GEMNET II – An Alternative Method for Grade Estimation

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ABSTRACT: Grade estimation is one of the most complicated aspects in mining. It also happens to be one of the most important. The complexity of grade estimation originates from scientific uncertainty, common to similar engineering problems, and the necessity for human intervention. The combination of scientific uncertainty and human judgement is common to all grade estimation procedures regardless of the chosen methodology.The GEMNET II system described in this paper was developed to provide a flexible but complete alternative method to existing grade estimation techniques, which takes into consideration the theory behind ore deposit formation while minimising the dependence on certain assumptions.

1 INTRODUCTION

This paper presents a complete system for grade estimation based on a very unique type of Artificial Neural Networks (ANNs), the Radial Basis Function (RBF) networks. ANN technology is introduced to more engineering problems as new network models and learning algorithms are being developed and older models prove their value on real and sometimes critical applications. Grade estimation, as it will be discussed in the following paragraphs, is one of the problems that can be approached successfully by ANNs and specifically by RBF networks. ANNs have several properties that establish them as a potential approach for grade estimation, such as: nonlinearity, adaptivity, generalization, and uniformity of analysis and design.

The modular neural network system for grade estimation, GEMNET II, presented in this paper has been developed over the past three years at the AIMS Research Unit of the University of Nottingham. The main objectives of the development of GEMNET II were defined as follows (Kapageridis 1999):

- To find a suitable neural network architecture for the problem of grade estimation.
- To take advantage of the function approximation properties of artificial neural networks.
- To break down the problem of grade estimation into less complex functions that can be modelled using these properties.
- To integrate the developed neural network architecture in a system which will be user-friendly and flexible.
- To provide means of validating the results of this system.
- To minimise the knowledge required for using the system.
- To compare the performance of the system with existing grade estimation techniques on the basis of estimation properties, usability and time requirements.

2 RADIAL BASIS FUNCTION NETWORKS AND THE ILL-POSED PROBLEM OF GRADE ESTIMATION

RBF networks inherit all those properties that make artificial neural networks a potential solution to the problem of grade estimation. They are however more suitable to this problem than other architectures due to their function approximation properties which are unique. RBF networks offer solutions to ill-posed problems, i.e. problems that do not satisfy one of the following conditions (Tikhonov & Arsenin, 1977; Morozov, 1993; Kirsch, 1996):

- ***Existence.*** For every input vector $x \in \aleph$, there does exist an output $y = f(x)$, where $y \in \wp$.
- ***Uniqueness.*** For any pair of input vectors $x,t \in \aleph$, we have $f(x) = f(t)$ if, and only if, $x = t$.
- ***Continuity.*** The mapping is continuous, that is, for any $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon)$ such that the condition $\rho_\chi(x,t) < \delta$ implies that $\rho_y(f(x),f(t)) < \varepsilon$,

where *ρ(,)* is the symbol for distance between the two arguments in their respective spaces. The property of continuity is also referred to as *stability*.

It is fairly straightforward to prove that the problem of grade estimation from exploration data is an ill-posed problem. Concentrating on the conditions of uniqueness and continuity, it is quite clear that the grade values as presented by the exploration data do not satisfy any of these two conditions. As far as uniqueness is concerned, there are always two input vectors representing two different grade samples that have the same grade (within a certain accuracy) while having different spatial co-ordinates, volume, or distance from the point of mapping. Therefore the condition of uniqueness is not satisfied.

Continuity is the one requirement of the conventional estimation techniques that makes them fail or not even apply to several cases of grade estimation. There is no doubt that the grade values presented through drillhole samples from an orebody do not satisfy the condition of continuity. This is a common problem that leads to the use of very simple and not particularly reliable methods of grade estimation.

In order to solve the ill-posed problem of grade estimation from exploration data, RBF networks can be used as they are based on a method that was developed specifically for solving this type of problems. This method is called regularisation and was proposed by Tikhonov in 1963. The idea behind regularisation is to stabilise the solution by embedding prior information about it (Haykin, 1999). Commonly the prior information involves the assumption that the input-output mapping is smooth, in the sense that similar inputs correspond to similar outputs. This is an assumption that can be and has to be applied if RBF networks are to be used for grade estimation.

