Physics 535 Project Two: Dynamics of the Solar System

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Physics 535
for Professor D. Hobill

Question One

(a) Below is a plot of the orbits of Earth and Jupiter.

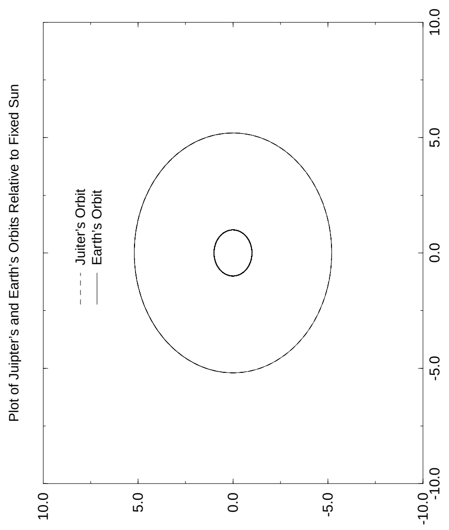

Figure 1: The orbits of Jupiter and Earth

Although the orbits of Earth and Jupiter seem to be confined closer inspection reveals that they are mutually influential as is evident by the fact that successive revolutions about the sun are not along the same path.

(b) Figure 2 depicts the orbit of Earth with the mass of Jupiter increased by 1000 times.

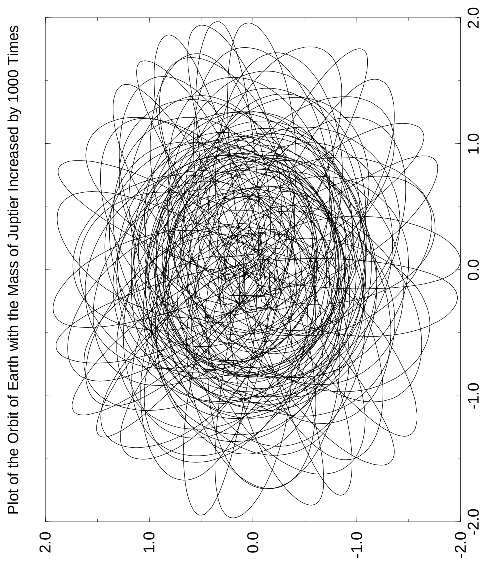

Figure 2: The orbit of Earth with the mass of Jupiter increased by 1000 times, fixed Sun

This change in the mass of Jupiter, and therefore the gravitational force on Earth, has changed the orbit of Earth from apparent regularity to apparent irregularity.

If we no longer hold the Sun fixed, compilation of the program to compute the orbit of earth is only possible for a time interval of three years. Below is a plot of Earth's orbit in this case:

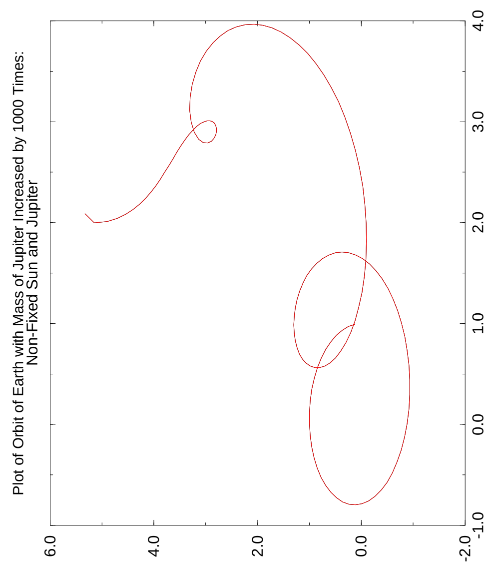

Figure 3: The orbit of earth with the mass of Jupiter increased by 1000 times, non-fixed Sun

Question Two

(a) The orbits of Jupiter and Asteroid Number 2 are plotted in Figure 4. To derive an expression for the length of the semimajor axis of a circular orbit (the radius of the orbit) we note that the centripetal force,

$$F_c = m\frac{v^2}{r} \tag{1}$$

must be equal to the graviational force,

$$F_g = \frac{GM_sM_a}{r^2}. \tag{2}$$

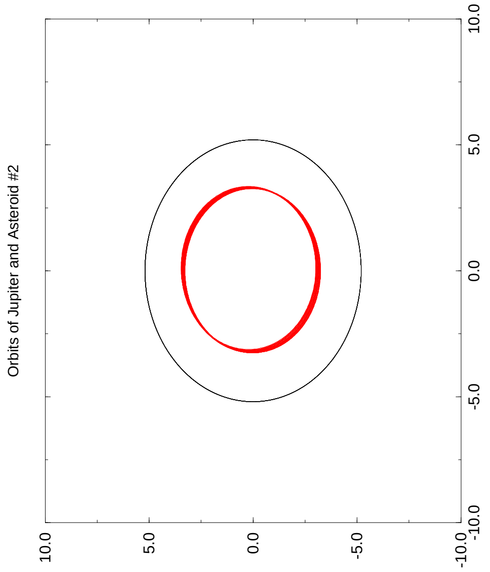

Figure 4: The Orbits of Jupiter and Asteroid Number 2

Equating expressions (1) and (2) we have,

$$mv^2 = \frac{GM_sM_a}{r} \Rightarrow r = \frac{GM_sM_a}{mv^2} = \frac{GM_sM_a}{2E}$$

Figure 5 is a plot of the variation in the semi- major axis length versus time for Asteroid 2

The smearing effect of the orbit can be measured by considering the difference in minimum semi-major axis length between successive orbits. In doing so, over ten Jupiter orbits we notice a smearing of Asteroid Number One's semi-major axis length of about $0.2AU$. However, we should note that this smearing affect is not linear. The amount of smearing that occurs between successive orbits increases because the asteroind is in resonance with Jupiter so any purturbations are continually amplified.

