Representation: Revisited

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The role of representation in teaching/learning activities has recently been realised. As a result, the National Council of Teachers of Mathematics in the United States has decided that representation will be a new "process standard" rather than simply a part of communication as it was in 1989 (Pape & Tchoshanov, 2001). In this essay, I would like to discuss representation from different perspectives; describe three types of representation and its role in mathematics learning.

Introduction

There are four main ideas in order to conceptualise representation. Firstly, within the domain of mathematics, representation may be a thought of internal-abstraction of mathematical ideas or cognitive schemata that are developed by the learner through experience (Pape & Tchoshanov, 2001). Secondly, representation can be explicated as "mental reproduction of a former mental sate"(Seeger, cited in Pape & Tchoshanov, 2001). Thirdly "a structurally equivalent presentation through pictures, symbols and signs" (Seeger, cited in Pape & Tchoshanov, 2001) also means to representation. Lastly, it is also known as "something in place of something" (Seeger, cited in Pape & Tchoshanov, 2001)

There is no unanimity in representing the term representation. Goldin tries to explicate it as hypothesised mental constructs and material-notations(Goldin & Shteingold, 2001). The former is an internal representation while the second is external representation (Kaput, 1999). Cifarrelli seems to use the word "representation" exclusively as mental representation whereas Evan used this word as material representation (Kaput, 1999). Similarly "fusion" is referred to emphasise on maintaining structure and orientation in time and in the space of actions and possibilities surrounded by a symbol rich experience (Kaput, 1999)-internalising the external representation. Representational capacity of early men has been believed to begin about 1.5 million years ago in the form of "mimetic"(Donald, 1991) .

Looking at the etymological point of view, representation is to identify, select and present something for something. For instance, in order to represent "five", the learner can select five tally bars or five unit cubes or five of any objects.

We may think of representation in terms of "presentation" and "re-presentation". To some extent, it may be true. However, if representation is regarded as presentation and re-presentation then mathematics learning-process will be oriented to reproduction of the ideas. Of course, representation is a part of the process of construction of knowledge, which can be performed either by sharing the ideas between two or more people or by constructing individually. In this light, we may consider the lower level of representation is to present or re-present the mathematical ideas per se.

Constructivism, constructionism and representation

Constructivism has been widely accepted in mathematics and science learning since 80s (Thompson, 1995). Education suffered a decline in the last 20 or 30 years (vonGlaserfled, 1995), which led for searching a different view of the learning process. In fact, the central idea of constructivism is to learn by constructing the knowledge rather than receiving from the teacher. The "perennial" concept regards the knowledge as an independent entity of the world, which does not help, for developing understanding of learning. Furthermore the constructivists' approach of knowing is to construct for an active representation of reality and to develop it as a part of an "internal mental network" (Hiebert & Carpenter, 1992)of the learner.

Looking at the historical perspective, we can find that the "skepticism" emerged in 500 BC (Eves, 1969)which did not accept the process of logical representation of "truth" (vonGlasersfeld, 1995). However, skeptics did not suggest an alternative way of representing knowledge instead of reiterating the argument to oppose the "rationalism"- the doctrine that knowledge is acquired and represented by reason without resort to experience (vonGlasersfeld, 1995). The knowledge which represents the real world, particularly, the "experiential world" through which one can relate with the abstract knowledge(vonGlasersfeld, 1995). The behaviorist's concept of "adaptation" (vonGlasersfeld, 1995) does not see any difference between human beings and other animals. However, for the constructivism, the most important is that the customary conception of truth as the correct representation of states or events of an external worlds replaced by the notion of the "viability" (vonGlasersfeld, 1995).Here, viability implies the adequacy in the contexts in which the concept, models and theories are created (vonGlasersfeld, 1995) and represented.

Generally, there are two views of knowledge known as “exogenic” and “endogenic” in which the previous deals with world-centered and the later deals with mind-centered knowledge (Gergen, 1995). In general, the views on representation of knowledge fall in the continuum of psychological-material reality. The exogenic tradition regards the external materials are the given while the endogenic regards the internal mental state is given(Gergen, 1995). The two different systems of representation of knowledge has a fundamental difference in viewing the learner in which the exogenic prefers to specify the learner as a “tabula rasa” while the endogenic focuses on rational capacities of individual (Bower & Hilgard, 1981; Gergen, 1995). In this light, the representation of knowledge from only one perspective does not give a practical solution. In one side, the problem of exogenic view of knowledge is how the external world is made manifest to the internal and how the subjectivities can ever record or ascertain the nature of the so-called objective world (Bower & Hilgard, 1981; Gergen, 1995). On the other side, the problem of endogenic view of knowledge is to understand or comprehend the subjectivities of the others and to ascertain whether the externalized source is the reflection of internal state (Bower & Hilgard, 1981; Gergen, 1995).

