Chapter Eleven: Index Number

When you completed this chapter, you will be able to:
- ✓ understand the uses of Index Numbers;
- ✓ identify the importance of selecting an appropriate base period;
- ✓ calculate index numbers by the various methods;
- ✓ distinguish the difference between Laspeyres and Paasche Index;
- ✓ recognize the limitations of Index Numbers.

Reference(s): Mason Chapter 17, Owen Chapter 8.
Exercise(s): Seminars 26 and 27, Mason Chapter 17 Exercises 5, 9, 13, 15.

At some time, everyone faces the question of how much something has changed over a period of time, and we use *index numbers* to measure the differences.

An **index number** measures *change in time series variable* in *comparison to a base year*.

Types of Index Numbers

1. Price index

This is the most frequently used one. It compares levels of price from one period to another. The familiar Consumer Price Index, measures overall price change of variety of consumer goods and services, and is used to define the cost of living.

e.g.1 The Consumer Price Index (CPI) measures changes in prices of goods and services, and is usually called the *Cost-of-Living Index*.

CPI measures the general level of consumer prices in comparison to the base-year (1985) index = 100. Thus a CPI equals to 200 in a given year (say 1994) means the level of prices is double that in the base-year.

Read Mason p626-630 for the special uses of Consumer Price Index (CPI)

2. Quantity Index

Quantity index measures how much the number of quantity of a variable changes over time.

3. Value Index

The value index measures changes in total monetary worth. That is, it measures changes in the dollar value of a variable. In effect, the value index combines price and quantity changes to a present a more informative index.

4. Composite Index

A single index that reflect a composite, or group, of changing variables. The Consumer Price Index measures the general price level for specific goods and services in the economy. It can combine the prices and quantities of goods to form a composite price index number.

Objectives of using Index Numbers

1. show ***changes*** in a series of data values over time

2. compare data values for ***different*** periods

3. compare the ***growth*** of manufacturing output

e.g.2 the price for product in year 5 was 25% above what it was in year 1 (base year)

index for the product in year 1 : 100
index for the product in year 5 : 125

Definition of Index Number

An Index number measures the value of a time series in a period *as a* ***percentage*** of the *time series in the* ***base*** *period*. (the period is usually in years)

The value of a variable expressed as 100 ± its *% deviation from the base period*.

$$\textbf{Index for a given year} = \frac{\textbf{value of that year}}{\textbf{value of the base year}} \textbf{ x 100}$$

e.g.3

	1982	1985	1992
Electric Power (billions of Kilowatt-hour)	1329	1614	1916
<u>Index</u> base year 1982 (1982 = 100)	1329/1329 x100 = 100	1614/1329 x 100 = 121.44	1916/1329 x 100 = 144.17
base year 1985 (1985 = 100)	1329/1614 x 100 = 82.34	1614/1614 x 100 = 100	1916/1614 x 100 = 118.71

e.g.4 Index the following data, taking 1987 as a base year

Year	1987	1988	1989	1990	1991	1992
Sales ($)	330	382	430	453	460	469
Index of sales (base year 1987)	330/330x100 = 100	382/330x100 = 116	130	137	139	142

Notes:

(i) a point increase in an index is a percentage increase if it is measured from the base year, e.g. the increase of 30 from 1987 to 1989 means 30%.

(ii) the increase from 139 to 142 from years 1991 to 1992 should not be confused with an increase of 3%, because 1991 is not a base year (base year is 1987).

Year	1991	1992
Sales($)	460	469
Index of Sales	460/460x100 = 100	469/460x100 =101.96

therefore, the increase from 1991 to 1992 is 1.96%

It is important to note that *index numbers only show changes relative to the base year*.

Base Year (the year whose index number is set at 100)

1. The choice of a base year (or base period) is so important.

2. Sometimes we will *change the base year*. Over a period of time, the fixed base year of an index will *become irrelevant* to the decision-making process. In 1992, for example, we are not interested in percentage changes compare to , say, 1960. The base year *needs to be changed to a more recent year*, e.g. 1980.

3. In statistical digest and similar publications, we often encounter series of index numbers in the middle of which a change of base occurs. There are two index values instead of one.

e.g.5 Changing the base year

	Year	Index
base year 1981, 1981 = 100	1981	100
	1982	116
	1983	138
	1984	160
	1985	**193**
base year 1985, 1985 = 100	**1985**	**100**
	1986	118
	1987	133
	1988	146

Criteria to determine base period

1. the base period should be ***fairly recent***, since an index number should help people compare present values with past values. If the comparison is to be meaningful, the past (base period) should be recent enough that make people remember its conditions. It is meaningless to tell that prices are 200% above what they were in the Middle age.

2. Base period should be *a period of* ***normal condition*** for the series whose index is sought. If a year of war is chose to be the base year, the consuming pattern may be abnormal in that year.

3. Select a base period that of ***comparability***. For comparisons to be valid, the indexes should have the same base period. e.g. Company A said the index of material cost was 105, company B said that it was 120. the comparison is meaningful unless the base period is the same.

