LES of Mixing Layer and Flow in Square Duct by the Second-Order Explicit Scheme

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Applicability of the 3D Cartesian-grid finite volume scheme to compressible LES is investigated. Main features of this scheme are 1) non-iterative approximate Riemann solver based on linearized differential relations along characteristics of one-dimensional hyperbolic conservation laws and 2) piecewise-parabolic reconstruction used in inviscid fluxes evaluation procedure. Simulation results for two simple test cases are presented: nearly uniform density mixing layer and turbulent flow in square duct. A good agreement with experimental data is demonstrated.

INTRODUCTION

The text below is a brief description of 3D Cartesian grid finite-volume scheme for compressible unsteady flow and its application to LES of two simple test cases. The main features of this scheme are:

- non-iterative approximate Riemann solver based on linearized differential relations along characteristics of one-dimensional hyperbolic conservation laws;
- piecewise-parabolic reconstruction used in inviscid fluxes evaluation procedure.

MATHEMATICAL MODEL

The governing equations are filtered compressible Navier-Stokes equations. After Favre filtering, dropping nonlinear terms, assuming subgrid turbulence behavior to be analogous to molecular dissipative mechanism, we obtain the following approximation to the filtered equations (in tensor notation):

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0, \\
&\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j u_i) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\sigma_{ij}, \\
&\frac{\partial \rho E}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j E + p u_j) = \frac{\partial}{\partial x_j} q_j + \frac{\partial}{\partial x_j}(u_i \sigma_{ij}),
\end{aligned} \tag{1}$$

$$\text{Here } \sigma_{ij} = \mu_{eff}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\left(\frac{\partial u_k}{\partial x_k}\right)\right) \text{ and } q_j = -\lambda_{eff}\frac{\partial T}{\partial x_j} \tag{2}$$

are effective stress tensor and effective heat flux vector expressed through partial derivatives of resolved quantities. The effective scalar transport coefficients are represented here simply as:

$$\mu_{eff} = \mu_{lam} + \mu_{sgs} \quad \text{and} \quad \lambda_{eff} = \lambda_{lam} + \lambda_{sgs} = \frac{\mu_{lam} c_p}{\mathrm{Pr}_{lam}} + \frac{\mu_{sgs} c_p}{\mathrm{Pr}_{sgs}},$$

where Pr_{sgs} is assumed to be constant and μ_{sgs} is modeled using constant coefficient Smagorinsky SGS model [1]:

$$\mu_T = 2C\rho\Delta^2 \left|\bar{S}\right|, \tag{3}$$

where $C = C_s^2$, $C_s = 0.1$,

Δ – spatial filter width (equal to cell size),

$$\left|\bar{S}\right| = \sqrt{2S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

NUMERICAL SCHEME

System (1) in *vector* notation reads:

$$\frac{\partial \bar{U}}{\partial t} + \frac{\partial \bar{F}}{\partial x} + \frac{\partial \bar{G}}{\partial y} + \frac{\partial \bar{H}}{\partial z} = 0 \tag{4}$$

Spatial derivatives of fluxes in (4) for Cartesian grid can be expressed by spatial finite differencing operator that retains conservation property:

$$L_h(\bar{U}^n_{i,j,k}) = \frac{\bar{F}^n_{i-\frac{1}{2},j,k} - \bar{F}^n_{i+\frac{1}{2},j,k}}{\Delta x} + \frac{\bar{G}^n_{i,j-\frac{1}{2},k} - \bar{G}^n_{i,j+\frac{1}{2},k}}{\Delta y} + \frac{\bar{H}^n_{i,j,k-\frac{1}{2}} - \bar{H}^n_{i,j,k+\frac{1}{2}}}{\Delta z}. \tag{5}$$

Inviscid part of fluxes in (5) is evaluated using monotone piecewise-parabolic approximation [2] via diagonalizing transformation of matrices 5*5 (see also [3]):

$[A] = [S^{-1}][\Lambda][S]$,

where [A] – Jacobian matrix,

[Λ] – diagonal matrix of eigenvalues of [A],

and linearized characteristic-based approximate Riemann solver [3]. Viscous fluxes are computed from the second order in space central differences.

Computations are advanced in time using two-step Runge-Kutta scheme [4].

$$\overline{U}^{(1)}_{i,j,k} = \overline{U}^{n}_{i,j,k} + \Delta t L_h(\overline{U}^{n}_{i,j,k}),$$
$$\overline{U}^{n+1}_{i,j,k} = \tfrac{1}{2}[\overline{U}^{n}_{i,j,k} + \overline{U}^{(1)}_{i,j,k} + \Delta t L_h(\overline{U}^{(1)}_{i,j,k})].$$

The results shown below are obtained with this time integration scheme. A minor modification to the source code can convert it to three- or higher order in time method.