It is necessary before carrying on to the application of RBF networks for grade estimation to examine their architecture and general operation. RBFs were initially used for solving problems of real multivariate interpolation. Work on this subject has been extensively surveyed by Powell (1990). The theory of RBFs is one of the main fields of study in numerical analysis (Powel 1981). RBF networks are very simple structures. Their design is in essence a problem of curve fitting in a high-dimensional space. Learning in RBF networks means finding the hyper-surface in multi-dimensional space that fits the training data in the best possible way. The universal approximation theorem for RBF networks, as stated by Park and Sandberg (1991), opened the way for their use in function approximation problems, which were commonly approached using Multi-Layered Perceptrons. The work of Park and Sandberg (1991, 1993), Cybenko (1989), and Poggio and Girosi (1990) led to a new model for function approximation based on generalised RBF networks. Specifically, the theorem can be stated as below:

Let $G{:}R^{mo} \rightarrow R$ is an integrable bounded function such that G is continuous and

$$\int_{R^{m0}} G(x)dx \neq 0$$

Let $\Im_G$ denote the family of RBF networks consisting of functions $F{:}R^{mo} \rightarrow R$ represented by

$$F(x) = \sum_{i=1}^{m_1} w_i G\left(\frac{x - t_i}{\sigma}\right)$$

where $\sigma > 0$, $w_i \in R$ and $t_i \in R^{mo}$ for $i = 1, 2, \ldots, m_1$. For any continuous input-output mapping function f(x) there is an RBF network with a set of centres $\{t_i\}_{i=1}^{m_1}$ and a common receptive field $\sigma > 0$ such that the input-output mapping function F(x) realised by the RBF network is close to f(x) in the L_p norm, $p \in [1,\infty]$.

The universal approximation theorem provides the theoretical basis for the design of RBF networks for practical applications.

A typical RBF network consists of three layers, an input layer, a single hidden layer, and an output layer. The processing elements in the hidden layer are quite different from other typical examples of ANNs like the Multi-Layered Perceptron (MLP). Each processing element is a Radial Basis Function with a varying receptive field and a varying centre location. These two parameters together with the linear weights of the connections between the hidden and output layer and the bias to the output are adjusted during training in order to provide the best possible mapping between the input vectors (e.g. drillhole samples) and the required output (grade).

3 GRADE ESTIMATION WITH GEMNET II

The ANN approach for grade estimation underlying the design of GEMNET II is based on RBF networks that treat the estimated variable (grade) as a hypersurface in the input vector space (Kapageridis 1999). This space takes two very distinctive forms:

- The 3D co-ordinate space of the drillhole samples;
- The space formed by the grade, distance, and volume of neighbouring samples.

In other words, GEMNET II treats grade as a function of the three spatial co-ordinates and the volume of drillhole samples or as a function of the grade, distance, and volume of neighbouring samples. This is achieved using a number of RBF networks each with a different function, the outputs of which are combined to provide a single grade estimate. The system comprises three RBF network modules responsible for the estimation and a data processing and control module that generates the training patterns for the networks by applying a search method developed specifically for GEMNET II.

It was necessary to develop a simplified 3D search method in order to cope with the geometrical characteristics of exploration sampling schemes. After considering a number of schemes, the author decided to use the simple search method shown in Fig. 1 (Kapageridis & Denby 1998).

