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THE DYNAMICS OF DETERMINISTIC CHAOS IN SINGLE MODE OPTICAL FIBRE LASER TRANSMISSION

A. Mary Selvam

Indian Institute of Tropical Meteorology, Pune 411 005, India.

ABSTRACT

A two stream (red and blue) energy cascade mechanism is proposed for large and small eddy generation from energy supply at a fundamental pump frequency. Unidirectional energy pulses radiating from the molecular/atomic aggregates of the non-linear laser optical medium are associated with corresponding vortex roll circulations analogous to convection cells in the atmospheric planetary boundary layer. Applying fluid dynamical concepts, persistent energy pulses integrated over large eddy length scales gives rise to low frequency pulses and also energises the higher frequency pulses (random noise) in the medium. Energy supply at any one frequency, in course of time generates a continuum emission which is scale invariant, self similar with fractal distribution characteristics and is associated with deterministic chaos. Quantum mechanical laws therefore imply fractal distribution characteristics and is observed in all natural phenomena.

INTRODUCTION

The universal self generating period doubling route to chaos has been observed in lasers and non-linear optical systems which are basically governed by quantum mechanical laws (Harrison and Biswas, 1986). Optical fibres which confine light beams to small cross-sections over long lengths makes it possible to study such non-linear processes at relatively low pump intensities and enables to understand the role of quantum mechanical laws in analogous natural phenomena. Experimentally, many diverse systems have been found to exhibit the characteristic behaviour associated with deterministic chaos, examples including chemical and biological systems as well as those from physics (Fairbairn, 1986). Mitchell Feigenbaum (1981) discovered that a few universal ratios-independent of any dynamical details-characterised all systems where periods doubled as they approached turbulence. At the point of infinite period doubling the orbits of Feigenbaum's system showed a complex behaviour in which one could discern a scale invariant or fractal structure (Levi, 1986). Phenomenological observation of fractal structures in nature represent the two fundamental symmetries of nature, namely, dilation ($r \longrightarrow ar$) and translation ($r \longrightarrow r+b$) and correspond respectively to change in unit of length or in the origin of the coordinate system (Kadanoff, 1986). A self similar object is identified by its fractal dimension D which is defined as $d\ln M(R)/d\ln R$ where $M(R)$ is the mass contained within a distance R from a typical point in the object. Self similar growth processes in nature lead to the observed universal fractal geometry of macroscopic structures in natural phenomena. However, the basic physical mechanism of the self organised fractal geometry in nature is not yet identified (Kadanoff, 1986). In the following it is shown that the universal period doubling route to chaos is the growth mechanism for macroscopic eddy energy structures from space-time averages of inherent microscale eddy energy production mechanisms, there being a two way ordered energy feedback between the larger and smaller scales, such that the energy continuum exists as a dynamic unified whole spanning several orders of magnitude in scales. Growth processes in nature occur by inherent self organisation in self similar structures implied in the period doubling route to chaos and is analogous to the 'Bootstrap' theory of Chew (1968) and the theory of implicate order envisaged by Bohm (1951).

MODEL FOR FRACTAL GEOMETRY IN NATURE

A striking example of self similar fractal geometry in nature is exhibited by the global cloud cover pattern. Macroscopically different shaped clouds are self similar fractals over a number of orders of magnitudes of length scales (Lovejoy and Schertzer, 1986), in the turbulent planetary atmospheric boundary layer (ABL). The observed self similar organisation of cloud growth phenomena in the ABL has been attributed to space-time integration of inherent self organised microscale energy production mechanisms and is manifested as the period doubling route to chaos (Mary Selvam, 1986; Mary Selvam and Murty, 1987). In summary, the physical theory involves self organised ordered two way energy cascade between larger and smaller scales in an eddy continuum which is inherently scale invariant and exists as a dynamic unified whole associated with a space time continuum energy display of diverse manifested natural phenomena appropriate to the characteristic of the medium for eddy energy propogation. In the following, universal unique quantitative relations are derived for the evolution of organised eddy energy structures by the period doubling route to chaos and applied to model atmospheric turbulence and, by analogy, interpret chaos in optical phenomena.

PHYSICS OF PERIOD DOUBLING ROUTE TO CHAOS

The period doubling route to chaos is basically a growth phenomena whereby large eddy growth is initiated from the turbulence scale in successive length step increments equal to the turbulence scale length. A representative example is the organised growth of large eddy or vortex roll circulations from the turbulence scale in the atmospheric planetary boundary layer and manifested as cloud rows and cloud streets in satellite pictures of global cloud cover (Eymard, 1985). The Rayleigh-Benard instability observed in laboratory experiments (Ananthakrishna, 1986) simulates analogous vortex roll circulations and is an example of a self organised system.

