Reprinted from Preprint Volume 15th Conference on Severe Local Storms, Feb. 22-26, 1988, Baltimore, Md. Published by the American Meteorological Society, Boston, Mass.

P5.5

DYNAMICS OF DETERMINISTIC CHAOS AND APPLICATIONS TO MODELING OF SEVERE LOCAL STORMS

A. Mary Selvam

Indian Institute of Tropical Meteorology,
Pune 411 005, India

1. INTRODUCTION

The atmospheric cloud/weather systems are large coherent structures which form in the turbulent shear flow of the planetary Atmospheric Boundary Layer (ABL) and are associated with very high Reynold's numbers up to 10^{10} to 10^{12}. The existence of coherent structures (seemingly systematic motion) in turbulent flows has been well established during the last 20 years of research in turbulence. It is still, however debated whether these structures are the consequences of some kind of instabilities (such as shear, or centrifugal instabilities) or whether they are manifestations of some intrinsic universal properties of any turbulent flow. Numerical cloud modeling has not been successful, because, limitations of available computer capacity has required severe truncation of the Navier-Stokes equations conventionally used for simulating turbulent fluid flows. An understanding and a quantitative description of the formation of organised cloud/weather systems in the apparent chaos of the turbulent ABL will be possible with the formulation of model equations simpler than the Navier-Stokes equations and amenable to computer simulations but still capable of describing the main features of large scale turbulence (Levich, 1987). In this paper, a universal theory of chaos is presented and applied to model macroscale atmospheric turbulence (eddy dynamics) and cloud systems with simple analytic equations describing steady state microscopic domain eddy energy transport processes in terms of non-dimensional quantities for relative length scales and therefore applicable for all macroscopic scales. The proposed numerical theory for chaos dynamics is analogous to the recently identified computational methods of (1) 'cellular automata' (Frisch, Hasslacher and Pomeau, 1986) where it is suggested that a microscopic pseudo molucular model is more easily computable than the corresponding equations for fluid motion, and would certainly provide all needed information about macroscopic flows (2) 'randomly exact methods' which make use of stochastic algorithms that are only weakly dependent on dimensionality with ease of transition between classical and quantum mechanical discriptions. (Doll and Freeman, 1987). The universal period doubling route to chaos or deterministic chaos is a signature of non-linearity and is found to occur in diparate physical, chemical and biological systems (Feigenbaum, 1980; Fairbairn, 1986; Delbourgo, 1986). Feigenbaum (1980) established that the route to chaos is independent of the non-linear equations describing the system. Lorenz (1963) showed that deterministic chaos is exhibited by the three coupled non-linear ordinary differential equations for a heat convective system obtained by severe truncation of Navier-Stokes equations. Lovejoy (1981) and Lovejoy and Schertzer (1986), by their valuable study of the fractal dimension of global cloud cover pattern have established conclusively the existence of deterministic chaos in the ABL.

Satellite pictures of global cloud cover pattern show cloud organisation in streets/rows, and Mesoscale Cloud Clusters (MCC) indicating the existence of helical vortex roll circulations in the ABL (Eymard, 1985). However, it is not clear how such long lived coherent cloud configurations are maintained in the disipative turbulent environment of the ABL (Tennekes, 1973). In the following it is shown that small scale turbulence (chaos), conventionally regarded as dissipative, on the contrary, contributes to the deterministic growth of coherent, macroscopic cloud systems. It is shown in this paper that organised helical vortex roll (large eddy) circulations grow by space-time integration of turbulence scale buoyant energy generation with implicit ordered two way energy flow between the larger and smaller scales and such a mechanism for growth of self similar structures is identified as the universal period doubling route to chaos or deterministic chaos. The author has first made a detailed discussion of the universal theory for deterministic chaos followed by its application to formation of severe weather systems in the atmospheric boundary layer.

2. DETERMINISTIC CHAOS IN ABL AND FRACTAL GEOMETRY OF CLOUDS.

Mitchell Feigenbaum (1980) discovered that a few universal ratios -independent of any dynamical details -characterised all systems where periods doubled as they approached turbulence. At the point of infinite period doubling the orbits of Feigenbaum's system showed a complex behaviour

