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FIRST STEPS IN A NEW RETURN STROKE MODEL BASED ON STATE-SPACE EQUATIONS

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Abstract – This paper presents the basic ideas of a new lightning return stroke model that describes the evolution of current in time along the channel by means of state-space variables. The channel is reproduced by a sequence of distributed circuit elements that are allowed to vary in height and time. The model substitutes the traditional injection of current into the channel by applying a initial condition for the difference of potential between the extremity of downward and upward leaders, just before their attachment. Only first developments are presented to show the possibility to reproduce the typical features of lightning currents measured at the bottom of the channel.

1 - INTRODUCTION

Return stroke current models are representations intended to describe the temporal and spatial evolution of lightning current along the channel.

Excluding the Physical Models, typically all the return stroke models uses the injection of current waves at the channel base as the input to begin the process regarding the evolution of current in time along the channel. Some models allow the injection point to be placed at the attachment point and others allow the current sources to be distributed along the channel.

Though the assumption of an injected current is in accordance with the general idea of a simplified process to represent the evolution of lightning current, the assumption of an initial condition represented by the existence of a difference of potential between the extremity of the charged downward and upward leaders in the instant just before their attachment is more realistic [1].

The authors' research group has been involved with investigations related to return stroke current models. Their approach began with a hybrid electromagnetic model HEM [2] and then evolved into a distributed circuit model [3] whose results match those of the more elaborate HEM model. Both models were able to allow the channel parameters to vary along the channel (with height), though they were unable to consider the time variation of such parameters. More recently the author created a new model DNUTL [4] that maintains the simplicity of a distributed circuit but that allows the channel parameters to have time and height variation. This model is able to match all the typical five signatures for electromagnetic fields generated by lightning currents. All these developed models have considered the evolution of lightning current from the injection of a current wave in an arbitrary position along the channel.

Thus, the research group investigation is directed to look for realistic representations of the return stroke current evolution that are able to achieve a commitment between simplifications and the physical aspects involved in the evolution of current along the lightning channel. In this perspective, a step ahead is the substitution of the current source that injects the current wave into the channel by the assumption of an initial voltage applied between downward and upward channel, just before their attachment. In this representation the current wave is the result of the dropping of charges deposited in the Corona sheath around the core, or equivalently deposited into the capacitances distributed along the channel.

This is the focus of this paper that describes the basic formulations of this new model, based on state-space variables. Due to the potential characteristics of such variables, the model is able to easily adopt any temporal and spatial variation of circuit parameters used to represent the channel. This paper describes only the first steps to formulate this model.

2 – BASIC CONSIDERATIONS

In this model, the channel is represented by a set of elements (Figure 1) each one of them presenting a specific longitudinal parameters (a resistance to represent losses in the channel core and inductance) and transversal parameters (a capacitance associate to the corona sheath and a resistance to represent losses during the process of current generation).

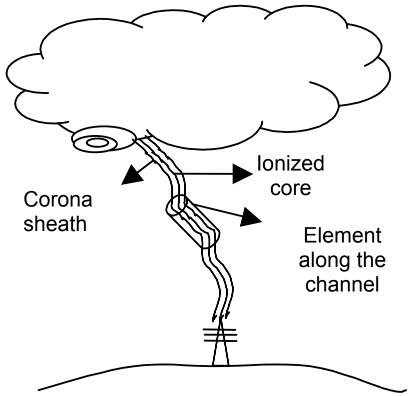

Figure 1 – Channel representation.

This results in the representation of the channel by a sequence of distributed circuits as indicated in Figure 2.

In this representation, the voltage at the capacitors and the current across the inductors are the state-space variables. If it is assumed that two adjacent elements are not connected and a difference of potential (dp) is applied between them, the capacitors above and below these elements are charged, according to both the dp value and their capacitances. After that, when you connect both elements, the capacitors begin to discharge composing the current flow along the channel. The current flowing across the segment connected to the soil represents the current at the channel base.