Turbulent eddies of frictional origin at the planetary surface possess an inherent upward momentum flux which is progressively amplified by buoyant energy supply from microscale fractional condensation (MFC) of water vapour on hygroscopic nuclei by deliquescence even in an unsaturated environment (Pruppacher and Klett, 1978). The exponential decrease of atmospheric density with height further accelerates the turbulence scale upward momentum flux. Therefore the unidirectional (upward) turbulence scale energy pump generates successively larger vortex roll circulations in the ABL. The larger eddies carry the turbulent eddies as internal circulations which contribute to their (large eddies) further growth. A conceptual model of the large and turbulent eddies in the ABL is given at Fig.1.

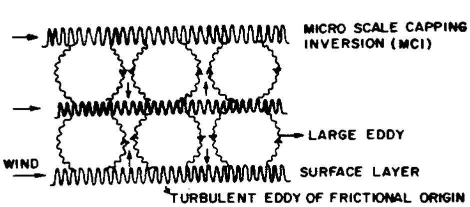

Figure 1 : Conceptual model of large and turbulent eddies in the ABL.

Townsend (1956) has investigated the structure and dynamics of large eddy formations in turbulent shear flows and has shown that large eddies of appreciable intensity form as a chance configuration of the turbulent motion as illustrated in the following example. Consider a large eddy of radius R which forms in a field of isotropic turbulence with turbulence length and velocity scales 2r and w respectively. The dominant turbulent eddy radius is therefore equal to r. The mean square circulation around a circular path of large eddy radius R is given by

$$W^2 = \oint \oint w w_1 ds\, ds_1$$

where w, w_1 are the tangential velocity components at the positions of the path elements ds and ds_1. If the velocity product falls to zero while the separation of ds and ds_1 is still small compared with the large eddy radius R, i.e., the motion in sufficiently separated parts of the flow is statistically independent

$$W^2 = 4 \pi R\, 2 r w^2$$

The root mean square (r.m.s.) velocity of circulation W in the large eddy of radius R is

$$W^2 = \frac{2}{\pi} \frac{r}{R} w^2 \qquad (1)$$

The above equation can be applied directly to derive the r.m.s. circulation speed W of the large eddy of radius R generated by turbulence scale energy pump. The scale ratio Z is equal to the ratio of the radii of the large and turbulent eddies. The environment of the turbulent eddies is a region of buoyant energy production by condensation (in the troposphere) and is therefore identified by a microscale capping inversion (MCI) layer on the large eddy envelope (Fig.1.). An incremental growth dR of large eddy radius equal to the turbulent eddy radius r occurs in association with an increase dW in large eddy circulation speed as a direct consequence of the buoyant vertical velocity w_* production per second by MFC. The MCI is thus a region of wind shear and temperature inversion in the ABL. The growth of large eddies from the turbulence scale at incremental length steps equal to r (turbulence length scale doubling) is therefore identified as the universal period doubling route to chaos. The scale invariant eddy energy continuum generated by the period doubling route to chaos growth process has inherent eddy energy circulations in a hierarchical nested logarithmic spiral vortex patterns of outward and compensating inward flow. Such a self organised ordered energy flow pattern is manifested as the strange attractor design in the region of chaos in various forms of manifestation, namely, the planetary rings, chao-tic emission in non-linear optical devices and the hurricane spiral cloud formation. The scale invariant eddy continuum exists as unified whole with ordered energy flow between its individual constituent members. Therefore the fine scale structure of the planetary rings originate from the microscale design in turbulent eddies at the planetary surface.