Eqn. (2) for large eddy growth from the turbulence scale energy pump at the planetary suraface and is given as $W = \frac{w_*}{k} \ln Z$. $k = 0.4$ for $Z = 10$. The energy flow structure in the eddy continuum is in the form of nested logarithmic spiral vortex roll circulations where particle trajectories follow Kepler's third law of planetary motion (Mary Selvam and Murty, 1987) since R^3/T^2 is a constant from Eqn. (1), T being the large eddy circulation time period. Further, the period doubling growth sequence which governs all natural phenomena being basically a space-time integration of microscale pertubation events leads to the following conclusions as a natural consequence (1) the commonly observed nomal distribution characteristics for the energy/geometrical structure in nature is consistent (2) the eddy energy continuum spectrum is the same as the cumulative normal probability distribution (3) the kinetic energy of unit mass of any component eddy of frequency ν of the scale invariant eddy continuum is equal to $H\nu$ where H is the spin angular momentum of the largest eddy in the continuum (Mary Selvam, 1987). Therefore, the eddy continuum energy structure follows laws analogous to quantum mechanical laws for subatomic phenomena. It follows that subatomic phenomena appear to possess the dual nature of wave and particles since one complete eddy energy circulation is inherently bidirectional and associated with corresponding bimodal form for manifested phenomena e.g., formation of cloud in updraft regions and dissipation of clouds in downdraft regions giving rise to discrete cellular structure to cloud geometry which is therefore basically a quantum mechanical phenomena in the macroscale.

In the following it is shown that the universal period doubling route to chaos growth phenomena in nature gives rise to geometrical structures which follow statistical normal distribution parameters as a natural consequence. The period doubling route to growth is initiated and sustained by the turbulent (fine scale) eddy acceleration w_* across unit cross section and propogates by inherent property of inertia of the medium. Therefore, the statistical parameters, mean, variance, skewness and kurtosis for the geometry of the pertubation field in the medium is given by w_*, w^2_*, w^3_* and w^4_* respectively. By analogy, perturbation speed w_* (motion) per second of the medium sustained by its inertia represents the mass, w^2_* the acceleration (or force), w^3_* the momentum (or potential energy) and w^4_* the spin angular momentum, since an eddy motion has an inherent curvature to its trajectory. The eddy motion is inherently symmetric with bidirectional energy flow and therefore, the skewness factor w^3_* is equal to zero for one complete eddy circulation, thereby satisfying the law of conservation of momentum. The moment coefficient of kurtosis, a measure of the intermittency of turbulence is given by $(dW)4/w4_*$ and is shown to be equal to 3 from (1) and (2) (Mary Selvam, 1987) indicating that period doubling growth phenomena results in a threefold increase in the spin angular momentum associated with the large eddy generation and is consistent since period doubling growth occurs on either side of the primary turbulent eddy.

4.1 Physical meaning of the universal Feigenbaum's Constants

The universal period doubling route to chaos has been studied extensively by mathematicians (Feigenbaum, 1980; Delbourgo, 1986) who found that two universal constants a and d describe the approach to turbulence independent of the details of the non-linear equations describing the physical system. Delbourgo's (1986) computations show that the universal constants a and d follow the relation $3d = 2a^2$ over a wide domain. The physical concept of the large eddy growth by period doubling process enables to derive Feigenbaum's universal constants a and d and their mutual relationship as functions inherent to the scale invariant eddy energy structure as follows. From Eqn. (1) the function a may be defined as

$$a^2 = (WZ/w_*)^2 = 2z/\pi \simeq 2Z/3 \qquad (3)$$

a is therefore equal to 1/k from Eqn. (2) where k represents the non-dimensional steady state fractional volume dilution rate of large eddy by turbulent eddy fluctuations across unit cross section on the large eddy envelope. Therefore a represents the non-dimensional total fractional mass dispersion rate by dilation and a^2 represents the corresponding fractional energy flux into the environment. Let d represent the ratio of the spin angular moments fo the total mass of the large and turbulent eddies.

$$d = W^4Z^3/w^4_* = 4Z/\pi^2 \simeq 4Z/9 \qquad (4)$$

Therefore $2a^2 \simeq 3d$ in agreement with Delbourgo's (1986) results. The values of a and d are respectively equal to 2.52 and 4.05 for organised eddy growth (Z=10) in the atmospheric boundary layer. $2a^2 \simeq 3d$ is a statement of the law of conversation of energy for the period doubling growth sequence since $2a^2$ represents the total bidirectional eddy energy flow into the environment and is equal to the three fold increase in the spin angular momentum of the large eddy. The property of inertia enables propogation of turbulence scale perturbation in the medium where translational kinetic energy is generated during dilation by inherent latent energy potential of the medium e.g., the buoyant energy generation by water vapour condensation in the updraft regions of the atmospheric boundary layer.

5. DITERMINISTIC CHAOS MODEL OF SEVERE LOCAL STORMS

Cloud growth occurs in the updraft regions of large eddy circulations under favourable conditions of moisture supply in the environment. The turbulent eddies inherent to the large eddies are amplified

in which one could discern a scale invariant or fractal structure. Phenomenological observations of fractal structure in nature represent the two fundamental symmetries of nature, namely, dilation ($r \rightarrow b\ r$) and translation ($r \rightarrow r + c$) and correspond respectively to change in unit of length or in the origin of the co-ordinate system (Kadanoff, 1986). A self similar object is identified by its fractal dimension D which is defined as $d\ln M(R)/d\ln R$ where M(R) is the mass contained within a distance R from a typical point in the object. Self similar growth processes in nature lead to the observed universal fractal geometry of macrocopic structures in natural phenomena. However, the basic physical mechanism of the self oganised fractal geometry in nature is not yet identified (Kadanoff, 1986).