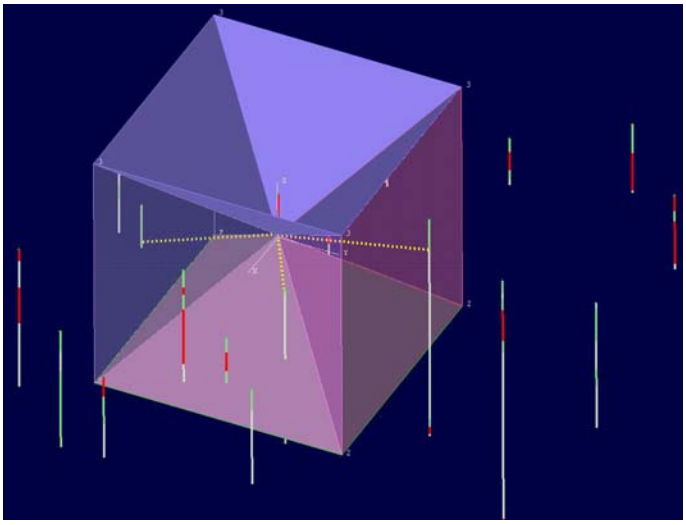

Figure 1. Simplified 3D search scheme used in GEMNET II.

There are only six sectors in this scheme: upper, lower, north, south, east, and west. These sectors are defined by the intersection of four planes: two planes vertical to the XZ plane at ±45° dip, and two planes vertical to the YZ plane at ±45° dip. In other words, these sectors look like pyramids of square base with their top at the estimation point. The advantage of this search scheme is not just the fact that it is very simple and affordable in computation terms. With this scheme, the drillhole where the current training point belongs is always within two opposite sectors. This allows easier control of the number of samples selected from this drillhole, which can help improve the results of estimation. Another advantage of this scheme is that it can handle any inclination of the orebody or the drilling scheme.

The data processing and control module accepts data in ASCII form and creates training pattern files for the RBF networks. The formation of training patterns is based on the search method described. Basically, for every training sample in the dataset, one neighbour sample is chosen from every sector – the one closest to the training sample. The grade of the neighbour sample, its distance from the training sample and its length are written as inputs on the training pattern file of the network responsible for the specific sector, while the training sample grade is written as the require output. Clearly, in some occasions there are no neighbour samples in some of the sectors. In those cases, the training sample is marked for estimation with the module that is trained on sample co-ordinates.

The first module consists of six RBF networks each trained on samples from one of the six sectors of the 3D search scheme. These networks have three inputs (neighbour sample grade, distance from the point of training/estimation, and neighbour sample volume) and one output (target grade at the point of training/estimation) (Fig.2).

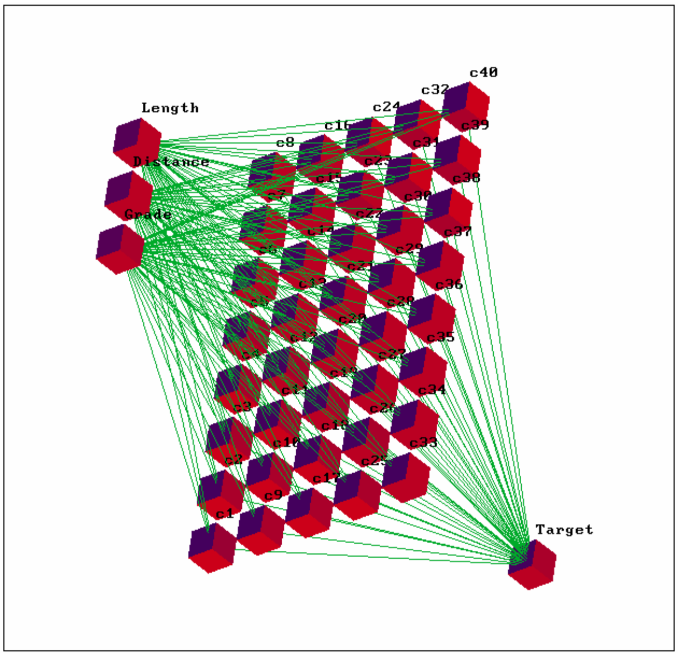

Figure 2. RBF network of the first module.

The second module is a single RBF network trained on the outputs of the six RBF networks of the first module (Fig. 3). This network performs the necessary averaging of the individual estimates is necessary as it was clear in some case studies that some of the RBF networks of the first module were consistently producing estimates closer to the actual values while others were consistently far from them. The number of hidden units normally varies between six and nine.

