Seminar 17 (Suggested Solution)

1. $\mu = E[X] = \sum x\, p(x)$
   $= (1 \times 0.07) + (2 \times 0.19) + ... + (5 \times 0.16) = 3.29$ *(2 decimal places)*

   $\sigma^2 = V[X] = \sum (x - \mu)^2\, p(x)$
   $= 0.07 \times (1 - 3.29)^2 + 0.19 \times (2 - 3.29)^2 + ... + 0.16 \times (5 - 3.29)^2$
   $= 1.3259$
   $\sigma = \sqrt{1.3259} = 1.15$ *(2 decimal places)*

2. E[strategy 1] = 0.15 x (10,000) + 0.85 x (-1000) = 650
   E[strategy 2] = 0.5 x (1,000) + 0.3 x (500) + 0.20 x (-500) = 550
   E[strategy 3] = 400
   Strategy 1 has the highest expected profit, however, the risk (variance of profit) for each strategy should also be taken into consideration.

3. Check conditions for Binomial Distribution.

   Let X be the no. of defective parts in a shipment.
   a) n = 18, P(defective parts) = p = 0.05, $X \sim B(n, p) \Rightarrow X \sim B(18, 0.05)$
      P(accept shipment) = P(X < 2)
      = P(X=0) + P(X=1)
      $= {}_{18}C_0 (0.05)^0 (0.95)^{18-0} + {}_{18}C_1 (0.05)^1 (0.95)^{18-1}$
      $= 1 \times 1 \times 0.3972 + 18 \times 0.05 \times 0.4181$
      $= 0.3972 + 0.3763$
      $= 0.7735$ *(4 decimal places)*

   b) n = 18, P(defective parts) = p = 0.1, $X \sim B(n, p) \Rightarrow X \sim B(18, 0.1)$
      P(accept shipment) = P(X < 2)
      $= P(X=0) + P(X=1) = {}_{18}C_0 (0.1)^0 (0.9)^{18-0} + {}_{18}C_1 (0.1)^1 (0.9)^{18-1}$
      $= 0.1501 + 0.3002 = 0.4503$ *(4 decimal places)*

   c) n = 18, P(defective parts) = p = 0.2, $X \sim B(n, p) \Rightarrow X \sim B(18, 0.2)$
      P(accept shipment) = P(X < 2)
      $= P(X=0) + P(X=1) = {}_{18}C_0 (0.2)^0 (0.8)^{18-0} + {}_{18}C_1 (0.2)^1 (0.8)^{18-1}$
      $= 0.0180 + 0.0811 = 0.0991$ *(4 decimal places)*

4. Check conditions for Binomial Distribution.

   Let X be the no. of engines fail.
   For a 4-engine plane; n = 4, prob. of fail = p = 0.4,
   $X \sim B(n, p) \Rightarrow X \sim B(4, 0.4)$

   Prob. (successful flight) = $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$
   $= {}_4C_0 (0.4)^0 (0.6)^{4-0} + {}_4C_1 (0.4)^1 (0.6)^{4-1} + {}_4C_2 (0.4)^2 (0.6)^{4-2}$
   $= 1 \times 1 \times 0.6^4 + 4 \times 0.4 \times 0.6^3 + 6 \times 0.16 \times 0.36$
   $= 0.1296 + 0.3456 + 0.3456 = 0.8208$ *(4 decimal places)*

4. For a 2-engine plane; n = 2, prob. of fail = p = 0.4,
   $X \sim B(n , p) \Rightarrow X \sim B(2 , 0.4)$

   Prob(successful flight) = $P(X\leq1) = P(X=0) + P(X=1)$
   $= {}_2C_0 (0.4)^0 (0.6)^{2-0} + {}_2C_1 (0.4)^1 (0.6)^{2-1}$
   $= 1 \times 1 \times 0.6^2 + 2 \times 0.4 \times 0.6$
   = 0.36 + 0.48 = 0.8400 *(4 decimal places)*
   The 2-engine plane has a slightly higher probability for a successful flight.

5. Check conditions for Binomial Distribution.

   Let X representing the no. of tires blowout.
   n = 20, prob. of blowout = p = 0.08,
   $X \sim B(n , p) \Rightarrow X \sim B(20 , 0.08)$

   a) P(from 3 to 6) = $P(3\leq X\leq6)$
   $= {}_{20}C_3 (0.08)^3 (0.92)^{20-3} + {}_{20}C_4 (0.08)^4 (0.92)^{20-4} + {}_{20}C_5 (0.08)^5 (0.92)^{20-5} + {}_{20}C_6 (0.08)^6 (0.92)^{20-6}$
   = 0.2114 *(4 decimal places)*

   b) $P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
   = 0.188693 + 0.328162 + 0.271091 + 0.141439
   = 0.9294 *(4 decimal places)*

   c) $P(X > 5)$
   $= 1 - P(X \leq 5)$
   $= 1 - P(X=0) – P(X=1) – P(X=2) – P(X=3) – P(X=4) – P(X=5)$
   = 1 – 0.188693 – 0.328162 – 0.271091 – 0.141439 – 0.052271 – 0.014545
   = 0.0038 *(4 decimal places)*

6. E[X] = np = 620 x 0.78 = 483.60 (2 decimal places)
   Var[X] = npq = 620 x 0.78 x 0.22 = 106.392
   $\sigma = \sqrt{106.392} = 10.31464977 = 10.31$ *(2 decimal places)*