FIRST TEST CASE: MIXING LAYER

Air flow in a planar turbulent mixing layer, experimentally studied in [5], was simulated on the grid of 120, 90 and 41 cells in x-, y- and z-directions, respectively. In order to speed up simulation the maximal Mach number was set to 0.53. To sustain turbulent flow regime, velocity field in BC was disturbed randomly in the vicinity of the *splitter* (seen in Fig. 1). Instantaneous fields of resolved vorticity

$$|rot(\vec{V})| = \sqrt{\varpi_x^2 + \varpi_y^2 + \varpi_z^2}\ , \text{ where } \varpi_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right), \text{ etc.,}$$

was plotted (Fig. 1). Also averaged velocity profiles u(y) in selected sections were obtained (Fig. 2). The following averaging rule was used:

$$\widetilde{u} = \frac{\int_{t}^{t+T} \rho(t)u(t)dt}{\int_{t}^{t+T} \rho(t)dt} \approx \frac{\sum_{n=N}^{n=M} \rho^n u^n}{\sum_{n=N}^{n=M} \rho^n}$$

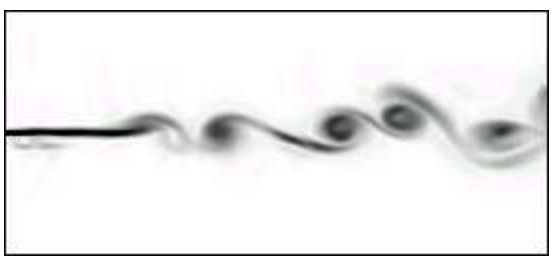

Fig. 1. Instantaneous resolved vorticity plot in a middle plane.

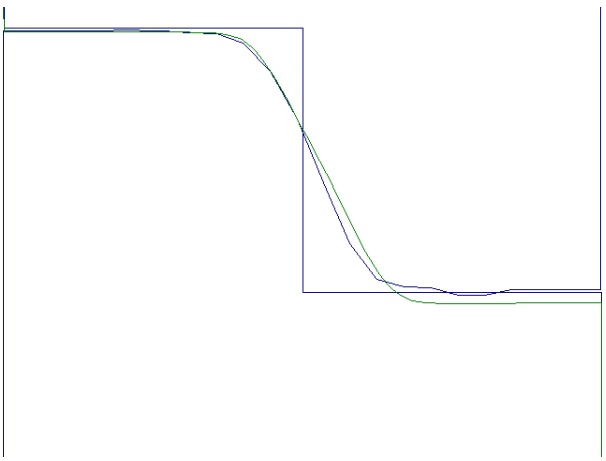

Fig. 2. Time-averaged simulated and measured [5] velocity profiles u(y).

Time-independent averaging was achieved after 18000 time steps, which took 15 days of computation on PowerPC 604e (233MHz) machine under OS AIX v. 4.3.1. As it can be seen, accordance between simulated and measured [5] profiles is quite satisfactory.

SECOND TEST CASE: A FLOW IN SQUARE DUCT

In this case the developed turbulent flow in a square duct at Re = 4800 was modeled. The case was taken from [6]. Similar to [6], following near-wall corrections to the model (3) was adopted:

$$C = 0{,}01 \cdot (1 - \exp(-(d^+ / 25)^3)),$$

where $d^+ = \dfrac{\rho u^* d}{\mu_M}$ – non-dimensional normal distance from the wall, expressed through friction velocity. To account for the corners, d was evaluated from expression [6]:

$$d = \frac{2yz}{y + z + \sqrt{y^2 + z^2}}.$$

In this case the mass force term was added to sustain the fluid motion. After nearly equilibrium flow was achieved (to 12000th time step), averaging of velocity profile and fluxes through some cross-section was started (it lasted during 8.2 characteristic times of large eddy turnover, up to 30000th time step) on the grid of 136, 45 and 45 cells in x-, y- and z-directions, respectively.

It was revealed that the averaged Re number was about 4866 while averaged Mach number was about 0.5. The computed mean wall friction was 4.3% less than from Nikuradse's formula for this Re number. As it can be seen from Fig. 4, aver-

aged velocity profile is rather symmetrical and looks fine.

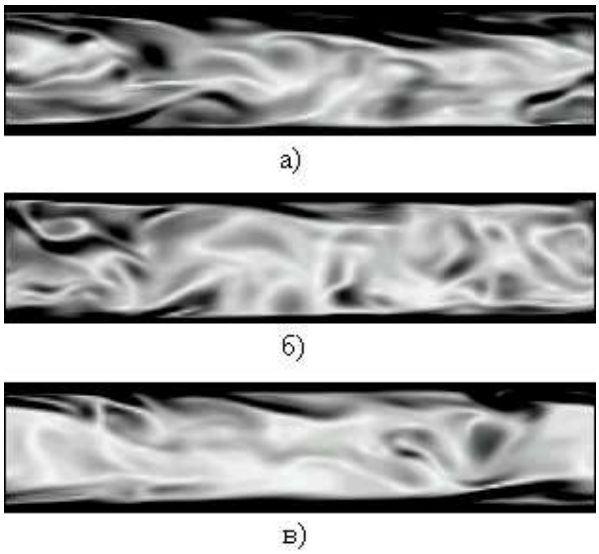

Fig. 3. Instantaneous vorticity fields in a middle plane of the square duct.

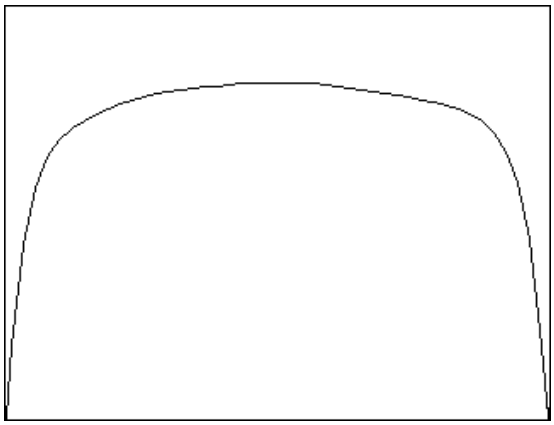

Fig. 4. Simulated averaged velocity in a middle plane.

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