

# A New Wide-Band Amplifier Technique

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**Abstract**—Precision dc-coupled amplifiers having risetimes of less than a nanosecond have recently been fabricated using the monolithic planar process. The design is based on a simple technique that has a broad range of applications and is characterized by a stage gain accurately determined by the ratio of two currents, a stage-gain-bandwidth product essentially equal to that of the transistors, and a very linear transfer characteristic, free from temperature dependence.

## I. INTRODUCTION

AMPLIFIERS having wide bandwidths, in excess of 100 MHz, with low amplitude distortion and stable gain, are of increasing importance in modern electronics. By far the most prevalent circuit configuration is the emitter-degenerated amplifier, which suffers a familiar problem: the current- and temperature-dependent emitter impedance causes distortion [1], [2] and gain instability. Negative feedback techniques over more than two stages are rarely applicable at these bandwidths due to the excess phase shift around the loop.

A further characteristic of such amplifiers is that voltage swings at the signal frequencies are present throughout the chain, and, consequently, the parasitic capacitances have a part in limiting the bandwidth. This problem is especially severe in monolithic (junction-isolated) circuits. In circuits using discrete components, inductive elements are often employed [3] to improve the bandwidth, but no satisfactory way of incorporating inductive elements in a monolithic circuit is available. Another approach is to make these parasitic capacitances part of a lumped constant delay line (the distributed amplifier [4]), but this, too, is not applicable to microcircuits. Interestingly, however, some of the amplifiers described later behave very much like distributed amplifiers in that each stage works at an  $f_t$ -limited bandwidth, and all stages contribute to the total output capability.

Suggestions have been made from time to time [5]–[7] to include compensating diodes in the collector load circuit to mitigate the nonlinearities introduced by the emitter diode, or even eliminate the linear impedance at this point altogether [7]. This is a useful technique, but has several limitations, and the motivation is still to produce a stage having *voltage gain*.

The main objective of the work reported here was

to develop a cascadable circuit form (a “gain cell”) that could provide dc-coupled temperature-insensitive sub-nanosecond *current gain* with the virtual absence of voltage swings, and a theoretically perfect transfer characteristic, having constant slope between the upper and lower overload limits. Conversion between voltage and current can then be made only where needed—at the input and output terminals.

It became apparent that, in addition to meeting these goals, the principle developed had several other useful properties. For example, the gain of each stage can be electronically controlled with precision over a wide range at nanosecond speeds, a property exploited in a new four-quadrant multiplier [8], [9]. The analysis also shows that the stage gain of certain configurations is independent of beta, even at gains *close to beta* in magnitude. This is something that, to the author’s knowledge, no other configuration permits.

## II. FOUNDATIONS OF THE TECHNIQUE

Two very common circuits, shown in Fig. 1, were married to produce a new configuration. The first, (a), is the “differential pair,” now widely used as a multiplier [10]–[13]. The second, (b), is the “current source” found in practically every linear IC. The married couple are shown in Fig. 3; this marriage has proven unexpectedly fruitful, and the prolific offspring will be described in this and later papers.

First, we will consider Fig. 1(a) in more detail. It can be used as a multiplier because the transconductance from base to collector is *proportional to the emitter tail current*  $I_E$ . However, it is far from being a precise element, because this transconductance is both nonlinear and temperature dependent.

In this paper, we will frequently use the junction-diode expressions [14].

$$I = I_s \exp \frac{qV}{mkT} \quad \text{or} \quad V = \frac{mkT}{q} \log \frac{I}{I_s} \quad (1)$$

where

- $I$  = forward conduction current,  $\gg I_s$
- $I_s$  = reverse saturation current
- $V$  = voltage across the junction
- $q$  = charge on the electron
- $m$  = a constant near unity [14]
- $T$  = absolute temperature.

The quantity  $mkT/q$  is about 26 mV at 300°K. Applying (1) to Fig. 1(a), and solving for the variable

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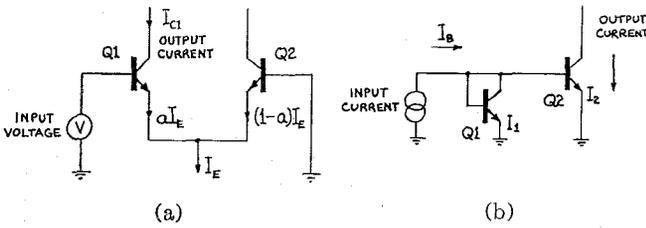


Fig. 1. Two common circuits. (a) The "differential amplifier." (b) The "current source."