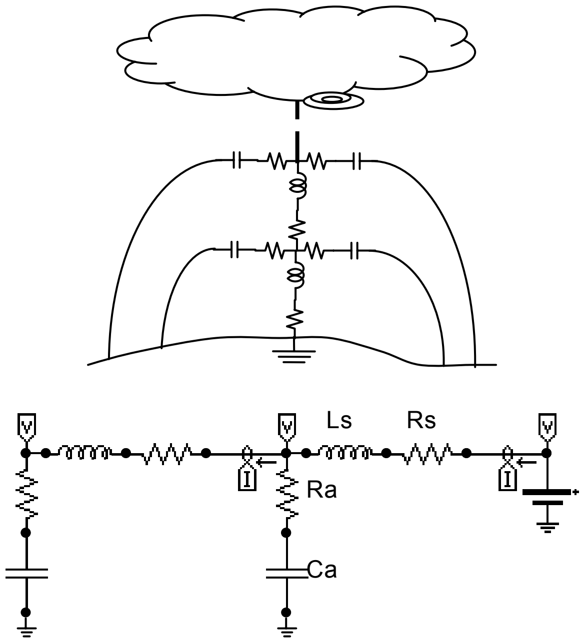

Figure 2 – Physical and circuital representation of parameters along the channel.

3 - DEVELOPMENTS

The developments presented in this paper are organized in three topics:

(i) Model formulation based in the circuit of Figure 2;

(ii) First results assuming the distributed parameters constant in space and time;

(iii) First results assuming the distributed parameters to vary in space and time.

3.1 MODEL FORMULATION

Based in the circuit of Figure 2,

$$C_{P1}\frac{dV_{C1}}{dt} = I_{C1} = I_1 - I_2$$

$$C_{PK-1}\frac{dV_{CK-1}}{dt} = I_{CK-1} = I_{K-1} - I_K$$

$$C_{PK}\frac{dV_{CK}}{dt} = I_{CK} = I_K - I_{K+1}$$

$$C_{PK+1}\frac{dV_{CK+1}}{dt} = I_{CK+1} = I_{K+1} - I_{K+2}$$

$$C_{PN}\frac{dV_{CN}}{dt} = I_{CN} = I_N$$

Complementary equations:

$$V_1 = R_{P1}I_{C1} + V_{C1}$$
$$V_{K-1} = R_{PK-1}I_{CK-1} + V_{CK-1}$$
$$V_K = R_{PK}I_{CK} + V_{CK}$$
$$V_{K+1} = R_{PK+1}I_{CK+1} + V_{CK+1}$$
$$V_N = R_{PN}I_{CN} + V_{CN}$$

After substituting state-space voltage variables in the complementary equations:

$$V_1 = R_{P1}I_1 - R_{P1}I_2 + V_{C1}$$
$$V_{K-1} = R_{PK-1}I_{K-1} - R_{PK-1}I_K + V_{CK-1}$$
$$V_K = R_{PK}I_K - R_{PK}I_{K+1} + V_{CK}$$
$$V_{K+1} = R_{PK+1}I_{K+1} - R_{PK+1}I_{K+2} + V_{CK+1}$$
$$V_N = R_{PN}I_N + V_{CN}$$

Substituting the equations above in the equations of currents:

$$L_{S1}\frac{dI_1}{dt} = -R_{S1}I_1 - R_{P1}I_1 + R_{P1}I_2 - V_{C1} + V_0$$

$$L_{SK-1}\frac{dI_{K-1}}{dt} = -R_{SK-1}I_{K-1} - R_{PK-1}I_{K-1} + R_{PK-1}I_K - V_{CK-1} + V_{K-2}$$

$$L_{SK}\frac{dI_K}{dt} = -R_{SK}I_K - R_{PK}I_K + R_{PK}I_{K+1} - V_{CK} + R_{PK-1}I_{K-1} - R_{PK-1}I_K + V_{CK-1}$$

$$L_{SK+1}\frac{dI_{K+1}}{dt} = -R_{SK+1}I_{K+1} - R_{PK+1}I_{K+1} + R_{PK+1}I_{K+2} - V_{CK+1} + R_{PK}I_K - R_{PK}I_{K+1} + V_{CK}$$

$$L_{SN}\frac{dI_N}{dt} = -R_{SN}I_N - R_{PN}I_N - V_{CN} + R_{PN-1}I_{N-1} - R_{PN-1}I_N + V_{CN-1}$$

Considering the equations above for the state-space variables (voltages at capacitors and current across inductors), it is possible to formulate a matrix equation to solve the problem:

$$\frac{d}{dt}\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} = [A]\cdot\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} + [B]\cdot[V_0]$$

$$\begin{bmatrix} I_K \\ V_K \end{bmatrix} = [C]\cdot\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix}$$

In the equations above, matrix A and vector B depends only on the parameters of the circuit that represents the channel. After charging the system by applying the initial voltage V_0, the system evolution in time describes the discharge of the channel that composes the return current.

