Nuclear Matter Critical Temperature and Charge Balance

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***Abstract*- An iterative algorithm was developed to fit Fisher Law for Heavy Ion Collisions with distinct charge balance, obtaining different critical temperatures in agreement with recent theoretical and experimental results. This way is confirmed the influence of charge balance on the caloric curve of nuclear matter.**

Keywords: Iterative Algorithm, Polynomial Fitting, Critical Temperature, Nuclear Caloric Curve

I. INTRODUCTION

Weiszäcker nuclear energy contains a term related to charge balance, namely the difference between the number of neutrons and protons, which should lead to a change in the limit temperature of the nuclear caloric curve[1]. Weiszäcker's expresion for nuclear energy is given by [2]:

$$E = -a_V A + a_S A^{2/3} + a_a \frac{(N-Z)^2}{A} + a_C \frac{Z(Z-1)}{A^{1/3}} \qquad (1)$$

where the first term represents bulk symmetric nuclear matter energy excluding coulombian interactions. The rest of the terms compensate nuclear energy disagreement with the asymmetric nuclear matter bulk limit. These terms are related to surface energy, to the asymmetry between neutrons N and protons Z and to coulombian repulsion, respectively.

Nuclear matter critical temperature is related to other variables such as system finite size and entropy [3], therefore the results hereby reported may help to reach a better understanding about the factors influencing the characteristics of the nuclear caloric curve. Other authors [4] have reported nuclear limit temperatures taking on account the balance between charge Z and the number of neutrons N, via the correlation between the factor N/Z for the fragment projectile and the isobaric ratio Y(3H)/Y(3He), obtaining a limit temperature in the range of 6 to 7 MeV. Souza et al. [5] have applied an improved statistical multifragmentation model (ISMM) to proton and neutron rich sources, obtaining a temperature plateau close to 6 MeV, with a minimum difference between the limit temperatures of the neutron and proton rich sources.

Fisher Law for nuclear matter can be written as [6]:

$$\ln\langle n_A \rangle = \ln q_0 - \tau \ln A + A\frac{\Delta\mu}{T} - \frac{c_0 \varepsilon A^{\sigma}}{T} \qquad (2)$$

using Fisher Liquid Droplet Model where q_0 is a normalization constant that depends only on the value of τ, τ is the critical topological exponent related to system dimensionality that can be computed through a tridimensional random walk in a closed surface, $c_0 \varepsilon A^{\sigma}$ is the free surface energy of a droplet of size A,

c_0 is surface energy coefficient, σ is the critical exponent related to the ratio of the surface dimensionality and volume dimensionality; and $\varepsilon = \left(1 - \frac{T}{T_C}\right)$ is the control parameter measuring the distance to the critical point whose temperature is given by T_C [7]. For a system with only a hundred of particles:

$$q_0 = 1 / \sum_{A=1}^{A_{\max}} A^{1-\tau} \tag{3}$$

where A is the fragment size.

An iterative algorithm was developed to fit Fisher Law using Heavy Ions Collisions with different charge balance (N-Z), obtaining critical temperature estimates whose difference is in the range of 1.2 MeV. This way is confirmed the minimal influence of the term (N-Z) on nuclear matter critical temperature.

II. METHODOLOGY

Forty thousand Heavy Ion collisions were simulated using Latino Model [8], where system evolves following a Newtonian dynamics via a Verlet algorithm. Internucleonic forces are computed with a Pandharipande potential. Fisher Law parameters were approximated by a fourth order polynomial in the excitation. The iterative algorithm floats the Fisher Law critical exponents, performing a Least Squares Fit for the polynomial coefficients as explained in the following. Fisher Law is written as:

$$\ln \frac{\langle n_A \rangle A^\tau}{q_0} = A \frac{\Delta\mu}{T} - \frac{c_0 \varepsilon A^\sigma}{T} \tag{4}$$

the difference of chemical potentials $\Delta\mu$ as well as the surface energy adimensional coefficient C_0 are approximated by polynomials on the excitation e^*:

$$\mu = P_{l.m}^{(1)}\left(e^*\right) \tag{5}$$

$$C_0 = P_{l.m}^{(2)}\left(e^*\right) \tag{6}$$

where:

$$P_{l.m}(x) = \sum_{n=0}^{m} a_n x^n \tag{7}$$

leading to a least squares problem whose solution is given by:

$$\mathbf{M}^T Y = \mathbf{M}^T \mathbf{M} A \tag{8}$$

where A is a vector containing the polynomial coefficients of $P_{l.m}$, $\mathbf{M}$ is a matrix and Y is a vector obtained from the system of equations:

$$\left[\ln \frac{\langle n_A \rangle A^\tau}{q_0}\right]_i = A\left[\frac{P_{l,m}^{(1)}}{T}\right]_i - \left[\frac{P_{l,m}^{(2)} \varepsilon A^\sigma}{T}\right]_i \tag{9}$$

where i is an index related to each collision replica.