The third module is a single RBF network that accepts 3D data in the form of vectors with four dimensions (easting, northing, elevation, and volume) and produces one output (target grade) (Fig. 4). This network replaces one or more of the RBF networks of the first module in the case were there are no neighbour samples in some of the search scheme sectors. The network of this module is however trained on all samples regardless of the results of the search process.

Figure 3. RBF network of the second module.

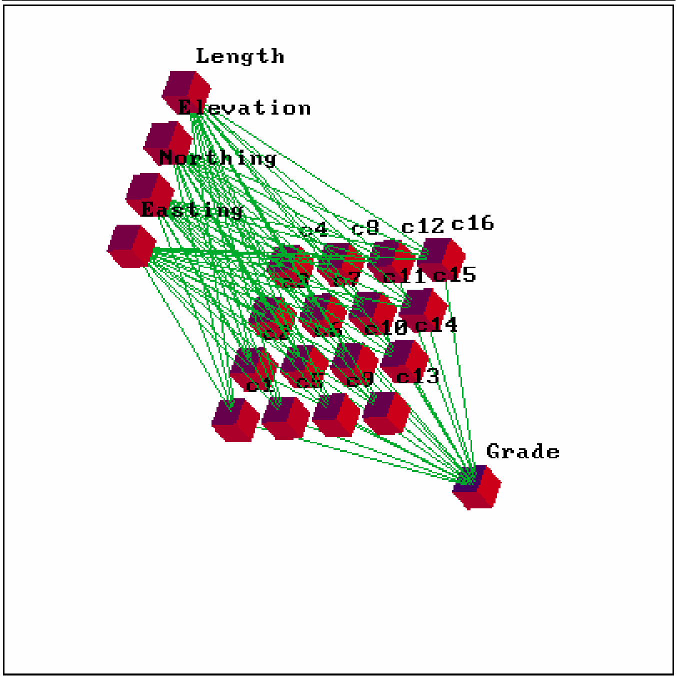

Figure 4. RBF network of the third module.

After training is complete the saved topologies are used for estimation. During training this is done on the basis of drillhole samples hidden from the training process in order to validate the learned mappings. During estimation the drillhole samples are mostly targeted on the training and validation process. Cross validation is used for testing the validity of the learned mappings and for comparing with other grade estimation techniques.

4 APPLICATION OF GEMNET II TO REAL EXPLORATION DATA

The case studies presented in this paper are the final tests of the GEMNET II architecture. Their purpose was to demonstrate the full potential of the approach and provide a complete comparison with other estimation techniques. They are presented in order of increasing complexity and difficulty. The number of available samples increases as well as the structural complexity of the deposits. The data used in these case studies come from real deposits. In some of them the 3D co-ordinates of the samples have been changed without affecting their relative locations. These studies are ideal for geostatistics and in fact have been used for demonstrating grade/reserve estimation using computer software. However, no results have ever been published using this data other than the papers written by the author during this project (Kapageridis *et al* 1999a, 1999b, Kapageridis 1999).

The deposits in the four case studies that follow present a complex 3D structure (Fig. 5). They all come with a complex geological model, which is used for constraining the estimation process. This geological model in some cases becomes even more complicated by the presence of faults and other discontinuities. This factor makes grade estimation an even more challenging task. In all of the case studies, a complete geostatistical study has been performed including the methods of kriging and inverse distance, the results of which are presented in this paper together with the study of GEMNET II application.

The four copper/gold deposits used for testing the estimation performance of GEMNET II have very little in common. Except from the type and possibly the way they have been formed, these deposits present a very different 3D picture and a very different estimation task. Their size and geometry varies significantly as does the grade distribution suggested by the available samples.

The available samples for each of the four deposits vary in number considerably. The drilling geometry is also different as is the assaying procedure. These differences ensured that GEMNET II would be tested on very different conditions and data and that the results would reflect its performance over a wide range of problems. Table 1 gives the main characteristics of the four deposits presented in this paper.