The construction and representation of knowledge according to Gergen is carried out by “social constructionist orientation of knowledge” (Gergen, 1995, p. 23) in which social interchange has a major role in constructing and representing knowledge (p.24). Explicitly speaking, the construction and representation of meaning is achieved through social interdependence which is context dependent and that serves communal functions (pp. 24-26).

Looking at the radical constructivism and social constructionism, we can notice that there is a difference between two theories in terms of representation of knowledge. The radical constructivism focuses on internal representation and social constructionism focuses on shared representation.

Vygotsky, Piaget and representation

Vygotsky argued that advanced concepts appear first in social interaction, and only gradually become accessible to an individual (Confrey, 1995). Vygotsky was influenced by the Marxism which explicates that knowledge is constructed as a consequence of pre-existed matter or tools (Confrey, 1995). Moreover, Vygotsky argued for the evolution of higher cognitive process from social to individual (p.189). In other words, the knowledge is "external" in the beginning, which is, "eternalised" later. Here, Vygotsky clarifies that internalisation is not the "transferal" of an external activity but it is a process of gaining control over external "sign" forms (p.190).

Piaget mentioned a different representational system from the representational system of Vygotsky. The focus of Piaget is on the subjectivity of representation, and the process of internalisation, according to him, is performed through interaction with the physical reality (Confrey, 1995). Furthermore, the internalisation according to Piaget is schemata that reflect the regularities of an individual's physical action (p.200). On the contrary, Vygotsky thinks the internalisation as a social process. Moreover the representational system in Vygotskian perspective is more shared in the beginning and internalised later.

Ethnomathematics and Representation

The term ethnomathematics is used to express the relationship between culture and mathematics (D'Ambrosio, 2001). It is a new idea of studying mathematical representation from different cultural perspective. Ethnomathematics is concerned by the connections that exist between the symbol, the representation and imagery (Vergani, 1998). Moreover, representation in ethnomathematical perspective has a wider scope since the different cultures have different types of representational system, which would be useful in mathematics learning.

It is apparent that the development of ethnomathematics has tried to transform the traditional concept of Euro-centered mono-representational system of mathematics to world centered multi-representational system of mathematics(D'Ambrosio, 2001). Ethnomathematics does not study only the number system and symbols of different ethnic groups but also studies about the representational system of different aspect of their culture. The representational system of a culture depends upon the types of mathematical knowledge it has. For instance some cultures have "logico-mathematical knowledge"(Kamii, Kirkland, & Lewis, 2001); some deals with "narrative knowledge(Seeger, 1998)"; and some culture deal with "paradigmatic knowledge" (Seeger, 1998).

The tradition of representation started from the ancient civilisations.

The representational system of early Babylonian was more mathematical while comparing with the Egyptians (Eves, 1969). The Mayan represented the number as a positional base-twenty system(Lara-Alecio, Irby, & Morish-Aldana, 1998). The Chinese represented "multiplicative" number system (Eves, 1969) while the Indian and Arabic represented the place value systems. The cultural artefacts, language, myths and literature help pinpoint the representational system of different cultures and civilisation.

Ethnomathematics has an important role in learning mathematics hence in representational system. Particularly, it is important to contextualise the representational system. The example of Maori based mathematics teaching (Aspin, 1995) is evident in this regard.

Three types of representation: two schools of learning theories

On the basis of aforementioned discussion we can distinguish representation into external, shared and internal representation. Epistemological link of external representation is with the school of behaviourism. The external representations are, for instance, mathematical symbols, signs, characters, and signals. Similarly the cultural artefacts, literature, number symbol and measuring tools are also external representation in ethnomathematics. The traditional external representational systems are static in the sense that they provide rules or frameworks for creating fixed external formulas, equations graphs and diagrams(Goldin & Shteingold, 2001). However, the use of microcomputer and graphing calculator has made the external representation system dynamic(Kaput, 1989).

The shared representations exist during the interaction process. In some instance, it has been regarded as external representation. However, there is a nuance difference between the external and shared representation. Furthermore, the external representation is carried out by virtue of external entities while the shared representation is performed through discourses. Moreover, this types of representation deals with the representation that are shared between the teachers and the learners (Goldin & Shteingold, 2001). The examples of shared representation are all external which are in shared mode.

The internal psychological representational system, on the other hand, plays a crucial role in mathematics learning. In order to understand a mathematical concept, that should be represented in internal mental network (Hiebert & Carpenter, 1992) of the learner. There are various forms of internal representation including verbal/syntactic, imagistic, formal notational, and affective(Goldin & Shteingold, 2001).

