Seminar 15 (Suggested Solution)

1. a. $P(A|C) = P(A \text{ and } C) / P(C) = (1/7) / (1/3) = 3/7$
   b. $P(C|A) = P(A \text{ and } C) / P(A) = (1/7) / (3/14) = 2/3$
   c. $P(B \text{ and } C) = P(B|C)\, P(C) = (5/21) \times (1/3) = 5/63$
   d. $P(C|B) = P(C \text{ and } B) / P(B) = (5/63) / (1/6) = 10/21$

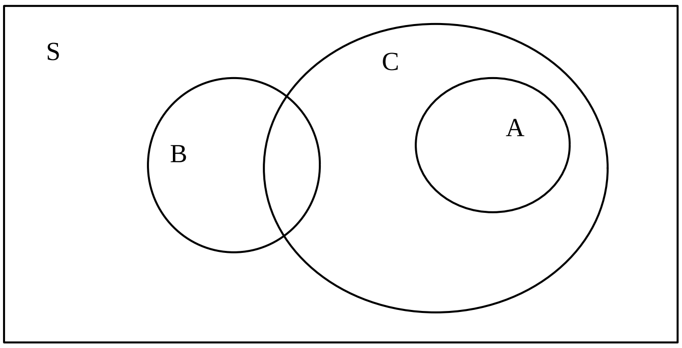
2. a. P(A or B or C)
   = P(A) + P(B) + P(C) - P(A and B) - P(B and C) - P(A and C) + P(A and B and C)
   since P(A) + P(B) + P(C) = 0.21 + 0.15 + 0.40 = 0.76 < 1
   and P(A and B and C) = 0.

   $\therefore$ P(A or B or C) < 1
   $\Rightarrow$ A, B and C are not collectively exhaustive

   b. $P(B|C) = P(B \cap C)/P(C) = 0.0225 / 0.4 = 0.05625 \neq P(B)$
   $\therefore$ events B and C are not statistically independent

   c. P(A and C) = P(A) + P(C) - P(A or C)
   $= 0.21 + 0.40 - 0.40 = 0.21 \neq 0$
   $\therefore$ events A and C are not mutually exclusive

   d.



> Tutors should remind the students to define necessary events first, and then use the defined events in all calculation.
> For example: use P(M) instead of P(shoplifter is male) in question 3.

3. Define M = shoplifter is male, W = shoplifter is female
   F = shoplifter is first-time offender, R = repeat offender

   a. P(M) = (77+55)/250 = 0.5280
   b. P(F|M) = P(F and M) / P(M) = (77/250) / (132/250) = 0.5833
   c. P(W|R) = P(W and R) / P(R) = (85/250) / (140/250) = 0.6071
   d. P(W|F) = P(W and F) / P(F) = (33/250) / (110/250) = 0.3000
   e. P(M and R) = 55/250 = 0.2200

4. **Define R = radiation leak occurs, F = fire occurs,**
M = mechanical failure occurs, H = human error occurs.

Event	P(Event)	P(R|Event)	P(R ∩ Event)	P(Event|R)
F	0.0010 / 0.2 = 0.005	0.20	0.0010	0.0010 / 0.0037 = 0.2703
M	0.0015 / 0.50 = 0.003	0.50	0.0015	0.0015 / 0.0037 = 0.4054
H	0.0012 / 0.10 = 0.012	0.10	0.0012	0.0012 / 0.0037 = 0.3243
			P(R)= 0.0037	

a. P(F) = 0.005, P(M) = 0.003, P(H) = 0.012

b. P(R) = 0.0037

c. P(F|R) = 0.2703, P(M|R) = 0.4054, P(H|R) = 0.3243

5 a. **Define D be the event of the disease.**
M be the event of positive test result.
** *N be the event of negative test result. (it is not necessary to define another event, N, for negative test result,* $\overline{M}$ should be used instead)

P(D) = 0.03, P($\overline{D}$) = 0.97, P(M|D) = 0.90, P(M|$\overline{D}$) = 0.02

P(M) = P(M|D) P(D) + P(M|$\overline{D}$) P($\overline{D}$)
= (0.90) (0.03) + (0.02) (0.97)
= 0.0464

b. Required probability:
= P(D|M)

$$= \frac{P(M \mid D)P(D)}{P(M)}$$

$$= \frac{(0.90)(0.03)}{0.0464}$$

= 0.5819 (correct to 4 decimal places)

c. P($\overline{M}$) = 1 - P(M) = 1 - 0.0464
= 0.9536

P(M|$\overline{D}$) = 0.02 ⇒ P($\overline{M}$ | $\overline{D}$) = 0.98

Required probability:
= P($\overline{D}$ | $\overline{M}$)

$$= \frac{P(\overline{M} \mid \overline{D})P(\overline{D})}{P(\overline{M})}$$

$$= \frac{(0.98)(0.97)}{0.9536}$$

= 0.9969 (correct to 4 decimal places)