social, philosophic and sentimental factors" (p.19).

Writers such as Bruner (1964) and Vygotsky (1978) showed us the significance of written language, and one of its particular conceptual 'tools', mathematical symbolism. Mathematics, as an example of a cultural phenomenon, has an important 'technological' component, to use White's terminology. White's schema also offered an opportunity to explore the ideology, sentiment and sociology driven by this symbolic technology.

Mathematics in this context is therefore conceived as a cultural *product*, which has developed as a result of various activities. Hence the focus of the analysis shifts to the activities themselves and the search eventually narrowed to six. These all relate to the physical and social environment in some way and, to paraphrase White's statement, the functions of this symbolic technology called mathematics are concerned with relating man to environment in a particular way.

What then are these six 'environmental' activities?

3. *Environmental activities and the symbolic technology of mathematics*

i) *Counting*

Even with so-called 'primitive' peoples there is plenty of evidence of counting. Harris' (1980) survey of Aboriginal mathematics shows us that while "almost all Australian languages contain only two or three cardinal numbers" (p.13) there is clearly much use made of body-counting—a possible forerunner of finger counting.

If we move to a country with many separate cultures we can find Lancy (1983) in Papua New Guinea classifying 225 different counting systems into the following four types:

Type I - a body tally system with the number of body parts varying from 12 to 68;
Type II - a tally system using counters, like sticks. The base number is usually between 2 and 5;
Type III - mixed bases of 5 and 20 using compound number names like "two hands and a foot" to mean 15;
Type IV - base 10 system with several discrete , rather than compound number names.

That work is being continued by Lean (1986) with over 500 counting systems.

These and other studies [see Menninger (1969), Ascher and Ascher (1981), and Zaslavsky (1973)] demonstrate that there are not just two systems of numbers—"civilized" and "primitive"—as used to be the conventional public wisdom, but a rich variety of systems, varying in line with the environmental need, and existing in all societies.

ii) *Locating*

This word is chosen to characterise the activities relating to finding one's way around, knowing one's home area, travelling without getting lost and relating objects to each other. As might be expected all societies have developed different ways to code and symbolise their spatial environment, and different societies find different aspects to be of significance.

Mapping, navigation and the spatial organisation of objects exist in all cultures and all develop important mathematical ideas. Compass points are almost universal, and the stars offer direction-seekers many clues. Astro-archeology and geomancy both have strong spatial foundations while the more scientific pursuit of astronomy was a very significant stimulus for mathematical development (see Michell, 1977; Pennick, 1979; Critchlow, 1979).

A study which looks in detail at another culture's way of conceptualising large-scale space was Pinxten's (1983) work about the Navajo philosophy, and phenomenology, of space, and provides us with good evidence that 'locating' is both a clearly universal activity and also is one which provides a rich set of geometrical concepts and language.

iii) *Measuring*

Measuring is another universally significant activity for the development of mathematical ideas. Measuring is concerned with comparing, with ordering and with valuing, and all societies value certain things. We must be careful not to search for precision, and systems of units though—these only develop in relation to particular environmental needs, and in particular societal contexts.

For example in Papua New Guinea, Jones (1974) collected data from several informants about quantities and measures which included statements like these: "The local unit of distance is a day's travel, which is not very precise". Harris' (1980) survey among the Aboriginal groups showed other features though which were equally revealing of both skills or needs: "People 'measure' via a mental picture or by 'eye'. There's hardly anyone here who can't buy a dress for a relative by looking at the dress—they nearly always buy the correct size" (p.52).

Zaslavsky (1973) refers us to body-measures used for length (the Ganda of Uganda refer to the *mukono*, like the cubit, the distance from the elbow to the tip of the outstretched middle-finger), a basket holding about ten pounds, a package of coffee beans, and a bundle of sweet potatoes, all 'standard' measures to the local people, but with that element of inaccuracy which allows for social and commercial negotiation! She quotes the old Ethiopian proverb "Measure ten times, tear the cloth once".