Table 8.1: Main characteristics of the four deposits used for testing the final GEMNET II architecture.

Code name	MAC_DEMO	THOR	SME	GEOST_GOLD
Number of samples	1361	3612	10,656	30,211
Estimated grades	Au, Cu	Au	Cu	Au
Number of orebodies	1	4	5	1

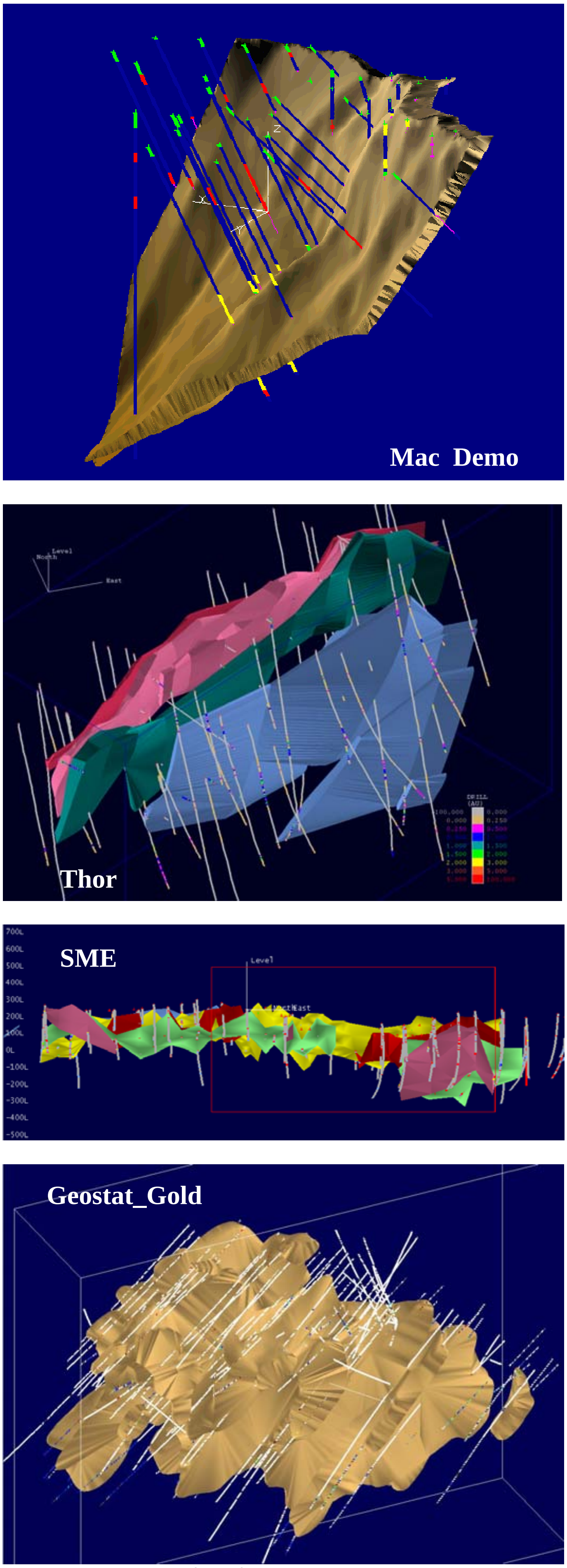

Figure 5. 3D views of the four copper/gold deposits used in the case studies (top to bottom: macdemo, thor, sme, geostat_gold) (screenshots from VULCAN/Envisage).

The measures of performance for the three approaches compared were the mean absolute error, the data fit diagram (scatter plot), and the estimated vs. actual grade distribution diagram. For GEMNET II, a guide to the quality of the produced estimates, the reliability indicator values, is also shown in slices through the estimated block model. The reliability indicator gives the variance of the individual estimates from the different RBF networks used by GEMNET II. The idea behind the reliability indicator is that the more the RBF networks disagree on one particular estimate the less reliable is the final estimate produced by the system.

