Seminar 18 (Suggested Solution)

1. Let X be the no. of accidents in any given month
   $X \sim P(3)$
   a) $P(X = 5) = e^{-3}\ 3^5 / 5! = 0.1008$ (4 decimal places)

   b) $P(X<3) = P(X=0) + P(X=1) + P(X=2)$
   $= e^{-3}\ 3^0 / 0! + e^{-3}\ 3^1 / 1! + e^{-3}\ 3^2 / 2! = 0.4232$ (4 decimal places)

   c) $P(X\geq 2) = 1 - P(X<2)$
   $= 1 - P(X=0) - P(X=1) = 0.8009$ (4 decimal places)

2. Let X be the no. of person die from the infection
   $X \sim B(2000, 0.002)$
   since $n = 2000 > 100$ is large enough, and $p = 0.002$ is close to zero,
   Poisson approximation can be used.

   $\lambda = np = 2000 \times 0.002 = 4 \Rightarrow X \sim P(4)$
   $P(X < 5) = P(X=0) + P(X=1) + ... + P(X=4)$
   $\cong e^{-4}\ 4^0 / 0! + e^{-4}\ 4^1 / 1! + ... + e^{-4}\ 4^4 / 4! = 0.6288$ (4 decimal places)

3. a) $\text{mean} = \mu = \lambda = np = 2000 \times 0.002 = 4$
   $\text{variance} = \sigma^2 = \lambda = 4$

   b) For $k = 2$, we have $\mu \pm 2\sigma = 4 \pm 2 \times 2 = (0, 8)$

4. a) Let X be the no. of vegetable contained in the salad.
   $X \sim P(5)$

   $P(X>5) = 1 - P(X\leq 5)$
   $= 1 - P(X=0) - P(X=1) - ... - P(X=5) = 0.3840$ (4 decimal places)

   b) Let Y be the no. of days that the salad contain more than 5 vegetables.
   $Y \sim B(4, 0.3840)$

   $P(Y=3) = {}_4C_3\ (0.3840)^3\ (1-0.3840)^{4-3} = 0.1395$ (4 decimal places)

5. Let X be the no. of form in error.
   $X \sim B(4{,}000 , 0.001)$

   since $n = 4000 > 100$ is large enough, and $p = 0.001$ is close to zero, (or $np = 4000 \times 0.001 = 4 < 5$)
   Poisson approximation can be used.
   $\Rightarrow \text{mean} = \lambda \cong np = 4000 \times 0.001 = 4$
   $X \sim P(4)$

   $P(\bar{X} = 6, 7 \text{ or } 8) = P(X=6) + P(X=7) + P(X=8)$
   $\cong e^{-4}\ 4^6 / 6! + e^{-4}\ 4^7 / 7! + e^{-4}\ 4^8 / 8! = 0.1935$ (4 decimal places)