Random variables η_x are generated to float Fisher Law critical exponents as well as the critical excitation:

$$\tau^{(t+1)} = \tau^{(t)} + \eta_\tau \tag{10}$$

$$\sigma^{(t+1)} = \sigma^{(t)} + \eta_\sigma \tag{11}$$

$$e_C^{*\,(t+1)} = e_C^{*\,(t)} + \eta_{e_C^*} \tag{12}$$

During the iteration the values of these critical exponents that optimize the statistical χ^2 are kept [9] until the best fit is attained. In this fashion were simulated twenty thousand replicas of Zn^{76}+Ca^{40} and Zn^{76}+Ar^{40} Heavy Ion Collisions. This way an estimator for the critical excitation for these collisions was obtained.

Once the best estimator was obtained, it was used to estimate the critical temperature via the following relation :

$$e_C^* = E_{x,c} / A = T_C^2 / 13 \tag{13}$$

This relation assumes a Fermi degenerate gas behavior and is in better agreement with experiment results than the empirical thermometers based on isotope ratios, as shown by Moretto et al. in a study about the evaporation in compound nucleus decay [10]. Nevertheless, other studies have criticized the hypothesis of a Fermi degenerate gas behavior close to the critical point, as long as the relation $E/A=aT^2$ holds for fragments produced at moderate low temperatures and in the case of intermediate energy collisions there are fast particles emitted in the final state from the region where the projectile and fragment overlap.

Heavy Ion collisions were simulated using LATINO semiclassical model where binary interaction is reproduced with a Pandharipande potential given by:

$$V_{nn} = V_{pp} = V_0\left(\frac{e^{-\mu_0 r}}{r} - \frac{e^{-\mu_0 r_C}}{r_C}\right) \quad (14)$$

and:

$$V_{np} = V_r\left(\frac{e^{-\mu_r r}}{r} - \frac{e^{-\mu_r r_C}}{r_C}\right) - V_a\left(\frac{e^{-\mu_a r}}{r} - \frac{e^{-\mu_a r_a}}{r_a}\right) \quad (15)$$

made up from linear combinations of Yukawa potentials whose coefficients are designed to reproduce nuclear matter properties and to fulfill Pauli Exclusion Principle [11].

Clusters are detected using an Early Cluster Recognition Algorithm that optimizes the configurations in energy space. Most Bound Partition is obtained minimizing the sum of cluster energies for each partition:

$$\{C_i\} = \arg\min\left[E_{\{C_i\}} = \sum_i E_{\text{int}}^{C_i}\right] \quad (16)$$

where the cluster energy is given by:

$$E_{\text{int}}^{C_i} = \sum_i \left[\sum_{ij\in C_i} K_j^{CM} + \sum_{j,k\in C_{iu},\, j\le k} V_{jk}\right] \quad (17)$$

in this expression the first sum is on the partition clusters, K_j^{CM} is the kinetic energy of particle j measured in the cluster mass center, and V_{ij} is the internucleonic potential. The algorithm uses the technique of "simmulated annealing" to find the most bound partition in energy space.

Ground states of neutron or proton rich sources, were built up starting from a random configuration with a given kinetic energy and confined in a parabolic potential. Nucleon speed was gradually reduced until the system was bound, afterwards the parabolic potential was supressed and a frictional method was applied until the system reached its theoretical binding energy (Fig. 1).

Projectile is boosted on target with a given kinetic energy for distinct impact parameters. System evolution was simulated using a Verlet algorithm [12], where two Taylor expansions are substracted, one of them forwards and the other backwards on time:

$$\vec{r}(t+\Delta t) = 2\vec{r}(t) - \vec{r}(t-\Delta t) + \vec{a}(t)h^2 \quad (18)$$

$$\vec{v}(t+\Delta t) = \vec{v}(t) + 0.5 * \left[\vec{a}(t+\Delta t) + \vec{a}(t)\right]h \quad (19)$$

$$\vec{a}(t+\Delta t) = -(1/m)\nabla V\left(\vec{r}(t+\Delta t)\right) \quad (20)$$