The following table (Table 2) summarises the results on all four case studies for the three approaches tested. GEMNET II was running from VULCAN/Envisage version 3.4. Geostatistics were running also from the same environment using GSLIB. Therefore the same computational overhead from VULCAN has been present while the various approaches were tested. GEMNET II has been integrated in VULCAN/Envisage in order to become more user friendly, more complete, and easier to compare its results with those from the other methods (Kapageridis *et al* 1999a).

Table 2. Summary of results from case studies.

Case Study	Mac_Demo Au/Cu	Thor Au	SME Cu	Geostat_Gold Au
Actual Mean Grade	2.34/4.01	0.9269	0.9154	4.1316
Kriging Mean Grade	2.26/3.72	0.8660	0.8698	3.9014
ID2 Mean Grade	2.54/3.69	0.8686	0.881	3.9264
GEMNET II Mean Grade	1.96/3.41	0.8291	0.8803	3.8907
Kriging ABS %	20.47/19.68	19.67	14.77	14.46
ID2 ABS %	22.47/20.06	23.66	18.33	19.78
GEMNET II ABS %	18.78/18.9	18.2	14.65	15.04

In all four case studies, GEMNET II performed very well even in comparison with the other already established techniques. It should be noted that inverse distance weighting has benefited from the geostatistical study that improved significantly the results obtained with this technique. The performance of the three estimators becomes clearer by examining the data fit and distribution graphs. Examples from the first case study are given in the following figures (Fig. 6, 7).

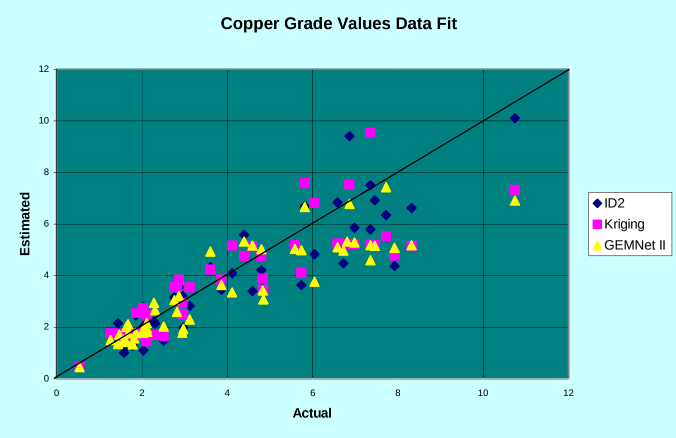

Figure 6. Data fit diagram of copper grade estimates produced by the three approaches.

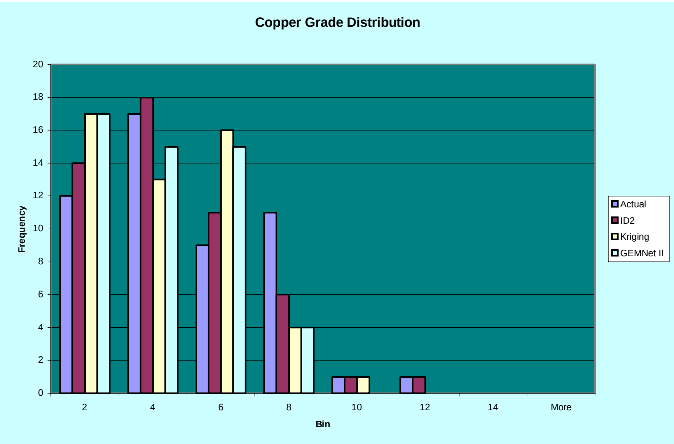

Figure 7. Actual and estimated copper grade distributions.

GEMNET II tends to underestimate high-grade samples but the overall estimation is not biased or affected by extreme values. Generally, the three techniques performed reasonably well with no particular problems. A major difference between them was, though, the time required for their application.

The time requirements for the application of the three methods were quite different, even though geostatistics were fairly straightforward in these case studies. GEMNET II required up to 10 hours to process the samples and block model centroids, develop the networks and perform grade estimation. The geostatistical studies required up to a week to complete. The time spent for grade estimation using inverse distance and kriging, once a geostatistical study was complete, was about 15 minutes.

