

Seminar 19 (Suggested Solution)

1. a) $P(Z > 2.58) = 0.00494 \cong 0.0049$ (4 decimal places)
b) $P(Z < -1) = 0.1587$ (4 decimal places)
c) $P(-1.5 \leq Z < 5) = P(-1.5 < Z < 5)$
 $= (0.5 - 0.0668) + (0.5 - 0)$
 $= 0.9332$ (4 decimal places)

2. a) $z = +0.35$
b) $z = -1.21$
c) the area to the right of $z = 0.5 - 0.4838 = 0.0162 \Rightarrow z = 2.14$
d) the area to the right of $+z = (1-0.95)/2 = 0.025 \Rightarrow z = 1.96$

3. $X \sim N(18, 2.5^2)$
 - a) $P(X < 15)$
 $= P((X-\mu)/\sigma < (15 - 18)/2.5)$
 $= P(Z < -1.2)$
 $= 0.1151$ (4 decimal places)

 - b) $P(X < k) = 0.2236$
 $\Rightarrow P((X-\mu)/\sigma < (k - 18)/2.5) = 0.2236$
 $\Rightarrow P(Z < (k - 18)/2.5) = 0.2236$ (4 decimal places)

 \Rightarrow from Normal table, $(k - 18)/2.5 = -0.76$
 $\Rightarrow k = 2.5 \times (-0.76) + 18$
 $= 16.1$

 - c) $P(X > k) = 0.1814$
 $\Rightarrow P((X-\mu)/\sigma > (k - 18)/2.5) = 0.1814$
 $\Rightarrow P(Z > (k - 18)/2.5) = 0.1814$

 \Rightarrow from Normal table, $(k - 18)/2.5 = 0.91$
 $\Rightarrow k = 2.5 \times (0.91) + 18$
 $= 20.275$

 - d) $P(17 < X < 21)$
 $= P((17-18)/2.5 < (X-\mu)/\sigma < (21-18)/2.5)$
 $= P(-0.4 < Z < 1.2)$
 $= 0.8849 - 0.3446$
 $= 0.5403$ (4 decimal places)

4. Let X be the mark on the statistics examination
 $X \sim N(74, 7^2)$

a) let k be the lowest passing mark

$$\begin{aligned} P(X < k) &= 0.1 \\ \Rightarrow P(Z < (k-74)/7) &= 0.1 \\ \Rightarrow (k-74)/7 &= -1.28 \\ \Rightarrow k &= 7 \times (-1.28) + 74 = 65.04 \\ \text{the lowest passing mark is } &66, \text{ a grade of } 65 \text{ is F} \end{aligned}$$

b) let k be the highest mark for grade B

$$\begin{aligned} P(X > k) &= 0.05 \\ \Rightarrow P(Z > (k-74)/7) &= 0.05 \\ \Rightarrow (k-74)/7 &= 1.645 \\ \Rightarrow k &= 7 \times (1.645) + 74 = 85.52 \\ \text{the highest mark for grade B is } &85, \text{ a grade of } 86 \text{ is A} \end{aligned}$$

c) let k be the lowest mark for grade B

$$\begin{aligned} P(X > k) &= 0.35 \\ \Rightarrow P(Z > (k-74)/7) &= 0.35 \\ \Rightarrow (k-74)/7 &= 0.39 \\ \Rightarrow k &= 7 \times (0.39) + 74 = 76.73 \\ \text{the lowest mark for grade B is } &77, \text{ a grade of } 76 \text{ is C} \end{aligned}$$

5. Let X be the lifetime of the small motors
 $X \sim N(10, 2^2)$

let k be the guarantee period

$$\begin{aligned} P(X < k) &= 0.03 \\ \Rightarrow P(Z < (k-10)/2) &= 0.03 \\ \Rightarrow (k-10)/2 &= -1.88 \\ \Rightarrow k &= 2 \times (-1.88) + 10 = 6.24 \text{ (years) (2 decimal places)} \end{aligned}$$

6. Let X be the examination mark

$$X \sim N(82, 5^2)$$

$$\begin{aligned} P(88 < X < 94) &= P((88-82)/5 < (X-\mu)/\sigma < (94-82)/5) \\ &= P(1.2 < Z < 2.4) \\ &= 0.1151 - 0.0082 \\ &= 0.1069 \end{aligned}$$

it means 10.69% of students get grade B, that is 8 students.

$$\begin{aligned} \text{total number of students took the examination} &= 8 / 10.69\% = 74.83629 \\ &\cong 75 \text{ (students)} \end{aligned}$$