Behaviourists also believe that representation plays a vital role in mathematics learning. However, they do not accept the role of shared representation. Instead, they think representation as a part of performance behaviours, which is expected to be achieved in the end of learning activities (Goldin & Shteingold, 2001). On the contrary, the cognitivists have focused on internal representational system since it is constructed by the learner. Broadly speaking, the constructivists have viewed representation as a subjective phenomenon whereas behaviourists have regarded it as an objective phenomenon.

The role of representation in mathematics learning

Broadly speaking, there are two general types of representations that affect children's understanding of and solution to, mathematics problems: 1) instructional representations (definitions, examples and models) that used by teachers to impart the knowledge to students and 2) cognitive representations that are constructed by the students themselves as they try to make sense of a mathematical concept or attempt to find a solution to a problem(Miura, 2001).On the one hand, the previous representation plays a role for developing a shared representational system. For instance, in order to represent a unit circle the teacher and students discuss about the equation $x^2 + y^2 = 1$, which is a way of constructing shared representation. On the other hand, the activities, such as sketching diagram for a mathematical concept; searching for a relationship between $x^2 + y^2 = 1$ and $x^2 + 2xy + y^2 = 4$; and writing a reflective journal about ongoing classroom activities are helpful in order to construct the internal representational system.

Looking at the historical development of mathematical systems, we can find that the formal representations, which we say now as external representation, were constructed as internal mental representation. Of course, it would be more radical to recommend for adopting the same process of developing representation as mathematicians did, in order to discover mathematics. However, we can derive some implications of constructing and cognizing representation from the historical perspective, which can be adopted in order to make effective learning process.

Apparently, information technology has helped for developing multiple representations of mathematical concepts, which is worthwhile on transforming from static representational system to more dynamic and constructive representational system. The concept of “mathematical modeling” also bears the meaning of the term representation (however, it is a sophisticated idea to say “modeling” in stead of representation) in many respects in which an important role of computer software is evident. In school mathematics, the Logo and mathGV and even other generic software for instance, word processor and spreadsheet have facilitated to develop more comprehensive representations.

Under these circumstances, the role of representation in classroom teaching and learning is apparently important. The following seven roles of the representation in teaching and learning process have been discussed in this article.

1. **Representation as a source of communication:**

Communication is one of the “process standard” of school mathematics (Pape & Tchoshanov, 2001). From sociological point of view, representation has been regarded as a part of communication. Of course, representation has a major role in promoting effective communication within and across the mathematical systems. It is apparent that effective communication requires an effective representational system.

In mathematics, communication is vital for successful learning. To some extent, mathematics is regarded as a language. In this light, the representations of mathematical ideas are the means of communication in mathematics as words do in language. For instance, Sally represents 2+3=? in terms of story as:

> Tom has $2 and Dave has $3. How much money will be there if they both put together?

The story (representation) now tells (communicates) the others about Sally’s ideas.

2. **Representation as an indicator of students’ attitude towards mathematics**

The affective representational system is a source of identifying students’ attitude towards mathematics. Particularly, the internal representational system is helpful for fostering students’ changing attitude towards mathematics. It is very important for a teacher to know about the students’ attitude and changing beliefs towards a particular mathematical concept in order to design learning activities.

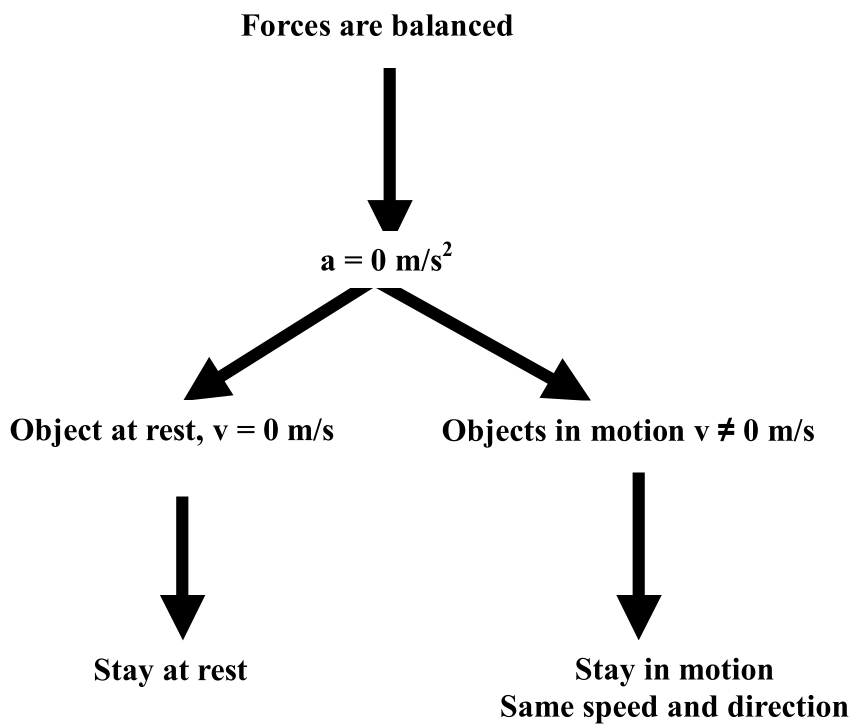
3. **Representation as evidence of probing understanding of learning**