The integration of GEMNET II inside VULCAN allowed the visualisation of the block model estimates, the reliability indicator values, and the locations of the RBF network centres. Using these visual tools it is possible to validate the approach and find potential problems that might come from the RBF network training or the sampling quality and quantity.

As shown in the following figures (Fig. 8, 9), the block model estimates of GEMNET II as well as the reliability indicator values, could be visualised in sections and together with any other type of data, e.g. a solid model of the orebody or the drillholes, using the graphical capabilities of VULCAN..

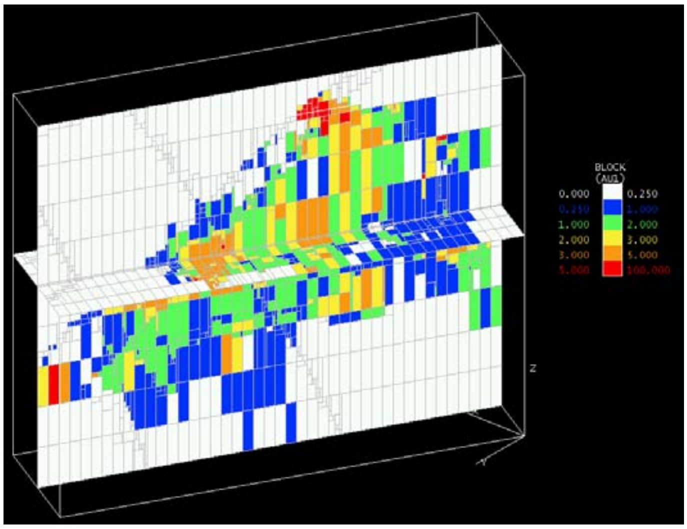

Figure 8. Vertical and horizontal section through block model coloured by the gold grade estimates of GEMNET II in VULCAN.

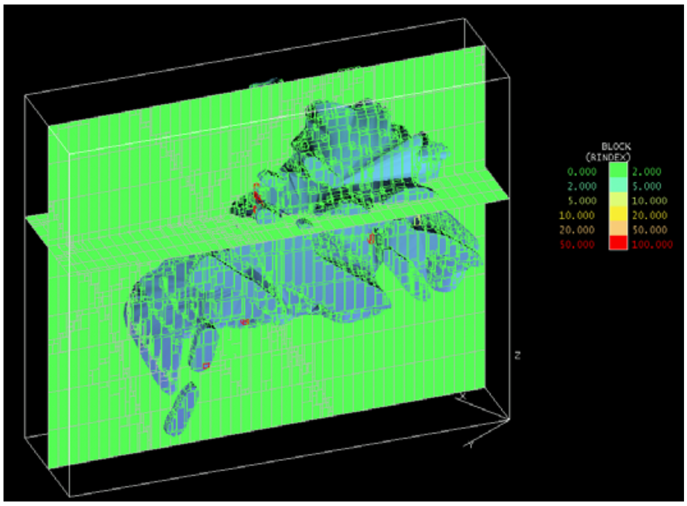

Figure 9. Vertical and horizontal block model section coloured by the reliability indicator values and solid model of the orebody.

The RBF centres location in the input vector space is absolutely crucial to the performance of an RBF network. The RBF centres visualisation tool has been developed specifically for GEMNET II in Envisage and allows the displaying of both the centres and the training samples of any RBF network from the modular architecture. This option loads the RBF centres using a special symbol on the screen and also the training samples as crosses. The correct input space is used, i.e. the 3D real world co-ordinates space for the third module and the neighbour sample grade, distance, and length input space for the first module. The following figures (Fig. 10, 11) give one example for each module.