After obtaining the state-space variables, to solve the system by simulation, the Runge-Kutta method (4th order) was chosen. But first, the algorithm to assemble matrices A and C was formulated. The number of segment N in the channel defines the dimension of the system, being matrices A and C 2Nx2N and vector B 2Nx1.

The equations above show three types of dependence relations among variables and parameters:

For the index equal to 1:

$$\frac{dI_1}{dt} = -\frac{(R_{S1} + R_{P1})}{L_{S1}}I_1 - \frac{1}{L_{S1}}V_{C1} + \frac{R_{P1}}{L_{S1}}I_2 + \frac{1}{L_{S1}}V_0$$

$$\frac{dV_{C1}}{dt} = \frac{1}{C_{P1}}I_1 - \frac{1}{C_{P1}}I_2$$

For the index varying from 2 to N-1:

$$\frac{dI_K}{dt} = \frac{R_{PK-1}}{L_{SK}}I_{K-1} + \frac{1}{L_{SK}}V_{CK-1} - \frac{(R_{SK} + R_{PK} + R_{PK-1})}{L_{SK}}I_K - \frac{1}{L_{SK}}V_{CK} + \frac{R_{PK}}{L_{SK}}I_{K+1}$$

$$\frac{dV_{CK}}{dt} = \frac{1}{C_{PK}}I_K - \frac{1}{C_{PK}}I_{K+1}$$

For the index equal to N:

$$\frac{dI_N}{dt} = \frac{R_{PN-1}}{L_{SN}} I_{N-1} + \frac{1}{L_{SN}} V_{CN-1} - \frac{(R_{SN} + R_{PN} + R_{PN-1})}{L_{SN}} I_N - \frac{1}{L_{SN}} V_{CN}$$

$$\frac{dV_{CN}}{dt} = \frac{1}{C_{PN}} I_N$$

Considering matrix C, for the index varying from 1 to N-1:

$$V_K = R_{PK} I_K + V_{CK} - R_{PK} I_{K+1}$$

And for the index equal to N:

$$V_N = R_{PN} I_N + V_{CN}$$

The fourth order Runge-Kutta method may be described:

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6}$$

The implementation of this method transforms the auxiliary variables ki into auxiliary vectors Ki :

$$[K_1] = h \cdot \left([A] \cdot \begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} + [B] \cdot [V_0]\right)$$

$$[K_2] = h \cdot \left([A] \cdot \left(\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} + \frac{1}{2}[K_1]\right) + [B] \cdot [V_0]\right)$$

$$[K_3] = h \cdot \left([A] \cdot \left(\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} + \frac{1}{2}[K_2]\right) + [B] \cdot [V_0]\right)$$

$$[K_4] = h \cdot \left([A] \cdot \left(\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix} + [K_3]\right) + [B] \cdot [V_0]\right)$$

$$\begin{bmatrix} I_K \\ V_{CK} \end{bmatrix}_{i+1} = \begin{bmatrix} I_K \\ V_{CK} \end{bmatrix}_i + \frac{1}{6}[K_1] + \frac{1}{3}[K_2] + \frac{1}{3}[K_3] + \frac{1}{6}[K_4]$$

$$\begin{bmatrix} I_K \\ V_K \end{bmatrix}_i = [C] \cdot \begin{bmatrix} I_K \\ V_{CK} \end{bmatrix}_i$$

The choice of the initial conditions for the system, of the sample interval h and of the number of interactions allows solve the state-space equations. It is suggested to choose h from the eigenvalue of matrix A with largest amplitude (λ_{max}):

$$h_{max} = \frac{1}{5 \cdot \lambda_{max}}$$

Since the eigenvalues of matrix A represent the poles of the dynamic system and the inverse of these values represents the time constant of the system, the last equation above suggests the sample interval to be five times smaller than the shortest time constant. Nevertheless, the sampling frequency may approach a little more the Nyquist frequency. In some cases it is reasonable to reduce the factor 5 to 4 or 3.