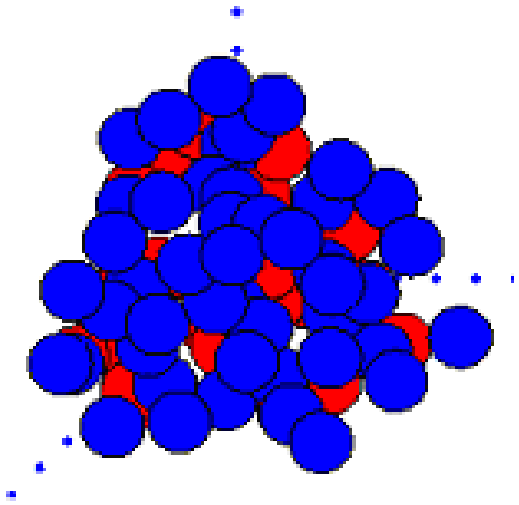

Fig 1.- Shows the ground state of the Heavy Ion Zn^{76} obtained starting from a random configuration, subsequently confined in a parabolic potential and finally cooled by a frictional method until it attains its theoretical binding energy.

Excitation is computed by the temperature attained by the projectile-target compound, when the maximal compression is reached. This temperature is estimated using Kinetic Theory for the n nucleons in the compound:

$$\frac{3}{2}nT = K^{CM} \quad (21)$$

Projectile energy is varied in the range going from 600 up to 2000 MeV and system evolves until its microscopic composition remains frozen (Fig. 2), although some monomers might be ejected. This time can be determined using the Microscopic Persistence Coefficient, defined as the probability of having two particles linked in a cluster of partition X still bound in a cluster of partition Y:

$$P[X,Y] = \frac{1}{\sum_{cluster} n_i} \sum_{cluster} n_i a_i / b_i \quad (22)$$

where b_i is equal to the number of particles that belong to cluster C_i of partition $X \equiv \{C_i\}$ and a_i is equal to the number of particle pairs belonging to cluster C_i of partition

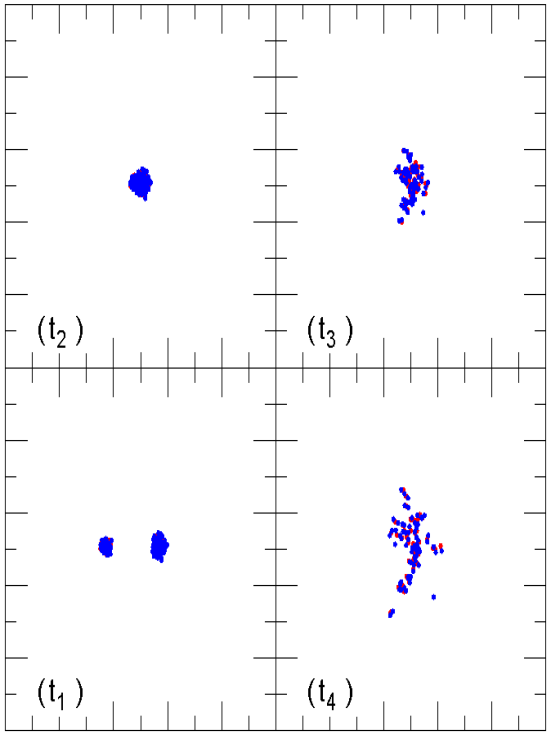

Fig 2.- Shows the evolution of central collision Zn^{76}+Ar^{40} for a projectile energy equal to 1600MeV, simulated with model LATINO.

$X \equiv \{C_i\}$ that also belong to a given cluster C_i' of partition $Y \equiv \{C_i'\}$. n_i is the number of particles in cluster C_i. Fig. 3 shows that persistence attains an asymptotic limit value once the biggest fragment size (FM), as well as the logarithmic derivative of the kinetic energy transported by light fragments and the logarithmic derivative of the number of intermediate fragments are altogether stable.

The ratio of isotope yields:

$$Y_2(N,Z)/Y_1(N,Z) \propto \exp(\alpha N + \beta Z) \qquad (23)$$

has been commonly used as a signature of thermodynamic equilibrium. Nevertheless Latino Model has been used elsewhere to prove that this signature holds at early stages of the Heavy Ion Collision, when the biggest fragment temperature is out of equilibrium [13]. Fig. 4 shows a χ^2 fit of the ratio of isotope yields for one hundred thousand replicas of $^{76}Zn + ^{40}Ar$ and $^{76}Zn + ^{40}Ca$ Heavy Ion Collisions using Latino Model, obtaining Isoscaling Parameters,

