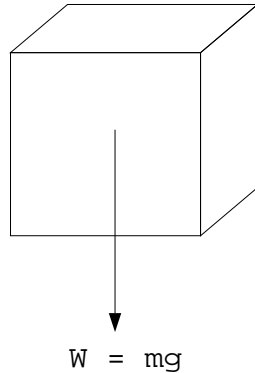


**Today's lecture:** Static Fluids: Properties of Materials which are not Solid

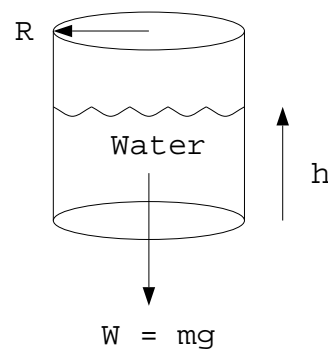
- **Solids** are rigid and don't deform under force



They are specified by either mass or weight. For example, I would like a pound of butter.

- **Fluids (liquids or gases)** are easily deformed by force and take the shape of the container. They are specified by volume (i.e. a gallon of milk).

Example: What is the weight of this volume of water?



We calculate the weight by using the measurement of  $\rho =$  **fluid density**.

$$\rho = \text{mass} / \text{volume}$$

$\rho_{\text{gas}} \lll \rho_{\text{liquid}}$  (because the mass of a gas is much less than the mass of a liquid, i.e., there are fewer molecules/unit volume in a gas than in a liquid)

$$W = mg = \rho V g \text{ (because mass = } \rho V \text{)} = \rho g (\pi R^2 h)$$

Therefore, the less dense something is, the less it weights too!

\*\*Note: Review your formulas for volumes of different shapes.

### Pressure

A fluid's weight is distributed over the area of the bottom of the container.

$$\text{The weight per unit area} \equiv \text{Pressure} = \text{Nt} / \text{m}^2 = \text{Pascal (Pa)}.$$

Other units of pressure include 1 atmosphere = 1 atm =  $1.01 \times 10^5$  Pa = 15 psi

$$\text{Pressure} = \text{Weight/area} = \rho g (\pi R^2 h) / \pi R^2 = \rho g h$$

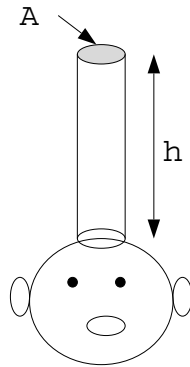
The pressure at a given depth in a fluid is the same everywhere in the fluid at that depth. Thus, the walls also feel a pressure outward which increases with depth.

Example: Air pressure.

Can we use this information to estimate how high the atmosphere is?

Air pressure at sea level ( $P_{\text{sea}} = 10^5 \text{ Nt/m}^2$ )       $1 \text{ Nt/m}^2 = 1 \text{ Pascal (Pa)}$

This is the weight of the atmosphere over the area of our head.



$$\text{Pressure} = W_{\text{air}} / A = m_{\text{air}} g / A = \rho_{\text{air}} (Ah) g / A = \rho_{\text{air}} h g$$

$$10^5 \text{ Nt/m}^2 = 1.2 \text{ kg/m}^2 (h) (10 \text{ m/s}^2) \Rightarrow h = 8 \times 10^3 \text{ m (8 km)}$$

This height calculation is only an estimate because we assume that the atmosphere is not a constant density. We know that density decreases as we go up into the atmosphere.

Example: How far down do you have to dive in order to feel an extra pressure of 1 atmosphere (over what you already feel at sea level)?

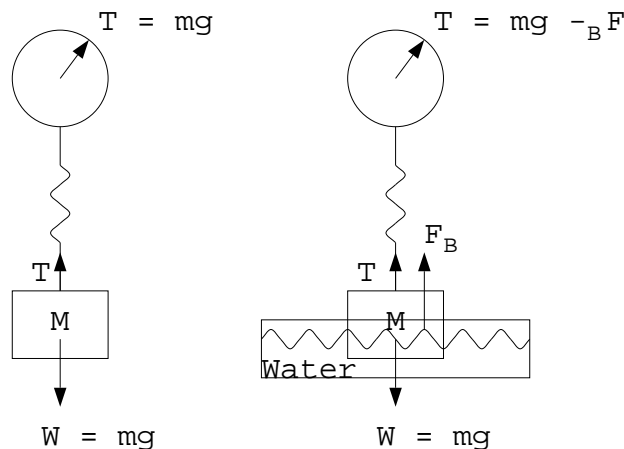
$$P = \rho_{\text{water}} g h = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$

$$\Rightarrow h = (1.01 \times 10^5 \text{ Pa}) / [(1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)] = 10.3 \text{ m} \approx 34 \text{ feet}$$

Thus, skin divers feel the pressure in their ears increase by 1 atmosphere for every 34 feet they dive. Note that the TOTAL pressure which a diver feels at 34 feet is actually 2 atm, not 1, because the pressure starts at 1 atm already at the surface of the water and increases from there.

