A New Return Stroke Model Based on State-Space Equations

S. Visacro, M. H. Murta Vale, A. P. Silva, F. H. Silveira

Lightning Research Center – Federal University of Minas Gerais,
Belo Horizonte - 31270-901 - MG - Brazil

ABSTRACT: This paper presents the basic ideas of a new lightning return stroke model that describes the evolution of return current by means of state-space variables. The channel is reproduced by a sequence of distributed circuit elements that are allowed to vary in height and time. The model allows substituting the traditional injection of current into the channel base by the formation of the return current from the dropping of charges deposited along the corona sheath into the channel core. Though only preliminary results have been obtained, this more realistic representation seems to have potential to reproduce the typical features of lightning currents measured at the channel base.

1. INTRODUCTION

The literature presents different approaches to formulate Return Stroke Current Models (RSCM). Due to the complexity of lightning processes, such models adopt different levels of simplifications to describe the temporal and spatial evolution of lightning current along the discharge channel. While the Physical models contemplates more realistic representation of the lightning processes, most of the other model types are focused only on matching their results with measured effects of lightning currents, such as the distant electric and magnetic fields generated by such currents.

The authors have proposed some contributions in this field [Visacro and Silveira, 2004], including a recent model DNUTL (Dynamic non-uniform transmission line model) [Visacro and De Conti, 2005], that is able to match all the typical five signatures for electromagnetic fields generated by lightning currents. This simplified distributed circuit model allows computing the variation of channel parameters in time and height, by means of a very expedite formulation.

Most RSCM, including DNUTL, assume the evolution of return current along the channel from the injection of an external current surge at the channel base (and rarely at the attachment point or at points distributed along the channel). Recently, trying to achieve a better commitment between simplifications and lightning physics, the authors proposed a more realistic approach based on the formation of the return current from the dropping of the charges deposited along the corona sheath into the channel core [Visacro et al., 2006]. The charge is supposed to be deposited at height dependent capacitances distributed along the channel, due to the difference of potential between the extremities of the charged downward and upward leaders in the instant just before their attachment. In this model, the channel parameters (R,L,G,C) are found using the same expedite approach adopted in DNTUL model and the current wave is the result of the discharge of the charges deposited in the capacitances distributed

Correspondence to:
S. Visacro, Lightning Research Center – Federal University of Minas Gerais, Belo Horizonte - 31270-901 - Brazil, Lrc@cpdee.ufmg.br

along the channel.

The proposed paper describes the basic assumptions and the mathematical formulation of this model, based on state-space variables (currents across inductances and voltages at capacitances). It also denotes that the results provided by such model are able to reproduce typical profiles of lightning currents measured at instrumented towers. Since it consists on a mathematical representation of the process involved in the lightning current evolution, the quality of the results provided by this model is dependent on the definitions adopted to represent the physical parameters involved in the process. Nevertheless, it allows easily representing any temporal and spatial variation of channel parameters, including non-linear effects, and this may be very useful once the knowledge about the physics of these parameters is being increased.

2. BASIC CONSIDERATIONS

The channel is represented by a set of elements (Figure 1), each one of them presenting a specific longitudinal parameters (a resistance to represent losses in the channel core and inductance) and transversal parameters (a capacitance associated to the corona sheath and a resistance to represent losses during the process of current generation).

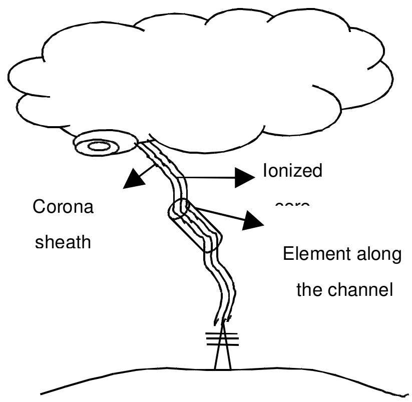


Figure 1. Channel representation.

Figure 2 shows the sequence of distributed circuit that represents the channel. In this circuit, the voltage at the capacitors and the current across the inductors are the state-space variables. If it is assumed that just before the attachment the first element is not connected to soil, the difference of potential (dp) applied between them, charges the capacitors above them according to their capacitance values. After that, when this element is connected to soil, the capacitors begin to discharge composing the current flow along the channel. The current across the element connected to the soil represents the current at the channel base.