4. Select a base period that of the ***availability of data***. The base period should be a period for which accurate and complete data are available. Sometimes people will choose the census year to be the base year.

Simple Aggregate Index

Most indexes cover *more than one item.*

e.g.6

Year	1982	1985	1992
Item	p_{i0}	p_{i1}	p_{i2}
Milk ($/quart)	0.26	0.31	0.56
Steak($/pound)	1.05	1.17	2.15
Butter ($/pound)	0.75	0.85	1.4
Pepper ($/pound)	2.5	2.2	2.6
Total	4.56	4.53	6.71
Price Index	100	99.34	147.15

Characteristics of Simple Aggregate Index:

1. the price may be given in *different units.*

2. there is *no indication of the relative importance* of each item.

$$\frac{\sum p_{in}}{\sum p_{i0}} \text{x}100$$

where p_{in} represents prices of item i (i = 1,2,3...) in year n,
and n = 0 (for base year 1982), 1 (for year 1985), 2 (for year 1992).

Weighted Aggregate Index
(Price Indexes with Quantity Weights)

A ***market basket*** : the total number of items of food with the quantities they were purchased.

p_{in} : price for item i in period n (if n = 0, it is base period)
q_{in} : quantity purchased for item i in period n (if n = 0, it is base year)

Equation for Weighted Aggregate Index

Base Year : $$\frac{\sum p_{i0}q_{i0}}{\sum p_{i0}q_{i0}} \text{ x } 100$$

Year n : $$\frac{\sum p_{in}q_{in}}{\sum p_{i0}q_{i0}} \text{ x } 100$$

e.g.7

Table 1: unit price per item

Year	1982	1985	1992
Item	p_{i0}	p_{i1}	p_{i2}
Milk ($/quart)	0.26	0.31	0.56
Steak($/pound)	1.05	1.17	2.15
Butter ($/pound)	0.75	0.85	1.4
Pepper ($/pound)	2.5	2.2	2.6

Table 2: Market Basket for 1982, 1985 and 1992

Year	1982	1985	1992
Item	q_{i0}	q_{i1}	q_{i2}
Milk	728qt	735qt	737qt
Steak	312lb	320lb	350lb
Butter	55lb	56lb	56lb
Pepper	0.3lb	0.3lb	0.3lb

Table 3: Total Expenditures for Yearly Market Baskets and Weighted Aggregate Index

Year	1982	1985	1992
Item	$p_{i0}\ q_{i0}$	$p_{i1}\ q_{i1}$	$p_{i2}\ q_{i2}$
Milk ($/quart)	189.28	227.85	412.72
Steak($/pound)	327.60	374.40	752.50
Butter ($/pound)	41.25	47.60	78.40
Pepper ($/pound)	0.75	0.66	0.78
Total	558.88	650.51	1244.40
Weighted Aggregate ***Price*** Index	100	116.40	222.66

Laspeyres Price Index

$$L_p = \frac{\sum p_{in} q_{i0}}{\sum p_{i0} q_{i0}} \times 100$$

The Laspeyres Price Index compares the cost of buying *base year quantities* at current year price with the cost of buying ***base year quantities*** at base year prices.

In other words, if we bought a shopping basket of goods last year, the Laspeyres Price Index will compare its cost with its cost last year.

e.g.8 The following table records the quantities purchased of X, Y and Z against the prices paid for them last year and current year.

Item	Quantity (Q_0)	Price	
		Last Year (P_0)	Current Year (P_1)
X	100	25	45
Y	50	120	200
Z	30	40	65

We use the quantities as weight since that is the best way to reflect their relative importance. The following table shows how the Laspeyres Price Index is evaluated.

Item	Q_0	P_0	Q_0P_0	P_1	Q_0P_1
X	100	25	2500	45	4500
Y	50	120	6000	200	10000
Z	30	40	1200	65	1950
Total			9700		16450

Laspeyres Price Index = (16450 / 9700) x 100 = 169.59

Paasche Price Index

$$P_p = \frac{\sum p_{in} q_{in}}{\sum p_{i0} q_{in}} \times 100$$

The Paasche Price Index compares the cost of buying *current year quantities* at current year price with the cost of buying ***current year quantities*** at base year prices.

In other words, if we bought a shopping basket of goods this year, the Paasche Price Index will compare its cost now with the cost it would have been last year.

e.g.9 We shall use the same data as before to construct the Paasche Price Index for our X, Y and Z with quantities of current year instead of that of last year.

Item	Quantity (Q_1)	Price	
		Last Year (P_0)	Current Year (P_1)
X	50	25	45
Y	75	120	200
Z	30	40	65

Again we use the quantities as weight since that is the best way to reflect their relative importance. The following table shows how the Paasche Price Index is evaluated.