**Objects Immersed in Fluids.**

Demonstration:



When you push up on the block with one hand, the apparent weight decreases. Likewise, when you push

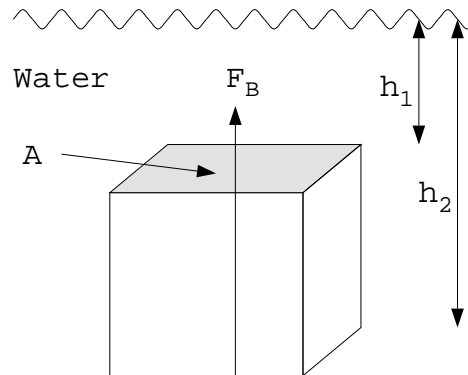
a container of water up against the block, the apparent weight also decreases. Therefore, water must exert some **buoyant force,  $F_B$ , on the block.**

**Archimedes Principle:**

$F_B$  is equal to the weight of fluid displaced by an object.

**Origin of  $F_B$ .**

Consider an object which is totally submersed in water.



As we saw from the demonstration of the water column, **the pressure at the bottom of the cube** is greater than the pressure at the top of the cube ( $P_2 > P_1$ ). Since  $P_2$  is larger, it must want to exert some force upward.

$$\Delta P = P_{h_2} - P_{h_1} = (\rho_{H_2O} g h_2) - (\rho_{H_2O} g h_1) = \rho_{H_2O} g (h_2 - h_1)$$

Notice that the pressure change is largest when there's a large difference in heights. This difference in pressure means that there will be a net force upward on the cube, which is just the buoyant force.

$$F_B = \Delta P A = \rho_{H_2O} (h_2 - h_1) g A$$

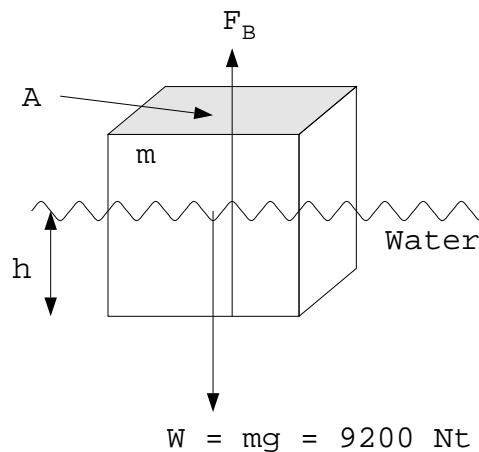
What is the weight of the water displaced by the cube?

$$W_w = m_w g = \rho_{H_2O} A (h_2 - h_1) g = \Delta P A = F_B$$

If we compare these two equations, we notice that the right sides of the equations are exactly the same. This is why the buoyant force ( $F_B$ ) is equal to the weight of the water displaced ( $W_w$ ).

Does the atmospheric pressure play any role in the buoyant force? Air does push down on the water, but no, atmospheric pressure is not significant, because the buoyant force is only determined by the *difference* of pressure on the top and bottom of the cube.

Example: Floating iceberg



If you have an iceberg floating in the water, how much of the ice is below the surface?

Let's say the iceberg is cubical in shape with a volume of  $1 \text{ m}^3$ . The density of ice is  $\rho_{\text{ice}} = 0.92 \times 10^3 \text{ kg/m}^3$ . So, the weight of the iceberg is

$$\begin{aligned} W &= \rho V g \\ &= (0.92 \times 10^3 \text{ kg/m}^3) (1 \text{ m}^3) (10 \text{ m/s}^2) \\ &= 9200 \text{ Nt} \end{aligned}$$

What is the height,  $h$ , of the iceberg submerged in water? The amount of water displaced by the iceberg increases with  $h$  so the buoyant force does too.

$$\begin{aligned} F_B &= \text{weight of the water displaced, } W_w \\ &= W_w = m_w g = \rho_w V g \\ &= \rho_w (A h) g \end{aligned}$$

The iceberg will be in equilibrium when  $W_{\text{ice}} = F_B$ , i.e.,

$$9200 \text{ Nt} = (10^3 \text{ kg/m}^3) (1 \text{ m}^2 h) (10 \text{ m/s}^2)$$

$$\Rightarrow h = 0.92 \text{ m}$$

92% of an iceberg is submerged!

### Conditions for an Object Not to Float (i.e., to sink)

$$\text{In general, } F_B = \rho_{\text{liq}} V g = W_w$$

$$\text{Object's weight } W = mg = \rho_{\text{solid}} V g$$

The sum of the forces in the vertical direction is the object's weight plus the buoyant force.

$$\begin{aligned} \sum F_y &= -W + F_B = ma \\ &= (-\rho_{\text{solid}} V g) + (\rho_{\text{liq}} V g) = m a_{\text{solid}} \end{aligned}$$

If the density of the fluid is *greater* than the density of the solid, then the acceleration of the solid is upward. As it rises, it will displace less water, until the buoyant force just balances the weight and it is in equilibrium. On the other hand, if the density of the fluid is *less* than the density of the solid, then the buoyant force can never be big enough to support the weight of the object and it will sink!