DILUTION OF LARGE EDDY VOLUME BY MASS EXCHANGE WITH ENVIRONMENT

The turbulent fluctuations mix overlying environmental air into the growing large eddy volume and the fractional volume dilution rate k of the total large eddy volume across unit cross section on its envelope is equal to

$$k = \frac{w_*}{dW} \frac{r}{R} \qquad (2)$$

where w_* is the unidirectional turbulent eddy acceleration and dW the corresponding acceleration of the large eddy circulation (Mary Selvam et al., 1985) during the large eddy incremental length step growth dR equal to r. $k > 0.5$ for $Z < 10$. Therefore organised large eddy growth can occur for scale ratio $Z \gg 10$ only since dilution by environmental mixing is more than half by volume and erases the signature of large eddies for scale ratio $Z < 10$. Therefore a hierarchical scale invariant self similar eddy continuum with semi-permanent dominant eddies at successive decadic scale range intervals is generated by the self organised period doubling route to chaos growth process. The large eddy circulation speed is obtained by integrating Eqn (2) for large eddy growth from the turbulence scale energy pump at the planetary surface and is given as

$$W = \frac{w_*}{k} \ln Z \qquad (3)$$

$k = 0.4$ for $Z = 10$. This is the well known logarithmic wind profile relationship in the ABL (Holton, 1979) and k is designated as the Von Karman's constant and its value as determined from observations is equal to 0.4 (Hogstrom, 1985). The period doubling route to chaos growth process therefore generates a scale invariant eddy continuum where eddy energy flow structure is in the form of nested logarithmic spiral vortex roll circulations, a complete circulation consisting of the outward and inherent compensating inward flow. The region of chaos is the dynamic growth region of large eddy by turbulence scale energy pumping and the nested vortex hierarchical continuum energy structure is manifested as the strange attractor design. The particles in the region of chaos follow laws analogous to Kepler's third law-planetary motion as shown in the following. The periods T and t the large and turbulent eddies are respectively given as $2\pi R/W$ and $2\pi r/w$. Substituting for W/w Eqn (1) gives

$$\frac{R^3}{T^2} = \frac{2}{\pi} \frac{r^3}{t^2} \qquad (4)$$

R^3/T^2 is a constant for constant turbulence scale energy pump and therefore large eddy circulations follow laws analogous to Kepler's third law of planetary motion

OUTWARD ENERGY FLUX FROM SOURCE

The rising large eddy gets progressively diluted by vertical mixing due to the turbulent eddy fluctuations and a fraction f of surface air reaches the normalised height Z given by (Mary Selvam et al., 1985).

$$f = \frac{W}{w_*} \frac{r}{R}$$

From Eqns (2) & (3)

$$f = \sqrt{\frac{2}{\pi Z}} \ln Z \qquad (5)$$

Therefore

$$W = w_* f Z$$

The steady state fractional energy flux from source is dependent only on the dominant turbulent eddy radius.

EDDY ENERGY SPECTRAL SLOPE

The eddy energy power spectrum is conventionally plotted as $\ln E$ versus $\ln \nu$ where E is the eddy energy and ν its frequency

$$E = \frac{1}{2} \cdot \frac{4}{3} \pi R^3 W^2$$

The spectral slope S of the scale invariant eddy energy continuum is given as

$$\begin{aligned} S &= \Delta \ln E / \Delta \ln \nu \\ &= -\ln (R^3 W^2 / r^3 w^2) / \ln (R/r) \\ &= -2 \text{ for large } Z \text{ from Eqn.(1)} \end{aligned} \qquad (6)$$

Therefore the universal period doubling route to chaos eddy growth mechanism gives rise to an eddy energy continuum spectral slope equal to -2. The universal scale invariant -2 power spectrum for eddy energy has been observed in the molecular scale of the Brownian motion, the Johnson thermal noise in resistors, the subatomic energy levels and in the macroscale atmospheric boundary layer turbulence (Feynman et al., 1964; Van Zandt, 1982).

QUANTUM MECHANICAL NATURE OF THE EDDY ENERGY STRUCTURE

The kinetic energy KE per unit mass of an eddy of frequency ν in the hierarchical eddy continuum is shown to be equal to the $H\nu$ where H is the spin angular momentum of unit mass of the largest eddy in the hierarchy. The circulation speed W of the largest eddy in the continuum is equal to the integrated mean of all the inherent turbulent eddy circulations. Let W_p be the mean circulation speed or the zero level about which all the smaller frequency fluctuations occur.

