Seminar 20 (Suggested Solution)

1. Let X be the number of defectives.
   $X \sim B(100, 0.1)$
   Checking:
   $np = 100 \times 0.1 = 10 > 5$, and $nq = 100 \times 0.9 = 90 > 5$
   Normal approximation can be used.

   $\mu = np = 100 \times 0.1 = 10$, $\sigma = \sqrt{npq} = \sqrt{100 \times 0.1 \times 0.9} = 3$
   $X \sim N(10, 3^2)$

   a) $P(X > 13 \mid X \sim B(100, 0.1))$
      $\cong P(X > 13 + 0.5 \mid X \sim N(10, 3^2))$
      $= P(X > 13.5) = P(Z > (13.5 - 10)/3)$
      $= P(Z > 1.17)$
      $= 0.1210$ (4 decimal places)

   b) $P(X < 8 \mid X \sim B(100, 0.1))$
      $\cong P(X < 8 - 0.5 \mid X \sim N(10, 3^2))$
      $= P(X < 7.5) = P(Z < (7.5-10)/3)$
      $= P(Z < -0.83)$
      $= 0.2033$ (4 decimal places)

2. For each toss, probability of having a total of 7 = 1/6
   $n = 180$, $p = 1/6$

   Let X be the number of a total 7. $X \sim B(180, 1/6)$

   Checking: $np = 180 \times 1/6 = 30 > 5$, and $nq = 100 \times 5/6 = 150 > 5$
   Normal approximation can be used.

   $\mu = np = 180 \times 1/6 = 30$, $\sigma = \sqrt{npq} = \sqrt{180 \times 1/6 \times 5/6} = 5$
   $X \sim N(30, 5^2)$

   a) $P(X \geq 25 \mid X \sim B(180, 1/6))$
      $\cong P(X > 25 - 0.5 \mid X \sim N(30, 5^2))$
      $= P(X > 24.5)$
      $= P(Z > (24.5 - 30)/5) = P(Z > -1.1)$
      $= 1 - 0.1357$
      $= 0.8643$ (4 decimal places)

   b) $P(33 \leq X \leq 41 \mid X \sim B(180, 1/6))$
      $\cong P(33-0.5 < X < 41+ 0.5 \mid X \sim N(30, 5^2))$
      $= P(32.5 < X < 41.5) = P((32.5-30)/5 < Z < (41.5 - 30)/5)$
      $= P(0.5 < Z < 2.3) = 0.3085 - 0.01072$
      $= 0.2978$ (4 decimal places)

   c) $P(X = 30 \mid X \sim B(180, 1/6))$
      $\cong P(30 - 0.5 < X < 30 + 0.5 \mid X \sim N(30, 5^2))$
      $= P(29.5 < X < 30.5) = P((29.5-30)/5 < Z < (30.5 - 30)/5)$
      $= P(-0.1 < Z < 0.1)$
      $= 0.0796$ (4 decimal places)

3. Let X be the number of accidents occur
   $\lambda = 10$, $X \sim P(10)$

   Checking: since $\lambda = 10 > 5$, Normal approximation can be used.
   $\mu = \lambda = 10$, $\sigma = \sqrt{\lambda} = \sqrt{10}$
   $X \sim N(10, \sqrt{10}^2)$

   a) $P(X < 15 \mid X \sim P(10))$
      $\cong P(X < 15 - 0.5 \mid X \sim N(10, \sqrt{10}^2))$
      $= P(X < 14.5) = P(Z < (14.5 - 10)/\sqrt{10}$
      $= P(Z < 1.42) = 1 - 0.0778$
      $= 0.9222$ (4 decimal places)

   b) $P(X \geq 10 \mid X \sim P(10))$
      $\cong P(X > 10 - 0.5 \mid X \sim N(10, \sqrt{10}^2))$
      $= P(X > 9.5) = P(Z > (9.5 - 10)/\sqrt{10})$
      $= P(Z > -0.16)$
      $= 1 - 0.4364$
      $= 0.5636$ (4 decimal places)

   c) $P(X = 5 \mid X \sim P(10))$
      $\cong P(5 - 0.5 < X < 5 + 0.5 \mid X \sim N(10, \sqrt{10}^2))$
      $= P(4.5 < X < 5.5) = P((4.5-10)/\sqrt{10} < Z < (5.5 - 10)/\sqrt{10})$
      $= P(-1.74 < Z < -1.42)$
      $= 0.0778 - 0.0409$
      $= 0.0369$ (4 decimal places)

4. Let X be the life-time of the light bulb
   $X \sim N(500, 10^2)$

   a) $P(X \geq 510)$
      $= P(X > 510) = P(Z > (510\ 500)/10)$
      $= P(Z > 1)$
      $= 0.1587$ (4 decimal places)

   b) Let Y be the no. of light bulb having at least 510 hours' lifetime.

      Checking conditions for Binomial Distribution,
      $Y \sim B(10, 0.1587)$
      $P(Y < 2) = P(0) + P(1)$
      $= {}_{10}C_0 (0.1587)^0 (1-0.1587)^{10-0} + {}_{10}C_1 (0.1587)^1 (1-0.1587)^{10-1}$
      $= 1 \times 1 \times 0.1776 + 10 \times 0.1587 \times 0.2111$
      $= 0.1776 + 0.3351$
      $= 0.5127$ (4 decimal places)