So accuracy is not necessarily to be valued highly, it depends on the purpose and importance of measuring. But all societies engage in plenty of measuring activities (see also Menninger, 1969; Gay and Cole, 1967; Ronan, 1981).

iv) *Designing*

Another universal and important source of mathematical ideas are the many aspects of designing pursued by all cultures. Where the locating activities refer to locating objects and orienting oneself mainly in the natural environment, the activities of designing concern the 'mental template' (Oswalt, 1976) for making man-made objects and artifacts which cultures use in their home life, for trade, for adornment, for warfare, for games and for religious purposes. In addition designing can concern larger-scale space, as with houses, villages, gardens, fields, roads and towns. (For interesting examples see Faegre, 1979; Temple, 1986; Gerdes, 1986; Zaslavsky, 1973 and Ellut, 1984.)

Designing concerns abstracting a shape from the natural environment. That is why I have chosen to focus (for mathematical purposes) on 'designing' rather than on 'making'. What is important mathematically is the plan, the structure, the imagined shape, the perceived spatial relationship between object and purpose, the abstracted form and the abstracting process.

Moreover the designed object often serves as the representation of the design by which other objects can be constructed. Man has of course developed other ways to represent designs, notably by drawing in the sand, or by constructing models, or later by drawing on paper and on electronic screens. All of these developments have been created by the need to consider aspects of the designed form without having to actually make the object. These in turn have developed important mathematical ideas concerning shape, size, scale, ratio, proportions and many other geometric concepts.

v) *Playing*

This may seem initially to be a rather curious activity to include in a collection of cultural activities relevant to the development of mathematical ideas, until one realises just how many games have mathemathical connections. It is even more important to include it, when considering mathematics from a cultural perspective, because of the vast documentation of games and playing around the world. One is forced then to realise just how significant 'play' has been in the development of culture. All cultures play and what is more important, they take their play very seriously!

Johan Huizinga (1949) in his classic book *Homo Ludens* says: "The spirit of playful competition is, as a social impulse, older than culture itself and pervades all life like a cultural ferment..." (p.173).

Certainly games, their description, analyses and roles, feature widely in the anthropological literature (see for example, Zaslavsky, 1973; Lancy, 1983; Falkener, 1961), and once the play-form itself becomes the focus, and a 'game' develops, then the rules, procedures, tasks and criteria become formalised and ritualised. Games are often valued in mathematics itself. It is not too difficult to imagine how the rule-governed criteria of mathematics have developed from the pleasures and satisfactions of rule-governed behaviour in games.

vi) *Explaining*

The sixth and final 'universal' activity I call 'explaining', and it is this activity which gives mathematics its meta-conceptual characteristic. Mathematics is a generalised and very powerful form of explanation. Explaining is the activity of exposing connections between phenonema and the "quest for explanatory theory" as Horton (1971) describes it "is basically the quest for unity underlying apparent diversity; for simplicity underlying apparent diversity; for simplicity underlying apparent complexity; for order underlying apparent disorder; for regularity underlying apparent anomaly" (p.209). But although classifying is a universal activity, the classifications obtained are not, and, the diversity of languages brings a diversity of classifications (see Lancy, 1983 for some interesting examples).

But what about explaining more complex phenomena, in particular dynamic phenomena, the processes of life, and the ebb-and-flow of events? Here the fundamental and universal representation is the 'story'. Every culture has its stories, its folk-tales, and its story-tellers, and the "once upon a time..." phrase is known everywhere, even if the actual wording is different. The 'story' is a universal phenomenon, but from our point of view, in thinking about mathematics in culture, its most interesting feature is the ability of the language to connect discourse in rich and various ways. In research terms, attention has focussed on the 'logical connectives' in languages which allow propositions to be combined, or opposed, extended, restricted, exemplified, elaborated, etc. From these the ideas of proof have developed along with criteria of consistency, elegance and conviction.