Therefore $2R = W_p / \nu$

$$KE = \frac{1}{2} W^2 = \frac{1}{2} \cdot \frac{2}{\pi} \cdot \frac{r}{R} w^2 \quad \text{from Eqn.(1)}$$

$$= H\nu$$

$$\text{when } H = \frac{2}{\pi} \cdot \frac{r w^2}{W_p} \nu = R W_p \qquad (7)$$

H is equal to the product of the momentum of unit mass of planetary scale eddy and its radius and therefore represents the spin angular momentum of unit mass of planetary scale eddy about the eddy centre. Therefore the kinetic energy of unit mass of any component eddy of frequency ν of the scale invariant continuum is equal to $H\nu$ which is analogous to quantum mechanical laws for the electron energy levels in the subatomic space. Further, in a later section in the following it is shown that the eddy energy probability density distribution is equal to the square of the eddy amplitude consistent with quantum mechanical phenomena. The energy manifestation of radiation and other subatomic phenomena appear to possess the dual nature of wave and particles since one complete eddy energy circulation structure is inherently bidirectional and associated with corresponding bimodal form of manifested phenomena e.g., formation of clouds in updraft regions and dissipation of clouds in downdraft regions giving rise to discrete cellular structure to cloud geometry.

Also from Eqn 7

$$\Delta E \cdot \Delta T = H = \text{constant} \qquad (8)$$

The above statement is analogous to Heisenberg's uncertainity principle for subatomic dynamics. In the context of the universal eddy energy continuum, Eqn (8) implies that large changes in eddy energy can occur only during short intervals of time and vice versa, illustrative examples being the hurricane systems on the one hand and climatic changes on the other.

STATISTICAL DISTRIBUTION CHARACTERISTICS OF THE EDDY CONTINUUM ENERGY FIELD

Fundamental classical statistical distribution functions commonly occurring in natural phenomena are shown to be inherent characteristics of the universal period doubling growth phenomena as follows. The distribution of means for sample size n has a variance W_2^2 which is related to the population variance W_1^2 as follows

$$W_2^2 = W_1^2/n$$

The above statistical relation may be derived from Eqn (1) in the context of variances of the eddy parameters for two different scale ratios Z_1 and Z_2 where $n = Z_2/Z_1$ as follows

$$W_1^2 = \frac{2}{\pi} \cdot \frac{r}{R_1} w^2 = \frac{2}{\pi} \frac{w^2}{Z_1}$$

$$W_2^2 = \frac{2}{\pi} \cdot \frac{r}{R_2} w^2 = \frac{2}{\pi} \frac{w^2}{Z_2}$$

Therefore $$W_2^2 = W_1^2/n$$

For eddy growth from smaller scale to the larger scale the ratio of eddy energy per unit mass of large eddy to that of the turbulent is equal to $1/n$ where n is the scale ratio. The eddy continuum generated by the successive integration of smaller scale circulations into large scale circulation patterns will therefore have an eddy energy spectral slope S by

$$S = -\frac{\ln(R_2^3 W_2^2 / R_1^3 W_1^2)}{\ln(R_2/R_1)}$$

$$= -2$$

S = -2 is in agreement with earlier derivation (Eqn 6) for eddy continuum generation from primary turbulent eddy scale r.

NORMAL DISTRIBUTION CHARACTERISTICS OF THE EDDY ENERGY PERTURBATION FIELD

Natural phenomena posses energy/geometrical structure which is found to follow statistical normal distribution characteristics. Such normal distributions conventionally attributed to random chance, are in reality the deterministic laws of the unified eddy dynamical processes, and therefore represent the implicit order in natural phenomena as shown in the following. Let P represent the probability of occurrence in themedium of bidirectional energy flux with characteristics of a particular large eddy radius R.

$$P = \frac{1}{2}\frac{W}{w_*} \text{ since } W \text{ originates from } w_*$$

$$= \frac{1}{\sqrt{2\pi}} \quad \frac{r}{R} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{KW}{w_*}} \qquad \text{from Eqn.(3)}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{r}{R}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{W^2}{w_*^2}} \quad \text{from Eqn.(8)}$$

Therefore W/w_* is distributed normally and is consistent since W is the integrated mean of w_* over the large eddy volume. Similarly since the large eddy is but the sum total of all the smaller scales, the large eddy energy content is equal to the sum of all its individual component eddy energies and therefore the energy E of any eddy of radius R in the eddy continuum expressed as a fraction of the energy content of the largest eddy in the hierarchy will represent the cumulative normal probability distribution. The eddy continuum energy spectrum is therefore the same as the cumulative normal probability distribution plotted on a log-log scale. The eddy energy spectral slope is derived from the cumulative normal distribution curve as follows.