3.2- FIRST RESULTS: CONSTANT PARAMETERS

The first evaluations were only intended to verify the capability of the model to reproduce typical profiles of lightning currents measured at the channel base. For this reason, a very simple configuration was assumed for a vertical channel 1000 m long, with 100 segments. The parameters were not supposed to vary with height and constant values were assumed for the capacitance and inductance distributed parameters. The values were calculated based in formulation developed for the DNTUL model [3] to determine the inductance at 40 m (height h_k = 40 m), assuming k = 4, and core radius r = 1 cm:

$$L_k = k \cdot \frac{1}{c}\left[60 \ln\left(\frac{h_k}{r}\right) + 90 \cdot \frac{r}{h_k} - 60\right] - (k-1) \cdot \frac{1}{c}\left[60 \ln\left(\frac{h_{k-1}}{r}\right) + 90 \cdot \frac{r}{h_{k-1}} - 60\right]$$

$$C_k = \frac{1}{v^2 L_k}$$

Two conditions concerned wave velocity were simulated: v = c/3 (c: light velocity) and v = c, considering the attachment at soil level and assuming the following parameters as indicated in Table 1.

<u>For velocity c/3</u>

Table 1 – Parameters adopted in simulation (v=c/3)

Circuit parameters	Channel parameters	Parameters of simulation
R_{S0} = 0.1 Ω/m	H_0 = 1000 m	N_S = 100
L_{S0} = 1.63 μH/m	t_{C0} = 2.5 μs	V_{C0} = 50 MV
R_{A0} = 0.1 Ω/m	r = 1 cm	
C_{A0} = 61.35 pF/m		

The monitored variable is the current at the channel base (first element along the channel).

<u>For velocity c</u>

Table 2 – Parameters adopted in simulation (v=c)

Circuit parameters	Channel parameters	Parameters of simulation
R_{S0} = 0.1 Ω/m	H_0 = 1000 m	N_S = 100
L_{S0} = 1.63 μH/m	t_{C0} = 2.5 μs	V_{C0} = 50 MV
R_{A0} = 0.1 Ω/m	r = 1 cm	
C_{A0} = 6.82 pF/m		

The results presented in the graphs below denote the model is able to reproduce the general aspects of measured lightning current profiles.

The sensitivity analyses performed for the case of velocity v denotes that increasing the series resistance Rs implies on decreasing three parameters: current amplitude, front-time and time to half value (Ra is assumed as constant). The effect of increasing Ra, (keeping Rs value constant) is the reduction of the current amplitude and the increase of front time.

When velocity is increased to the speed of light, the same behavior is observed due to the increase of Rs. Nevertheless, both the amplitude and front time are significantly smaller than in the case of velocity c/3.

3.3- COMPLEMENTARY EVALUATIONS

The research is still under development. The authors have already tested in two additional steps how the variation of parameters in time affects the current profile and also the influence of their variation with the height along the channel. The corresponding results will be presented soon, since the idea of this paper was only to show the basis of the new model.