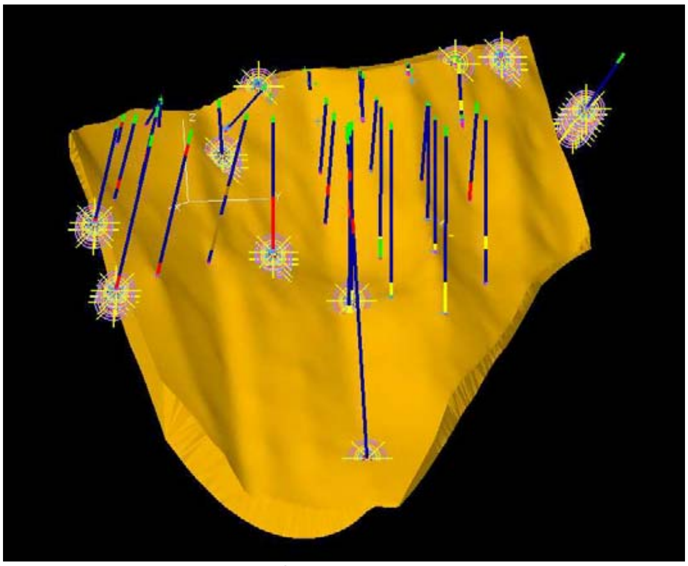

Figure 10. RBF centres of the third module visualised together with the solid model of the orebody and the drillholes.

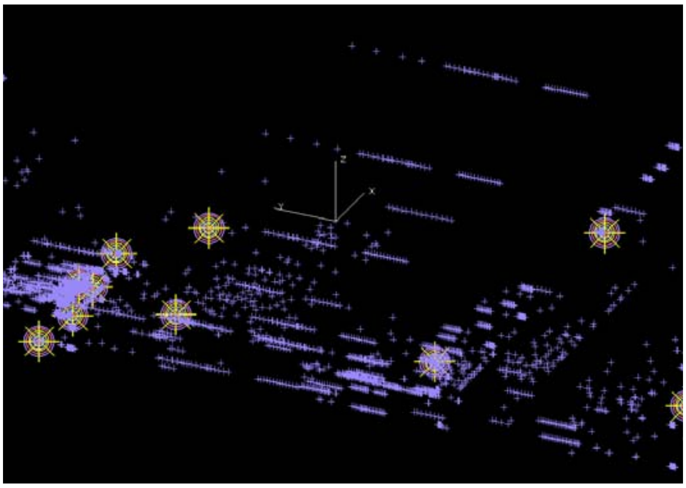

Figure 11. RBF centres from one of the six RBF networks of the first module visualised together with the training samples of that module.

By looking at the positions of the RBF centres, one can decide whether the network initialisation procedure is efficient and whether the learned mapping is reliable. A well spread distribution of centres in the input space with a high density of centres in areas where ore grade seems to present a complex behaviour suggest that the network has been properly developed. High density of centres in areas with very few or even no samples means that the initialisation and training process needs to be modified.

5 CONCLUSIONS

In this paper an in-depth discussion was given on GEMNET II, the integrated system for ore grade estimation based on artificial neural networks. The benefits of the approach were explained and in particular the advantages of the integration with the resources modelling package, VULCAN.

The system has many advanced features that can establish it as a commercial product. It provides validation tools that can help build confidence to the estimates while it removes most of the problems found in other grade estimation techniques. GEMNET II makes very few assumptions about the grade distribution. Its operation does not depend on the user's knowledge of geology, geostatistics, or even neural networks. It should be noted though that knowledge of neural networks could improve sometimes the results but not significantly. Generally, the system adjusts to the data presented to it to achieve the best possible estimation.

Even though it is based on artificial neural networks, GEMNET II is not a 'black box' approach. The technique is fairly understandable as it is based on established principles of ore grade spatial behaviour. The validation tools provided with GEMNET II and the exhaustive monitoring of the network development also help the user to understand how it works and why.

In the four case studies presented briefly in this paper, GEMNET II performed well in comparison with the other already established techniques. The results obtained have shown that it is a reliable and fast grade estimation system. GEMNET II has shown its potential as a valid alternative that can handle large amounts of data quickly and without being prone to extreme values.

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