The period doubling sequence for eddy growth gives $$\frac{r}{R} = \frac{dR}{R} = \frac{1}{2}$$

The standard deviation σ of the eddy fluctuations is equal to r. Therefore the period doubling sequence generates a large eddy of radius R where largest fluctuation corresponds to one standard deviation σ with a cumulative probability of occurrence equal to 100 dR/(R+dR) = 33 %. The cumulative normal probability distribution also gives 32% probability at one standard deviation in close agreement with the statistical parameters generated by the period doubling sequence. Further, the slope of the log-log plot of the cumulative normal probability curve at one standard deviation is equal to -1.8 in agreement with the computed (see Eqn 6) slope of -2 for the eddy energy spectrum.

PHYSICAL MEANING OF NORMAL DISTRIBUTION PARAMETERS

The statistical distribution characteristics of natural phenomena commonly follow normal distribution associated conventionally with random chance. The normal distribution is characterised by the moment coefficient of kurtosis equal to zero, signifying symmetry. The moment coefficient of Kurtosis is equal to 3 and represents the intermittency of turbulence. In the following it is shown that the universal period doubling route to chaos growth phenomena in nature gives rise to the observed statistical normal distribution parameters as a natural consequence. The period doubling route to growth is initiated and sustained by the turbulent (fine scale) eddy acceleration w_* which then propogates by the inherent property of inertia of the medium. Therefore, the statistical parameters mean, variance, skewness and Kurtosis of the perturbation field in the medium is given by w_*, w_*^2, w_*^3 and w_*^4 respectively. By analogy, the perturbation speed w_*(motion) per second of the medium sustained by its inertia represents the mass, w_*^2 the acceleration (or force), w_*^3 the momentum (or potential energy) and w_*^4 the spin angular momentum since an eddy motion has an inherent curvature to its trajectory. Because the eddy motion is inherently symmetric with bidirectional energy flow, the skewness factor w_*^3 is equal to zero for one complete eddy circulation thereby satisfying the law of conservation of momentum. The moment coefficient of Kurtosis which represents the intermittency of turbulence is shown in the following to be equal to 3 as a natural consequence of the growth phenomenon by the period doubling route to chaos.

From Eqn : 3 $dW = \frac{w_*}{k} d(\ln Z)$

$$\frac{(dW)^4}{w_*^4} = \frac{[d(\ln Z)]^4}{k^4} = \left(\frac{dZ}{kZ}\right)^4$$

$(dW)^4 / w_*^4$ represents the statistical moment coefficient of Kurtosis. Organised eddy growth occurs for scale ratio equal to 10 and identifies the large eddy on whose envelope period doubling growth process occurs. Therefore for a dominant eddy

$$k = \frac{w}{W} \cdot \frac{r}{R} = \sqrt{\frac{\pi}{22}} \quad \text{since} \quad \frac{r}{R} = \frac{1}{11}$$

$(dZ/Z) = 2$ for one length step growth by period doubling process since $Z = dZ + dZ$. Therefore moment coefficient of Kurtosis

$$= \left(\frac{1}{2}\right)^4 \times 49 \approx 3$$

In other words, period doubling growth phenomena result in a three fold increase in the spin angular momentum of the large eddy for each period doubling sequence. This result is consistent since perod doubling at constant pump frequency involves eddy length step growth dR on either side of the turbulent eddy length dR.

PHYSICAL MEANING OF THE UNIVERSAL FEIGENBAUM'S CONSTANTS OF THE PERIOD DOUBLING ROUTE TO CHAOS

The universal period doubling route to chaos has been studied extensively by mathematicians. The basic example with the potential to display the main featuresof the erratic behaviour is the Julia model (Delbourgo, 1986) given below

$$X_{n+1} = F(X_n) = L X_n (1 - X_n)$$

The above non-linear model represents the population values of the parameter X at different time periods n, and L parameterises the rate of growth of X for small X

The Eqn (1) representing large eddy growth as integrated space time mean of turbulent eddy fluctuations given as $W^2 = \frac{2}{\pi} \cdot \frac{r}{R} w^2$

is analogous to the Julia model since large eddy growth is dependent on the energy input from the turbulence scale. Therefore the well established abstract mathematical results for the Julia model can be interpreted in terms of physical processes occurring in nature as follows. Feigenbaum's research showed that the following two universal constants a and d are independent of the details of the non-linear equation for the period doubling sequence :

$$a = \lim_{n \to \infty} (DX^*_{2^{n+1}}) / (DX^*_{2^n})$$

$$= -2.5029$$

$$d = \lim_{n \to \infty} (L_{2^{n+2}} - L_{2^{n+1}}) / (L_{2^{n+1}} - L_{2^n})$$

$$= 4.6692$$

a and d therefore denote the successive spacing ratios of X and L respectively for adjoining period doublings.