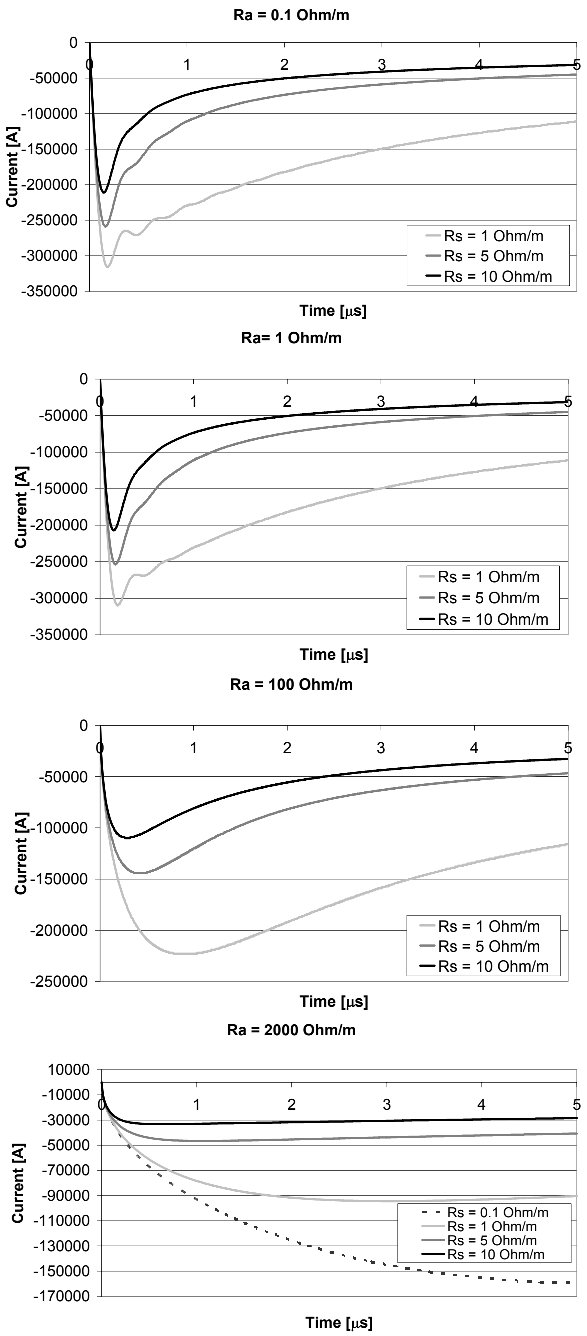

Figure 3 – Current at channel base (v = c/3).

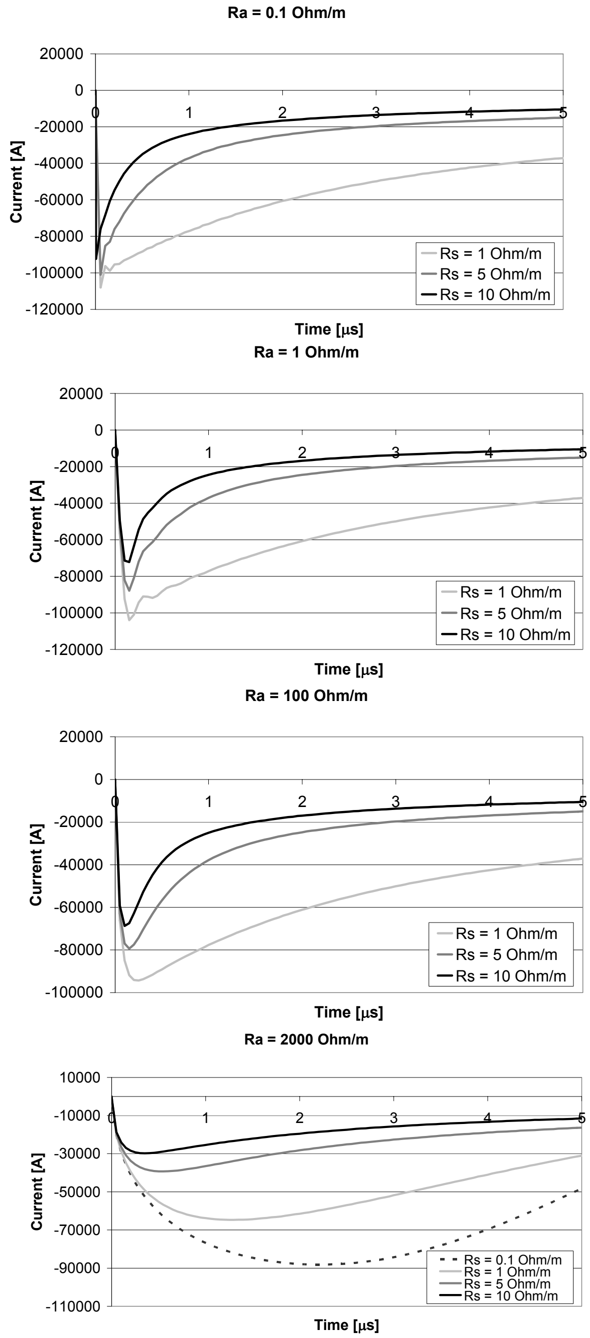

Figure 4 – Current at channel base (v = c).

4 - CONCLUSIONS

The first results provided by this model indicate its potentiality. Of course it is a simple mathematical representation of the process involved in the return stroke current evolution. In this respect the quality of this model is entirely dependent on the support provided by definitions about the representation of the physical parameters involved in the process.

Nevertheless, in spite of its complex formulation and relatively long processing time, a model based on state-space variables allows a great freedom to easily represent very accurately the variance of channel parameters in space and time, including the computation of non-linear effects.

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