These then are the six 'universal' environmental activities which it is suggested are significant for mathematical development. The symbolisations which have evolved through these activities, and reflections on them, are what we call mathematics. Moreover since these are universal activities, the corollary is that all cultures develop their own mathematics. *Mathematics is a pan-human phenomenon.* So different cultures create different symbolisations, as we have seen, and the phenomenon of 'Western' or 'European' Mathematics, which is known world-wide, must now be understood as only one kind of mathematics. Indeed it would be wrong to think of 'Western' Mathematics as only emanating from the 'West', and we must clearly begin the process of reclassifying and renaming that phenomenon. (Nevertheless from time to time I will in this paper use this convenient label, with the capital 'M').

As well as recognising the existence of other symbolisations which we can call mathematics, it is likely that different values also develop along with the different conceptual structures. Let us therefore consider the values element, a much more problematic task.

4. Mathematical values and culture

As White proposes, within cultural development, technology drives the ideology, sentiment and sociology of a culture, and these clearly involve different kinds of values.

We can begin to see what these could look like in the case of Mathematics in the writings of people like Kline (1972) and Wilder (1978). These however, it must be remembered, relate only to what we are calling 'Western' Mathematics. Other cultures' mathematical structures may well foster other sets of values, but we have yet to begin documenting those.

So, focussing on Mathematics, as the 'Western' cultural phenomenon which is now internationally known, we can see that it cannot be separated from 'Western' cultural and social history. It is a product of interactions between man and his particular physical and social environment over many years and in many societies.

We must first recognise that so much of the power of Mathematics comes from the security and *control* that it offers. Mathematics, through science and technology, has given Western culture strong feelings and sentiments (in White's terms) of security in knowledge—so much so that people can become very frustrated at natural or man-made disasters which they feel shouldn't have happened. The inconsistency of a Mathematical argument is a strong motive for uncovering the error and getting the answer 'right'. The mathematical valuing of 'right' answers informs society which also looks (in vain of course) for right answers to its problems. Western culture is fast becoming a Mathematico-Technological culture.

Where control and security are sentiments about things remaining predictable, another sentiment relates to *progress*. A method of solution for one Mathematical problem is able, by the abstract nature of Mathematics, to be generalised to other problems. The unknown can become known. Knowledge can develop. Progress, though, can become its own reward and change is now inevitable. Alternativism is also strongly upheld in Western culture and as with all the values described here, contains within itself the seeds of destruction. It is therefore important to recognise that it is the interactions and tensions between values such as these which allow cultures to survive and to grow.

If those are the main sentiments driven by the Mathematical technology then the principal ideology associated with Western Mathematics is *rationalism*. If one were searching for only one identifiable value, it would be this one. It is logic, rationalism and reason which has guaranteed the pre-eminence of Mathematics within Western culture. It is not tradition, not status, not experience, not seniority, but logic which offers the major criterion of Mathematical knowledge. With the advent of computers the ideology is extending even further, if that is possible.

European languages have incredibly rich vocabularies for logic—Gardner (1977) in his (English) tests used over 800 different logical connectives. The rise of physical technology has also helped this development, in that 'causation', one of the roots of rational argument, is developed much easier through physical technology than through nature—the time-scales of natural process are often too fast or too slow. It was simple physical technological devices which enabled man to experiment with process and to develop the formidable concept of direct 'causation'.

A complementary ideology which is clearly identifiable is *objectism*, since Western culture's world-view is dominated by material objects and physical technology. Where rationalism is concerned with the relationship between ideas, objectism is about the genesis of those ideas. One of the ways Mathematics has gained its power is through the activity of objectivising the abstractions from reality. Through its symbols (letters, numerals, figures) Mathematics has taught people how to deal with abstract entities, *as if they were* objects.

The final two complementary values concern White's 'sociology' component, the relationships between people and Mathematical knowledge. The first is called *openness* and concerns the fact that Mathematical truths are open to examination by all, provided of course that one has the necessary knowledge to do the examining. Proof grew from the desire for articulation and demonstration, so well practised by the early Greeks, and although the criteria for the acceptability of a proof have changed, the value of 'opening' the knowledge has remained as strong as ever. Openness in Mathematics is also therefore a dehumanised openness.