The universal constants a and d assume different numerical values for period tripling, quadrupling etc and the appropriate values are computed by Delbourgo (1986). The prediction $3d = 2a^2$ was made by Eckmann, Epstein and Witter on analytical grounds for certain restricted domains and Delbourgo's (1986) computations show that the relation $3d = 2a^2$ has a much wider validity.

The physical concept of large eddy growth by the period doubling process enables to derive the universal constants a and d and their mutual relationship as functions inherent to the scale invariant eddy energy structure as follows.

From Eqn (1) the function a may be defined as

$$a^2 = \left(\frac{W}{w_*} Z\right)^2 = \frac{2Z}{\pi} \approx \frac{2}{3} Z \tag{9}$$

a is therefore equal to 1/k from Eqn (3) where k is the Von Karman's constant representing the non dimensional steady state fractional volume dilution rate of large eddy by turbulent eddy fluctuations across unit cross section on the large eddy envelope. Therefore a represents thenondimensional total fractional mass dispersion rate and a^2 represents the fractional energy flux into the environment.

Let d represent the ratio of the spin angular moments for the total mass of the large and turbulent eddies

$$d = \frac{W^4}{w_*^4} Z^3 = \frac{4}{\pi^2} Z \approx \frac{4}{9} Z \tag{10}$$

Therefore $2a^2 = 3d$ from Eqns (9) and (10). $2a^2$ represents the total eddy energy flux into the medium.

The spin angular momentum of the resulting large eddy accounts for the observed value of three for the moment coefficient of Kurtosis of the normal distribution. Therefore the above equation relating the universal constants is a statement of the law of conservation of energy i.e. the period doubling growth process generates a three fold increase in the spin angular momentum of the resulting large eddy and propogates outward as the total large eddy energy flux in the medium. The property of inertia enables propogation of turbulence scale perturbation in the medium by release of the latent energy potential of the medium. An illustrative example is the buoyant energy generation by water vapour condensation in the updraft regions in the atmospheric boundary layer.

A no scale mass M can now be defined for the eddy mass with respect to the primary eddy perturbation mass w_* from Eqn (1) as follows : $M = (1/2)W$ since phenomenological manifestations of mass occur only during one half cycle of the eddy perturbation. Therefore for the basic period doubling growth process R = 2r and

$$M = \frac{1}{2}\sqrt{\frac{2}{\pi}\cdot\frac{r}{R}}\quad w_* = m_p/\sqrt{8\pi}$$

where $m_p = w_*/\sqrt{2}$ represent the mean of all possible period doubling perturbations corresponding to the w_* population in the medium. The above concept is analogous to the no scale super gravity model (Lahanas and Nanapoulos, 1987) where M, the super-Planck mass is given in terms of the Planck scale m_p ($\approx 10^{19}$ GeV). The virtues of the no scale super gravity model are automatically vanishing cosmological constant (atleast at the classical level), dynamical determination of all mass scales in terms of the fundamental planck scale m_p and acceptable low energy phenomenology. The no scale structure is super symmetric since it fuses together the non-trivial internal symmetries of the inherent small scale eddies with the space time (Poincaré) symmetries of the unified eddy continuum structure and accounts for the observed fractal geometry in nature.

FINE STRUCTURE OF EDDY ENERGY LEVELS FOR THE PERIOD DOUBLING SEQUENCE

The increase in energy (Δ E) associated with each period doubling sequence in the large eddy growth process is equal to (4/3)Z as shown earlier. The large eddy energy fine structure ΔE is computed and shown in the following to be equal to 137 for a scale ratio Z = 100. Since the large eddy of scale ratio Z = 100 consist inherently of the dominant small scale eddies at decadic scale range intervals, period doubling at Z = 100 involves energy inputs from (1) period doubling at Z = 100 (2) the statistical mean of all period doubling at Z = 10 and (3) the integrated mean of all period doubling of the primary large eddy Z = 2.

Therefore $E = (4/3)\left(100 + \frac{10}{\sqrt{2}}\right) + \frac{2}{\pi}$

≈ 137

Therefore the eddy energy structure constant for period doubling at Z = 100 is equal to 137.