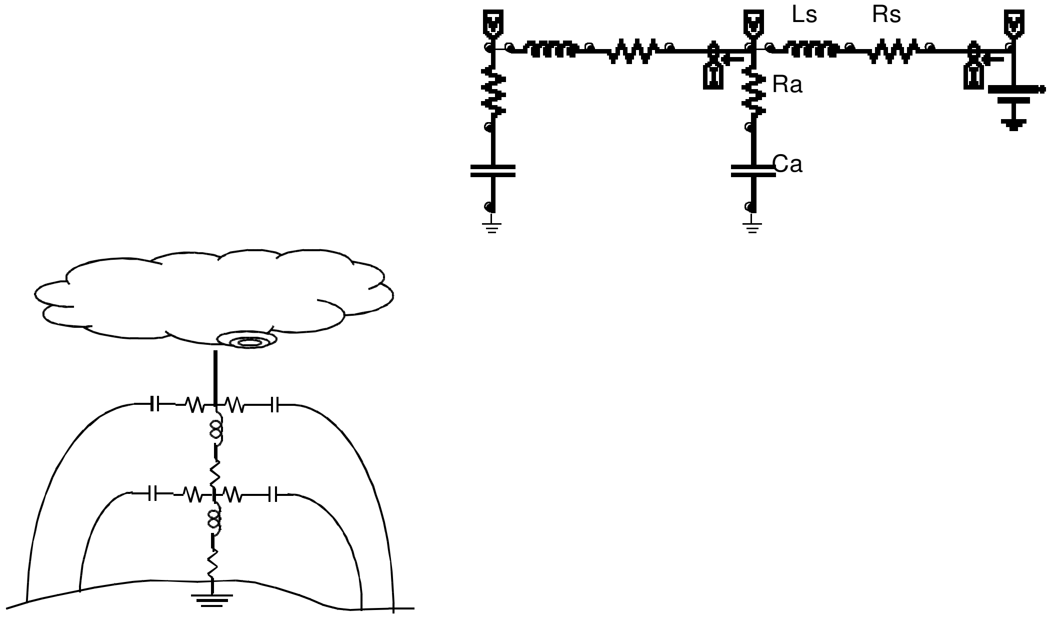


Figure 2. Physical and circuital representation of parameters along the channel.

3. MODEL FORMULATION AND PRELIMINARY RESULTS

Based in the circuit of Figure 2, from the state-space variables (voltages at capacitors and current across inductors), it is possible to formulate a matrix equation to solve the problem:

$$\frac{d}{dt}\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} = [A]\cdot\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} + [B]\cdot[V_0]$$

$$\begin{bmatrix} I_K \\ V_K \end{bmatrix} = [C]\cdot\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix}$$

In the equations above, matrix A and vector B depends only on the parameters of the circuit that represents the channel (see details in [Visacro et al.,2006]). After charging the system by applying the initial voltage V_0, the system evolution in time describes the discharge of the channel that composes the return current.

The first evaluations were performed only intended to verify the capability of the model to reproduce typical profiles of lightning currents measured at the channel base. Thus, a very simple configuration was assumed: a vertical channel 1000 m long, with 100 segments and 1 cm core radius. The parameters were not supposed to vary with height and constant values were assumed for the capacitance and inductance distributed parameters. The values were calculated based in formulation developed for the DNTUL model to determine the inductance value at 40 m. Two conditions were simulated for current wave velocity: v = c/3 (c: light velocity) and v = c, considering the attachment at soil level and assuming the parameters as indicated in Table 1. The current wave at the channel base was the monitored variable (first element along the channel).

Table 1 – Parameters adopted in simulation

	Circuit parameters	Channel parameters	Parameters of simulation
velocity V = c/3	R_{S0} = 0.1 Ω/m	H_0 = 1000 m	N_S = 100
	L_{S0} = 1.63 μH/m	t_{C0} = 2.5 μs	V_{C0} = 50 MV
	R_{A0} = 0.1 Ω/m	r = 1 cm	
	C_{A0} = 61.35 pF/m		
velocity V = c	R_{S0} = 0.1 Ω/m	H_0 = 1000 m	N_S = 100
	L_{S0} = 1.63 μH/m	t_{C0} = 2.5 μs	V_{C0} = 50 MV
	R_{A0} = 0.1 Ω/m	r = 1 cm	
	C_{A0} = 6.82 pF/m		

