Distributed Arithmetic Theory

Jamil Khatib

May 23, 2000

1 Introduction

Distributed arithmetic is a computation algorithm based on Look-up table (LUT). It performs the multiplication using the LUT. It is used for sum of products calculations "multiply accumulate (MAC)" such as dot products and digital filters.

2 Derivation

The sum of products calculations can be represented by:

$$y(n) = \sum_{k=1}^{K} A_k X_k(n) \tag{1}$$

Where

y(n):	result at time n
$X_k(n)$:	kth input at time n
A_k:	kth constant factor

Assume unsigned binary representation of the numbers, so X_k can be represented by equation 2

$$X_k = \sum_{b=0}^{B-1} X_{xb} 2^b \tag{2}$$

so by substituting equation 2 in 1 we get:

$$y = \sum_{k=1}^{K} A_k [\sum_{b=0}^{B-1} X_{xb} 2^b] = \sum_{k=1}^{K} \sum_{b=0}^{K} x_{kb} A_k 2^b \tag{3}$$

$$
\begin{array}{ccccc}
x_{10}A_1+ & x_{20}A_2+ & ...+ & x_{K0}A_K & \\
x_{11}A_1+ & x_{21}A_2+ & ...+ & x_{K1}A_K & 2^1 \\
... & ... & ... & ... & ... \\
x_{1(B-1)}A_1+ & x_{2(B-1)}A_2 + & ...+ & x_{K(B-1)}A_K & 2^(B-1)
\end{array}
$$

This equation 3 represents AND operation between a bit of the input and all bits of the constant. The sum represents addition and 2^b represents shifting operation. A table that is addressed with the inputs bits can be used to store sum of terms.
The contents of the LUT represents the result of the partial sum and anding operation with the input bits(address)

Address (inputs)	value (partial sum)
000	0
001	C0
010	C1
011	C0+C1
100	C2
101	C0+C2
110	C1+C2
111	C0+C1+C2

References

[1] The rule of distributed arithmetic in FPGA-based signal processing "http://www.xilinx.com"

[2] The scientest and engineer's guide to Digital Signal Processing by Steven Smith "http://www.'