The aim of mathematics learning is to develop understanding of learning. For teachers, representation can be useful for probing understanding of mathematics learning. The recent epistemology of understanding believes that understanding of learning is possible when its representation becomes a part of mental network(Hiebert & Carpenter, 1992). Different types of representation, for instance, diagrammatic representation, verbal representation, and sometimes representation of concrete objects can help as the means of probing understanding.

4. Representation as a means of establishing links between the concepts

Representation is not a single entity of something. It is a multifaceted idea of expressing mathematical relations, concepts and principles. Moreover it helps establish and visualize relationships between concepts. Representation of Newton's second law of motion requires a link between the concept of rest, balance and motion. The following figure is a representation of Newton's second law of motion. This representation has tried to link among the concepts of balance, acceleration, rest, motion and velocity.

Figure 1



5. Representation as a developmental process that exists in the procedural – conceptual continuum

According to Karmilloff –Smith, implicit information, which is already stored in mind in a certain form of internal representations and it, is embedded in a special - purpose procedure, is subject to iterative process of redescription (Voutsina & Jones, 2001)

The following table represents an extract of representation redescription (RR) model (Voutsina & Jones, 2001)

Table-1

Recurrent 3-phase model

Phase	Represent	Relates to	Goal of
Procedural	Performance	Algorithm, mnemonic, facts, formula	Success orientation
Meta Procedural	Internal knowledge	Network of facts formula and previous experiences	Organization oriented behavior
Conceptual Phase	Regulated knowledge by internal mental network	Relational and conceptual knowledge	Establishing control on external-internal continuum

According to RR model, in procedural phase, students are more success oriented and represent their algorithmic performance. In Meta procedural stage, the nature of representation is different from the previous level. Learners represent meta-procedural construct for example the interpretation of the algorithm and rationale of the procedure. In the third level, they establish the control over external-internal continuum in which the representation is regulated in internal mental network of the learner. For instance, the learner can represent a problem-situation in terms of what is asked; what process is to be used; and what might be the solutions in respect to involved concepts in the problem.

6. **Representational system as a means of overcoming cognitive obstacles**

"Cognitive Obstacle" is a piece of knowledge of the students that has in general been satisfactory for a time for solving certain problems, and so becomes anchored in the mind but subsequently when faced with new problems it proves to be inadequate and difficult to adapt (Tall, 1994). Such obstacles can be overcome through increasing power of a representational system (Goldin & Shteingold, 2001). Broadly speaking, since the representational systems are linked to each other then the obstacles can be overcome. For instance, the representation of Newton's second laws of motion as mentioned in figure 1 can help understand the law by relating the concept of rest, motion, acceleration and velocity. Similarly, if the representation were developed through wider perspective, it would be more helpful future learning. For instance, if the multiplication system of "whole number" were discussed and represented as a change of "scale" then it would be easy to deal the multiplication of fraction.

7. **Representation is not a method but as a part of process or means of constructing mathematical ideas.**

Representation is neither a method of teaching nor a theory of learning. It is a means of constructing mathematical ideas. It can be helpful to develop mathematical ideas explicitly through the different representational system. By representation students consolidate their ideas in a systematic ways. Furthermore, the representational system can help develop the categories and subcategories of the ideas of what they represent. Broadly speaking, representation helps on simplification of paradigmatic structure of learning of mathematical knowledge.

Conclusion

Representation is one of the indispensable and recently realized means of mathematics learning. We can categorize representation into three types, external representation, shared representation and internal representation. External representations are symbols, signs and signals while shared representation are the representations those occur in shared mode. Moreover, internal representations are verbal/syntactic, imagistic, formal notational and affective. The role of representation in classroom learning can be enlisted as a source of communication; as an indicator of students' attitude towards mathematics; means of probing understanding; means of establishing links between the concepts; as a part of the developmental process of procedural-conceptual continuum; as a means of overcoming cognitive obstacles; and as a process or means of constructing mathematical ideas.

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