EDDY ENERGY/MASS RATIOS FOR THE PERIOD DOUBLING SEQUENCE

The eddy energy/mass ratio is computed for a period doubling sequence at Z = 10 and shown to be equal to 1833. Period doubling involves length step growth of large eddy on either side of the turbulent eddy length and therefore involves a threefold increase in the total eddy perturbation length in the medium. Since Z = 10 consists of the smaller dominant eddies at Z = 1 and 0.1, the fine structure scale ratio with respect to the primary eddy is equal to 33.3/1.1 = 30.27. The eddy energy ratio corresponding to the scale ratio 30.27 is equal to 916.4 from the scale invariant spectral slope value of -2. Therefore the total large eddy circulation energy (mass) is equal to 1833.

CURVATURE OF TRAJECTORY INHERENT TO THE SELF ORGANISED EDDY GROWTH PHENOMENA IN NATURE

The self organised eddy growth in nature involves successive radial growth steps equal to turbulence length scale along with a corresponding eddy angular rotation from origin. The eddy growth originating from O (Fig. 2) follows the spiral

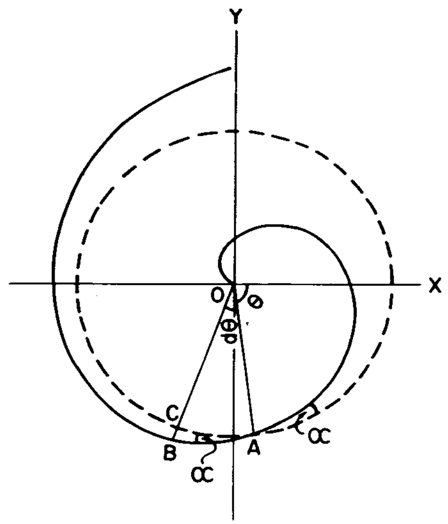

Figure 2 : The unique logarithmic spiral eddy eneregy flow track

curve OAB because of the inherent logarithmic energy flow trajectory at Eqn 3. The angular rotation from the origin at location A is measured with respect to axis OX. Let OA and OB denote the locations of the large eddy radii R and R + dR for a growth period of one second.

The angular rotation $d\bar{\theta}$ is given by

$$d\bar{\theta} = \frac{W}{R} = w_* f / r_R$$

AB is the tangent at A to the circle drawn with centre O and radius R so that

$AC = R d\bar{\theta}$

$BC = d\bar{R} = W = w_* f R / r_R + dR$

AB will also represent the tangent to the spiral at A for limited range. The angle BAC between the logarithmic spiral and its tangent is called the crossing angle α of the spiral.

$$\tan\alpha = BC/AC = d\bar{R}/R d\bar{\theta}$$

Substituting $b = \tan\alpha$ and integrating for eddy growth from r to R, the above equation gives

$$R = r e^{b\theta}$$

This is the equation for an equiangular logarithmic spiral when the crossing angle α is a constant. At any location A, the horizontal energy flow path into the eddy continuum system follows a logarithmic spiral track. A striking example is the hurricane spiral airflow track.

HORIZONTAL STRUCTURE OF ENERGY FLOW PATTERN OF THE UNIFIED EDDY CONTINUUM

The eddy energy continuum growth by the period doubling process involves radial growth dR equal to the dominant turbulent eddy radius r and accompanying angular rotation $d\theta$. Therefore

$$dR = r \quad \text{and} \quad R = \sum r$$
$$d\theta = t d\theta \quad \text{and} \quad \theta \propto \sum f$$

Horizontal width of energy inflow channel $= R d\theta$

The dominant turbulent eddy radius determines the angular turning $d\theta$ and incremental large eddy growth length dR and therefore the eddy continumm energy flow trajectory has different crossing angles and path widths at different locations with respect to the origin. A representative example is the hurricane spiral cloud cover pattern with different crossing angles and cloud band widths at different locations, the crossing angle decreasing near the storm centre where towering clouds indicate enhanced buoyant energy production. The mesoscale cloud clusters (MCC) of synoptic weather systems are macroscale manifestations of quantum mechanical laws with apparent wave-particle dual nature because of the different phenomenological forms of energy display in the inflow and outflow trajectories of one complete eddy cycle.

GROWTH TIME OF THE EDDY SPECTRUM

The eddy growth time T for an eddy of radius R is computed as follows.

$$T = \int dR/W = \frac{r}{w_*}\sqrt{\frac{\pi}{2}}\left(li\sqrt{Z}\right)_2^Z$$

where li is the logarithm integral or the Soldner's integral (Mary Selvam and Murty, 1985).