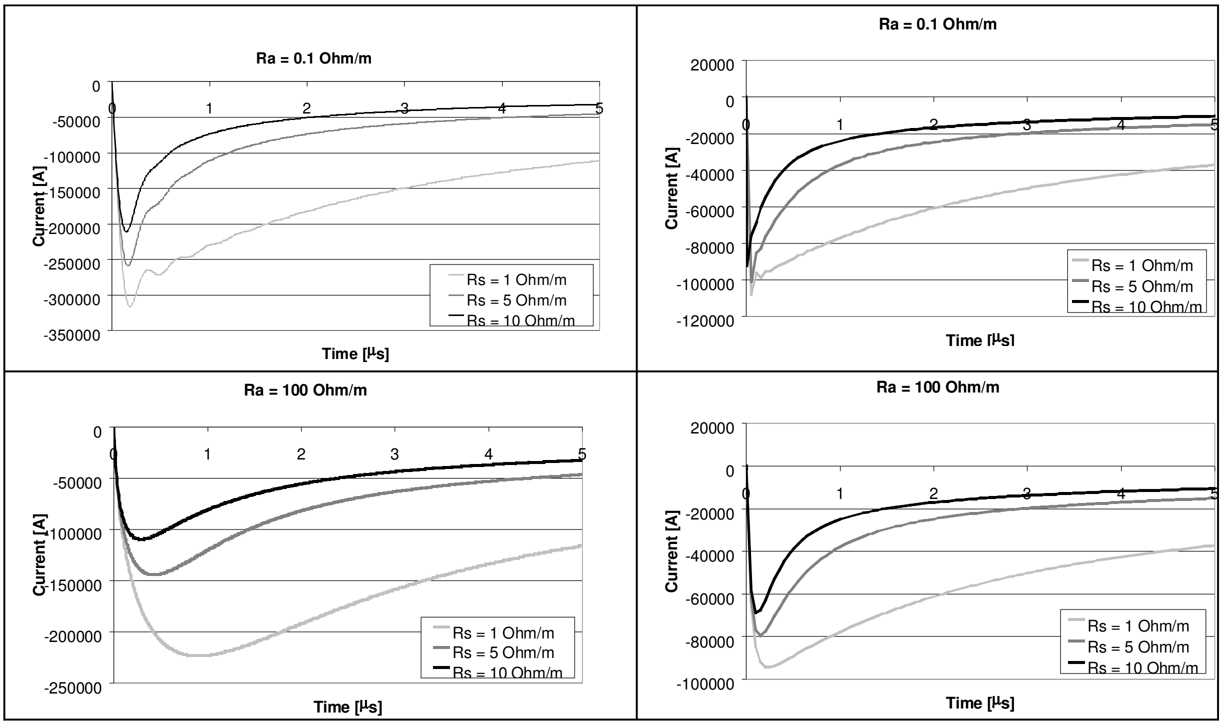
The results presented in the graphs below denote the model is able to reproduce the general aspects of measured lightning current profiles.

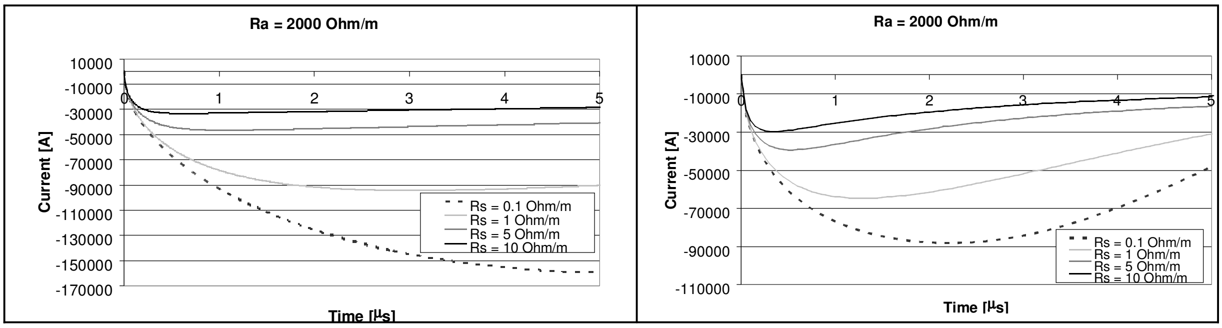
The research is still under development. The authors have already tested two additional steps concerning the variation of parameters in time and also the influence of their variation with the height along the channel.

4. CONCLUSIONS

Though only preliminary results have been obtained, this representation seems to have potential to reproduce the typical features of lightning currents measured at the channel base.

As the most attractive aspect, this model allows eliminating the need of injecting surge current into the channel base that corresponds to an unrealistic assumption.





(a) velocity = c/3 (b) velocity =c

Figure 3. Resulting current at the channel base.

Of course, the model is a simple mathematical representation of the process involved in the return stroke current evolution. The quality of the results it provides is entirely dependent on the definitions about the representation of the physical parameters involved in the process. Nevertheless, in spite of its complex formulation and relatively long processing time, a model based on state-space variables allows a great freedom to easily represent very accurately the variance of channel parameters in space and time, including the computation of non-linear effects.

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