Therein lies the root of the last value of significance—which I call *mystery*. Despite the openness, there is a mysterious quality about Mathematical ideas. Certainly everyone who has learnt Mathematics knows this intuitively, whether it is through the meaningless symbol-pushing which many children still unfortunately experience, or whether it is in the surprising discovery of an unexpected connection. The basis of the mystery again lies in the abstract nature of Mathematics—abstractions take one away from a context, and decontextualised knowledge is literally meaningless.

Of course Mathematical ideas offer their own kind of context, so it is possible to develop meanings within Mathematics. But where these notions and ideas come from is a mystery to all. One argument however is that they don't come from anywhere. Mathematical knowledge, like all knowledge is invented and created through social and interpersonal interaction.

These then are the three pairs of values relating to Western Mathematics. They are shaped by, and also have helped to shape, a particular set of symbolic conceptual structures and together with those structures would constitute the cultural phenomenon which is often labelled as 'Western Mathematics'. We certainly know that different symbolisations have been developed in different cultures and it is very likely that there are differences in values also, although evidence on this is not readily available at the present time. How unique these values are, or how separable a technology is from its values must also remain open questions.

5. Some implications of this analysis

White's (1959) view of culture has enabled us to create a conception of Mathematics different from that normally drawn. It is a conception which enables mathematics to be understood as a pan-cultural phenomenon, but which develops different symbolisations in different cultures. Western Mathematics is therefore to be recognised as one Mathematics among several, just as for example, Christianity is one religion among several, and English is only one language among many.

Education, in this context, is concerned with inducting the young into significant aspects of their culture, and Mathematics Education should concern the process of inducting the young people into Mathematical aspects of their culture. However the anthropological literature sensitises us to two rather different induction processes.

On the one hand we have enculturation, which concerns the induction of the young child into the home or local culture, while on the other we have acculturation, which is to do with the induction of the person onto a culture which is in some sense alien, and different from that of their home background.

In the context of this new journal some other remarks are also worth making. The dynamics of a culture can be related to various internal and external influences, and the balance of these is determined both by the particular culture under consideration and also by the societal and educational contexts in which the culture resides.

Mathematics, as part of culture, is also subject to these two sets of influences. Internally, the activities with which mathematics is concerned interact with each other—numbers can be applied to shapes and to locations, playing with number patterns develops algebraic ideas, and theories are devised and structured to explain regularities as might for example be met in measures. Mathematics is as much concerned with developing internal consistency and with creating new conceptual structures as it is with responding to external demands.

Nevertheless mathematicians do not, and cannot, behave independently from the rest of society, although they have often in their history tried to protect their limited independence as much as possible. Mathematical developments even recently have been triggered by such diverse 'external' activities as gambling, code-breaking, routing traffic and landing a man on the moon. Furthermore, making war has always demanded assistance from any quarter, and from artillery range-finding to mega-death calculations, the mathematicians' products have become increasingly valued. Mathematics, as a cultural product, is now strongly shaping Western culture as a whole.

Mathematics Education too plays its part in the dynamics of Mathematical culture. It has primarily a stabilizing influence, where it seeks to enculturate and thereby ensure the survival of the culture. The demands are both for objectifying the culture in order that it can be transmitted, and for careful consideration of the process of enculturation, in order that the values aspects are also fostered. 'Stabilizing' is though perhaps the wrong word to use because, in relation to cultural dynamics, each generation *redefines* its culture in order to enculturate the young. Equally it can never just be a matter of transmitting culture, as one might transmit a paper dart. The act of enculturation is an interpersonal one and, inevitably, the cultural products will be 'received' and redefined by the young in their turn.

Perhaps though the most dramatic influence on cultural development is the intentional acculturation process, wherever it occurs. As has already been argued, that influence is already being felt in present-day Mathematics education. It certainly played its part in the history of the development of Western Mathematical thought, and there is every reason to believe that as the understanding of mathematics as a cultural product spreads, so culture conflicts in mathematics education will be seen to have an increasingly powerful influence on the development of Mathematics in the future. We have seen this happen with language and we can see it beginning with medicine and science. We will see it happening also in Mathematics.

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