A striking example of self similar fractal geometry in nature is exhibited by the global cloud cover pattern. Macroscopically different shaped clouds are self similar fractals over a number of orders of magnitude of length scales in the turbulent planetary atmospheric boundary layer (Lovejoy and Schertzer, 1986).

3. PHYSICS OF DETERMINISTIC CHAOS IN THE ABL

The period doubling route to chaos is basically a growth phenomena where by large eddy growth is initiated from the turbulence scale in successive length step increments equal to the turbulence scale length (Mary Selvam, 1987). In summary, turbulent eddies of frictional origin at the planetary surface possess an inherent upward momentum flux which is progressively amplified by buoyant energy generation from Microscale Fractional Condensation (MFC) of water vapour on hygroscopic nuclei by deliquescence even in an unsaturated environment (Pruppacher and Klett, 1978). The exponential decrease of atmospheric density with height further accelerates the turbulence scale upward momentum flux. Therefore, the unidirectional (upward) turbulence scale energy pump generates successively larger vortex roll circulations in the ABL. The larger eddies carry the turbulent eddies as internal circulations which contribute to their (large eddies) further growth. Such a process of large eddy growth is analogous to the emission of anti-Stokes laser emission triggered by laser pump during chaos in optics (Harrison & Biswas, 1986).

Townsend (1956) has investigated the structure and dynamics of large eddy formations in turbulent shear flows and has shown that large eddies of appreciable intensity form as a chance configuration of the turbulent motion. Considering a large eddy of radius R which forms in a field of isotropic turbulence with turbulence length and velocity scales 2r and w respectively. It may be shown that the root mean square (r.m.s) velocity of circulation W in the large eddy of radius R is

$$W^2 = \frac{2}{\Pi} \frac{r}{R} w^2 \qquad (1)$$

The above equation can be applied directly to derive the r.m.s circulation speed W of the large eddy of radius R generated by the turbulence scale energy pump. The scale ratio Z is equal to the ratio of the radii of the large and turbulent eddies. The environment of the turbulent eddy is a region of buoyant energy production by condensation (in the troposphere) and is therefore identified by a Microscale Capping Inversion (MCI) layer on the large eddy envelope. An incremental growth dR of large eddy radius equal to the turbulent eddy radius r occurs in association with an increase dW in large eddy circulation speed as a direct consequence of the buoyant vertical velocity w_* production per second by MFC. The MCI is thus a region of wind shear and temperature inversion in the ABL. The growth of large eddies from the turbulence scale at incremental length steps equal to r-turbulence length scale doubling - is therefore identified as the universal period doubling route to chaos in the ABL.

The physics of deterministic chaos therefore enables to identify turbulence as topology dependent and intrinsic to boundary layer flows, the temperature inversion and wind shear being manifestations of large eddy growth from turbulent energy generation processes. This concept is in direct contrast to the conventional view that inversion layers act as atmospheric lids suppressing convective activity and that boundary layer turbulence e.g., Clear Air Turbulence (CAT) is generated by wind shear in inversion layers.

4. UNIVERSAL THEORY OF CHAOS FOR ABL AND QUANTUM MECHANICAL LAWS

The turbulent fluctuations mix overlying environmental air into the growing large eddy volume and the fractional volume dilution rate k of the total large eddy volume across unit cross sections on its envelope is equal to

$$k = \frac{w_*}{dW} \frac{dR}{R} \qquad (2)$$

where w_* is the unidirectional turbulent eddy acceleration and dW the corresponding acceleration of the large eddy circulation (Mary Selvam, 1987) during the large eddy incremental length step growth dR equal to $r.k > 0.5$ for $Z < 10$. Therefore organised large eddy growth can occur for scale ratio $Z \geq 10$ only since dilution by environmental mixing is more than half by volume and erases the signature of large eddies for scale ratio $Z < 10$. Therefore, a hierarchical, scale invariant self similar eddy continuum with semi-permanent dominant eddies at successive decadic scale range intervals is generated by the self organised period doubling route to chaos growth process. The large eddy circulation speed is obtained by integrating

inside the clouds due to enhanced cloud water condensation and form "cloud top gravity (buoyancy) oscillations". The cloud top gravity oscillations are responsible for (1) cloud vertical mixing and dilution. Downward transport of stratospheric Ozone may also occur in deep convective systems as reported by several workers (2) cloud electrification by downward transport of naturally occurring negative space charges from above cloud top regions to the cloud base and simultaneously the upward transport of positive space charges from lower troposphere to the cloud top regions thereby generating the observed vertical positive dipole cloud charge. The Travelling Ionospheric Disturbances (TIDS) have been attributed to cloud top gravity oscillations in deep convective systems. The cauliflower like surface granularity to the cumulus cloud is a signature of the innumerable turbulent eddies which form the cloud top gravity oscillations. Severe local storms occur in regions where there is generation of massive quantities of turbulence scale buoyant energy and therefore associated with large scale ratios for the dominant turbulent eddy and the cloud scale large eddy.