HORIZONTAL PROFILE OF EDDY CONTINUUM ENERGY FIELD

The turbulence scale energy pumping rate w_* maintains a potential energy structure in the medium where energy propogates by the property of inertia of the medium. Eddy perturbation releases the latent energy potential inherent to the medium and contributes to the sustenance of the eddy growth mechanism. Therefore for a scale invariant eddy continuum with maximum large eddy radius R, the potential energy generated at the origin with respect to the undisturbed environment at X (Fig. 2) by the turbulence scale energy pump is equal to $w_* \rho T_R$ where ρ is the density of the medium and T_R^* is the time for large eddy growth from the origin starting from the turbulence scale. Therefore the potential energy corresponding to any intermediate location N where the large eddy radius is R_N will be equal to $w_* \rho T_N$ where T_N is the time taken for large eddy propogation from N to X. Considering the potential energy perturbation of the eddy continuum with respect to the undisturbed environment at X, the normalised potential energy departure at the intermediate location N with respect to the maximum potential energy departure at the origin is computed as NPED $= T_N / T_R$

Earlier it was shown that the horizontal flow of eddy energy circulation follows the logarithmic law and depends only on the turbulent eddy radius as given by Eqn (5) $W = w_* fZ$. Therefore the normalised eddy energy circulation speed W/w_* is given by the nondimensional product fZ and is shown along with normalised horizontal profile of the eddy continuum potential energy field with respect to the undisturbed environment as given by NPED in Fig.3 with respect to the normalised length scale and gives the universal unique potential energy and circulation speed anomaly pattern in the scale invariant eddy continuum generated by the period doubling growth process.

EDDY CONTINUUM ENERGY EXCHANGE WITH ENVIRONMENT

The scale invariant eddy continuum is in continuous dynamic energy exchange with the environment in the turbulence scale and the relevant exchange parameters k and f are discussed in the earlier sections. Therefore the self organised scale invariant eddy energy structure does not exist in isolation, but is in constant two way communication with the environmental dynamic processes and therefore inherently linked with the total external environment.

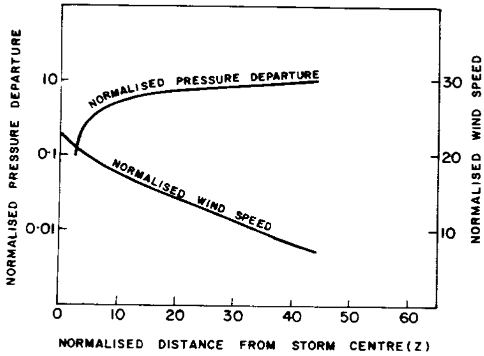

Figure 3 : The unique pattern of the no scale eddy energy flow speed and eddy potential energy field in the context of hurricane systems.

THE ROLE OF DETERMINISM AND CHANCE IN THE EDDY EVOLUTION MECHANISM

Eqn (1) which represents the formation of large eddy circulation from space-time integrations of small scale perturbations implies self consistent self organised two way energy feedback between the larger and smaller scales. Therefore at any instant in time the future evolution of any eddy scale is determined by (1) the integrated mean energy levels of the total scale invariant eddy continuum represented by the zero point energy level and therefore a function of all the past eddy perturbations in the space-time domain. The future eddy evolution will then originate from this predetermined zero point energy level and present moment growth is determined by the available latent energy potential of the medium, e.g., the buoyant energy supply in the troposphere. Therefore, in addition to chance, volitional direction of energy supply events during current moment in time along with all the past events determines the future energy base level. The dynamic space-time eddy energy continuum has an inherent curvature natural to eddy circulations and forms part of all energy/matter configurations in the universe.

CONCLUSION

The universal period doubling route to chaos is the self organised ordered growth mechanism of macroscale energy structures by space-time integration of inherent microscale eddy energy generation in the medium. The scale invariant eddy energy continuum generated by the period doubling growth sequence has a unique scale invariant spectral slope equal to -2. Therefore signal transmitted at any pump frequency can be retrieved with amplification of fine structure at the lower frequencies of the eddy continuum or vice versa. The no scale unique quantification of the period doubling fractal energy structure discussed above may help design of signal transmission devices which beneficially utilise the unified eddy energy continuum as transmission channels with amplification of signal fine structure, two disparate examples of possible applications being in laser signal transmission and in identifying the biological signal of malignant growths. The two way energy feedback inherent to the eddy continuum may enable beneficial control of growth processes in natural phenomena.

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