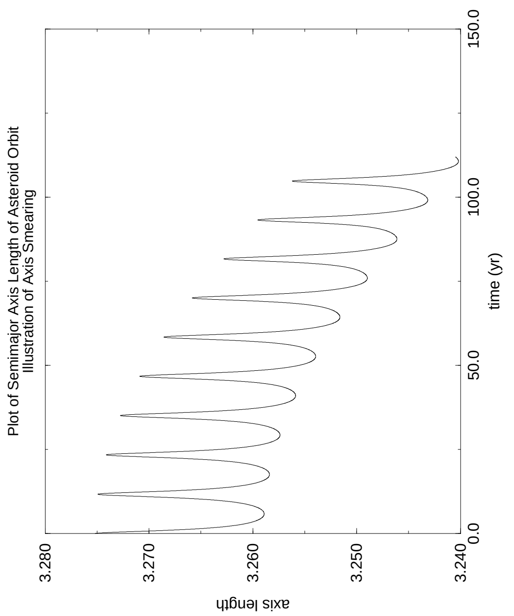

Figure 5: A plot of the semi-major axis length versus time for Asteroid 2

(*b*) The orbits of asteroid one and Jupiter are plotted below.

Figure 7 shows the time variance of the semi-major axis length of the orbit of asteroid 1.

We should notice that there is no noticeable smearing occuring in this orbit. The orbits of Asteroids three and the time evolution of its semimajor axes are plotted in the following graphs. Neither of the graphs of the semi-major axes of Asteroid one and three indicate smearing of the axis lengths. We can attribute this to the higher order resonance of each orbit wherein perturbations are not amplified. We might be inclined to conclude that Jupiter's effect on each orbit is noticeable only when the resonance of the Asteroid's orbit with Jupiter's is of low order.

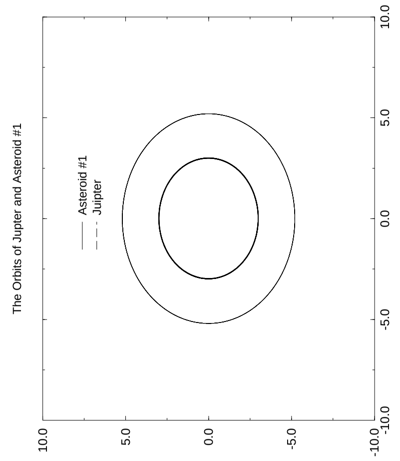

Figure 6: The orbits of Jupiter and Asteroid 1

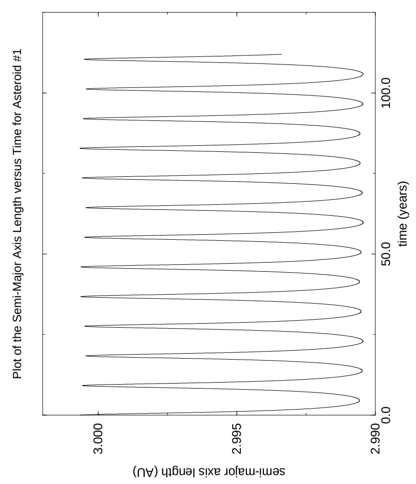

Figure 7: Variations in the semi-major axis length of the orbit of Asteroid 1

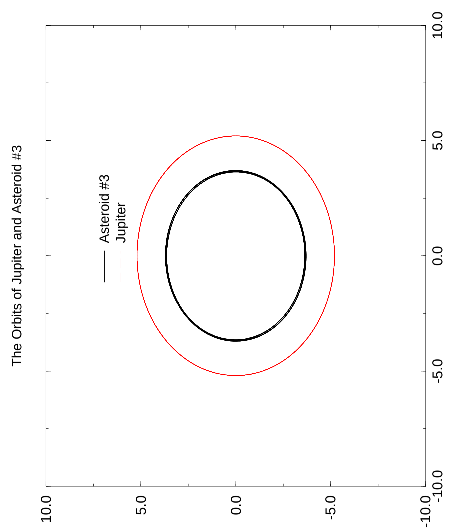

Figure 8: The orbits of Asteroid three and Jupiter

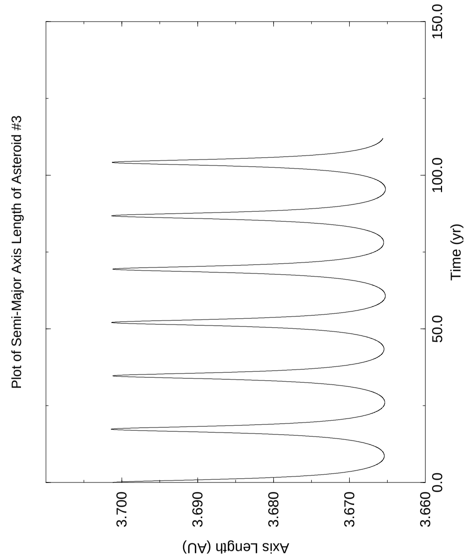

Figure 9: The time evolution of the semimajor axis length of asteroid three