Item	Q_1	P_0	Q_1P_0	P_1	Q_1P_1
X	50	25	1250	45	2250
Y	75	120	9000	200	15000
Z	30	40	1200	65	1950
Total			11450		19200

Paasche Price Index = (19200 / 11450) x 100 = 168.69

If using price as weighting to calculate Laspeyres and Paasche ***<u>Quantity</u>*** Index, we have the following formulas:

Laspeyres Quantity Index

$$L_q = \frac{\sum p_{i0} q_{in}}{\sum p_{i0} q_{i0}} \times 100$$

Paasche Quantity Index

$$P_q = \frac{\sum p_{in} q_{in}}{\sum p_{in} q_{i0}} \times 100$$

Read Mason P.620 for Fisher's Ideal Index

Compare Laspeyres with Paasche Indexes

1. Paasche Index requires the quantities to be measured each year and this can be a ***costly*** exercise. Laspeyres Index only requires them for the base year.

2. The denominator $\sum p_0 q_n$ in the Paasche Index ***changes*** each year, we *can only compare one year's Paasche Index with the base year*. For Laspeyres Index, the denominator $\sum p_0 q_0$ is fixed then each year's index can *be compared with any other year's index*.

3. Because of (2) above, *Laspeyres Index number for several different years can be* ***directly compared***, whereas with the Paasche Index comparisons can only be drawn directly between the current year and the base year.

4. Paasche Index keeps current purchasing patterns ***updated*** as it continually update the items in the shopping basket. The weights for Laspeyres Index *become out of date.*

Chain Base Index

With a chain base index, the *base year progresses a year* at a time so that each index is measured ***relative to the previous*** *year*.

The following example illustrates this. The table records the weekending price of a share quoted on the London Stock Exchange over a period of 7 weeks.

Week	Price	Index
1	28p	100
2	32p	(32/28)x100 = 114
3	34p	(34/32)x100 = 106
4	45p	(45/34)x100 = 132
5	54p	(54/45)x100 = 120
6	63p	(63/54)x100 = 117
7	84p	(84/63)x100 = 133

The Chain Base Index shows *how the rate of change is changing* as well as the extend of the change over the pervious week.

Fixed Base Vs Chain Base

The chain index is calculated with respect to the ***immediately preceding*** time point. This approach must be used *when the basic nature of the commodity* (or the components of the index) *changes over the whole time period*.

Limitations of Index Numbers

Index Numbers are *easy to understand* and fairly *easy to calculate*, so it is not surprising that they are *frequently used*. However, they are not perfect:

1. Index Numbers are usually only approximation of changes in price or quantity over time, and must be interpreted with care.

2. Weightings become out of date as time passes. Unless a Paasche Index is used, the weightings will gradually cease to reflect current reality.

3. New products or items may appear, and old ones cease to be significant. For example spending has changed in recent years, to include new items such as domestic computers and video recorders, whereas demand for black & white televisions has declined. These changes would make the weightings of a retail prices index for consumer goods out of date and the base of the index would need revision.

4. Sometimes, the data used to calculate index numbers might be incomplete, out of date, or inaccurate. For example the quantity indices of imports and exports are based on records supplied by traders who may be prone to error or even falsification.

5. The base year of an index should be a normal year, but there is probably no such thing as a perfectly normal year. Some error in the index will be caused by untypical values in the base period.

6. The "basket of items" in an index is often selective. For example the Retail Prices Index (RPI) is constructed from a sample of households and, more importantly, from a basket of only about 600 items.

7. A national index cannot necessarily be applied to an individual town, or a region. For example, if the national index of wages rises from 100 to 115, we cannot assume that the wages of people in Glasgow have gone by 15%.

8. An index may exclude important items; for example, the RPI excludes payments of income tax out of gross wages.

9. It does not reflect the quality of products.

Class : ____________ **Name** : __________________ **No.** : ___________

Class Exercise:

The following table records the quantities purchased of X, Y and Z against the prices paid for them last year and current year.

Item	Quantity		Price	
	Last Year (Q_0)	Current Year (Q_1)	Last Year (P_0)	Current Year (P_1)
X	70	100	40	50
Y	40	50	150	200
Z	50	40	50	70

Using last year as the base year, calculate the Laspeyres Price Index, Laspeyres Quantity Index, Paasche Price Index, and Paasche Quantity Index for the current year.

Solution:

Item	Q_0	Q_1	P_0	P_1	P_0Q_0	P_0Q_1	P_1Q_0	P_1Q_1
X	70	100	40	50	2800	4000	3500	5000
Y	40	50	150	200	6000	7500	8000	10000
Z	50	40	50	70	2500	2000	3500	2800
Total					11800	13500	15000	17800

$$L_p = \frac{\sum p_{in} q_{i0}}{\sum p_{i0} q_{i0}} \times 100 = 15000/11800 \times 100 = \mathbf{127.12}$$

$$P_p = \frac{\sum p_{in} q_{in}}{\sum p_{i0} q_{in}} \times 100 = 17800/13500 \times 100 = \mathbf{131.85}$$

$$L_q = \frac{\sum p_{i0} q_{in}}{\sum p_{i0} q_{i0}} \times 100 = 13500/11800 \times 100 = \mathbf{114.41}$$

$$P_q = \frac{\sum p_{in} q_{in}}{\sum p_{in} q_{i0}} \times 100 = 17800/15000 \times 100 = \mathbf{118.67}$$