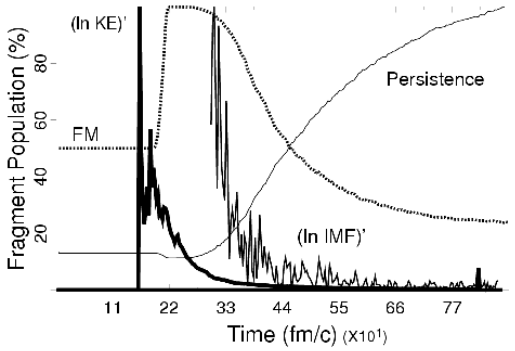

Fig 3.- Persistence attains an asymptotic limit value once the biggest fragment size (FM), as well as the logarithmic derivative of the kinetic energy transported by light fragments and the logarithmic derivative of the number of intermediate fragments are altogether stable. Central Collision Zn^{76}+Ar^{40} with a projectile energy equal to 1600MeV.

$\alpha = 0.35$ and $\beta = -0.40$, in agreement with those experimentally reported by Liu et al. in [14].

III. Results

Applying an iterative algorithm that floats Fisher Law critical exponents, an estimator is obtained for the critical excitation that optimizes Fisher Law fitting using data from Heavy Ion Collisions simulations for distinct charge balances (N-Z).

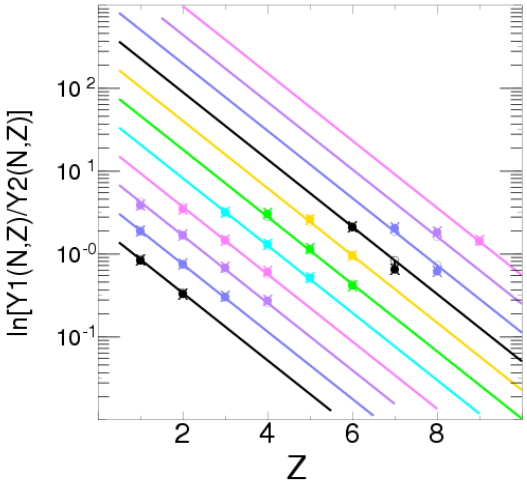

Fig. 4.- Best χ^2 fit of the ratio of isotope yields for one hundred thousand replicas of $^{76}Zn + ^{40}Ar$ and $^{76}Zn + ^{40}Ca$ Heavy Ion Collisions using Latino Model

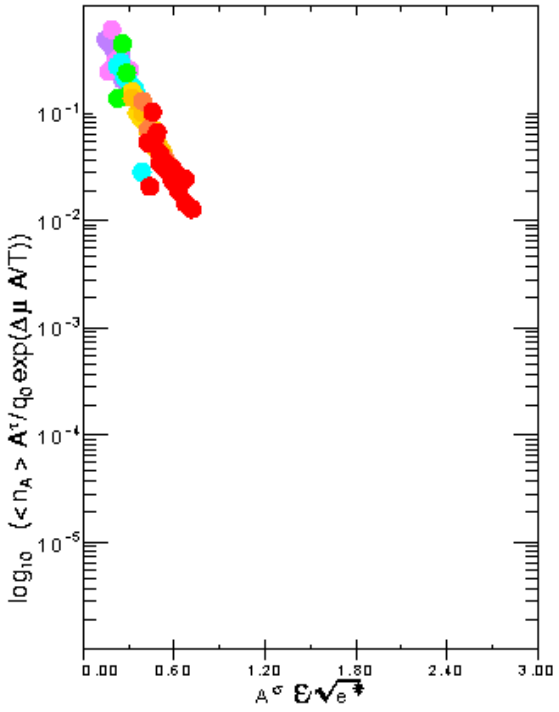

Fig 4.- Best χ^2 fit of Fisher Law, when twenty thousand replicas were performed for Zn^{76}+Ca^{40} collision using LATINO model.

This way, critical temperature estimates equal to 7.5MeV for collision Zn^{76}+Ca^{40} (Fig. 5) and 8.7 MeV for collision Zn^{76}+Ar^{40}, were obtained. These values are close to those obtained by Tapas Silk et al. [15], using as criticality signature the maximum of the constant volume heat capacity. And the difference between the critical temperatures hereby estimated comes out to be minimal, in agreement with the results reported by Souza et al. [5].

IV. Conclusions

An iterative algorithm was developed to obtain computational evidence about the influence of the term (N-Z) on nuclear matter critical temperature, comparing estimates for collisions with distinct values of (N-Z). The difference between the critical temperatures estimated was of about 1MeV for collisions Zn^{76}+Ca^{40} and Zn^{76}+Ar^{40}, in agreement with experimental and theoretical results recently reported about the minimal influence of isospin on nuclear matter critical temperature.

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