The deterministic chaos model enables a universal no scale (scale invariant) quantification of the steady state cloud dynamical, mirophysical and electrical processes (Mary Selvam and Murty, 1987) as listed in the following. (1) The ratio of the actual cloud liquid water content (q) to the adiabatic liquid water content (q_a) is equal to f, the fraction of surface air which reaches the normalised height Z after dilution by vertical mixing due to turbulent eddy fluctuations (2) the vertical profiles of the vertical velocity W and the total cloud liquid water content q_t are respectively given by $W = w_* fZ$ and $q_t = q_* fZ$ where t represents the total values and * represents cloud base value (3) the cloud growth time $T \simeq li\,(\sqrt{Z})\,\frac{z}{2}$ where li is the logarithm integral (4) the cloud dropsize spectrum follows the naturally occurring Junge aerosol size spectrum and (5) the computed raindrop size spectrum closely resumbles the observed Marshall - Palmer raindrop size distribution at the surface, (6) the electric field at the surface due to the cloud dipole charge, th strength of the cloud dipole, the cloud electrical conductivity, the point discharge current are expressed in terms of the basic non-dimensional parameters f and Z. Tha above quantitative relations are universal for all clouds and depends only on the scale ratio Z.

6. CONCLUSION

The universal theory of chaos is developed in detail for the turbulent ABL and applied for the simulation of cloud microphysical, dynamical and electrical characteristics. The quantum mechanical nature of the coherent cloud structures is emphasised. The vast amount of knowledge acquired in quantum mechanics and stochastic processes may therefore be beneficially applied for the study and prediction of weather phenomena, in particular severe local storms.

REFERENCES

Delbourgo, R., 1986: Universal facets of chaotic processes. ASia-PAcific Physics News, 1, 7 - 11.

Doll, J.D., and D.L. Freeman, 1986: Randomly exact methods, Science, 234, 1356 - 1360

Eymard, L., 1985: Convective orgaisation in a tropical boundary layer. An interpretation of doppler radar observations using Asai's model. J. Atmos. Sci., 42, 2844 - 2855.

Fairbairn, W., 1986: Organised chaos: A signature of non-linearity. Phys. Bull., 37, 300 - 302.

Feigenbaum, M.J., 1980: Los Alamos Sci., 1,4.

Frisch, U., B. Hasslacher and Y. Pomeau, 1986: Lattice-gas automata for the Navier-Stokes equation. Phys. Rev., Letts., 56, 1505 - 1508.

Harrison, R.G., and D.J. Biswas, 1986: Chaos in light. Nature, 321, 394 - 401.

Kadanoff, L.P., 1986: Fractals: where's the Physics? Phys. Today, Feb., 6 - 7.

Levich, E., 1987: Certain problems in the theory of developed hydrodynamical turbulence. Phys. Rep. Nos. 3,4,151, 129 - 238.

Lorenz, E.N., 1963: Deterministic non-periodic flow. J. Atmos. Sci., 30, 130 -

Lovejoy, S., 1981: A statistical analysis of rain areas in terms of fractals. 20th Conf. Radar Meteorol., Amer. Meteorol. Soc., Boston, 476 - 484.

Lovejoy, S., and D. Schertzer, 1986: Scale invariance, symmetries, fractals and stochastic simulations of atmospheric phenomena. Bull. Amer. Meteorol. Soc., 67, 21 - 32.

Mary Selvam, A., and A.S.R. Murthy, 1987: Simulation of urban effects on cloud physical parameters Proc. Int'l. Conf. Energy Transf. Interactions with small and meso-scale atmospheric processes. Switzerland, 2 - 6 March.

Mary Selvam, A., 1987: Remote sensing of geomagnetic field and applications to climate prediction. Proc. Int'l. Geoscience and Remote Sensing Symposium (IGARSS '87), 18 - 21 May, University of Michigan, USA.

Pruppacher, H.R., and J.D. Klett, 1978: Microphysics of clouds and precipitation. D.Reidel Publishing Co., Boston, USA, 714 pp.

Tennekes, H., 1973: In: Workshop on micrometeorology, Amer. Meteorol. Soc., Boston, USA.

Townsend, A.A., 1956: The structure of turbulent shear flow. Cambridge University Press, Cambridge, 115 - 130.