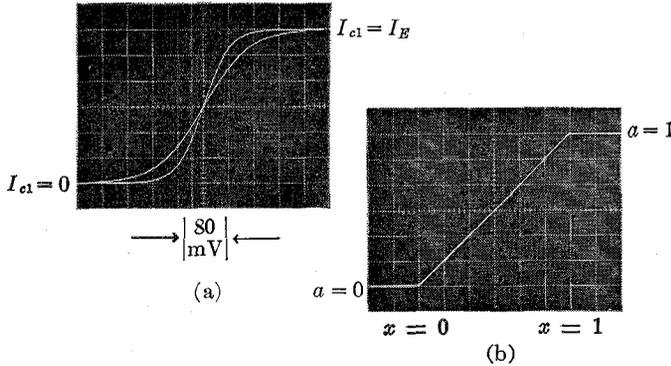


Fig. 2. Comparison of transfer curves for (a) Fig. 1(a) amplifier; (b) new circuit shown in Fig. 3. In each case, the curves for  $-50^\circ\text{C}$  and  $+125^\circ\text{C}$  are compared; in (b), they are virtually co-incident.

we find

$$a = \frac{\exp \frac{qV}{mkT}}{\lambda + \exp \frac{qV}{mkT}}, \quad (2)$$

independent of  $I_B$ , where

$$\lambda = \frac{I_{s2}}{I_{s1}} = \frac{\text{area of } Q2 \text{ emitter}}{\text{area of } Q1 \text{ emitter}}, \quad (3)$$

and  $I_{s1}$ , and  $I_{s2}$  are the reverse saturation currents of the emitter diodes of Q1 and Q2. This area ratio is more usually expressed as an equivalent offset voltage,

$$V_0 = \frac{mkT}{q} \log \lambda, \quad \text{or} \quad \lambda = \exp \frac{qV_0}{mkT}. \quad (4)$$

Thus, in terms of  $V_0$ , we have

$$I_{c1} \simeq aI_E = \frac{I_E}{1 + \exp \frac{q}{mkT} (V_0 - V)}. \quad (5)$$

Characteristic transfer curves are shown in Fig. 2(a) for two temperatures. The transconductance is clearly a nonlinear, temperature-dependent quantity. Notice that an emitter-area mismatch merely shifts the transfer curve by an amount  $V_0$ , without changing its shape. This matter of area mismatch will be raised again in connection with the improved circuits, where the effects are very different.

The circuit of Fig. 1(b) can also be used as an amplifier, in which case the signal input is the current  $I_B$ . With

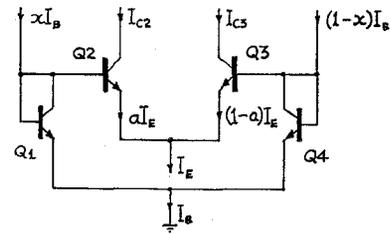


Fig. 3. The new circuit form. The text shows that the factors  $a$  and  $x$  are equal, for any  $I_B$  and at all temperatures.

reference to the figure it will be apparent that

$$\frac{mkT}{q} \log \frac{I_1}{I_{s1}} = \frac{mkT}{q} \log \frac{I_2}{I_{s2}} \quad (6)$$

or

$$I_2 = \frac{I_{s2}}{I_{s1}} \cdot I_1 = \lambda I_1. \quad (7)$$

This is a linear, temperature-insensitive relationship. By scaling the area of Q2 emitter relative to that of Q1, a stage gain can be realized; now emitter-area mismatches cause a gain-error, but do not jeopardize linearity.

Amplifiers built from directly cascaded stages like this have been successfully tested, and have the same high-speed properties as the circuits to be described—there is very little voltage swing, and, hence,  $f_t$  is the dominant factor in determining bandwidth. However, they lack the advantages of the differential configuration and have a fixed gain.

### III. THE IMPROVED CIRCUIT PRINCIPLE

We can now examine Fig. 3, which can be considered as a differential amplifier in which the base-drive voltages are derived from a pair of junctions that, like the diode of Fig. 1(b), are current-driven. However, in this case there are two drive currents (the signal input), which are of the form

$$I_{B1} = xI_B \quad (8)$$

$$I_{B2} = (1-x)I_B$$

where  $0 < x < 1$ , and will be termed the "modulation index" of the bias current  $I_B$ .

Temporarily ignoring the effects of junction area differences, finite beta and ohmic resistances, and summing the emitter voltages around the Q1-Q4 loop, we have

$$\begin{aligned} \frac{mkT}{q} \log \frac{xI_B}{I_s} - \frac{mkT}{q} \log \frac{aI_E}{I_s} \\ - \frac{mkT}{q} \log \frac{(1-x)I_B}{I_s} + \frac{mkT}{q} \log \frac{(1-a)I_E}{I_s} = 0 \end{aligned} \quad (9)$$

which collapses to

$$a = x. \quad (10)$$

Thus, the magnitudes of the output currents are simply

$$I_{c2} = xI_E \quad (11)$$

$$I_{c3} = (1-x)I_E,$$

and the dc stage gain is

$$G_o = \frac{I_{C2}}{I_{B1}} = \frac{I_E}{I_B}. \quad (12)$$

Typical transfer curves for this circuit are shown in Fig. 2(b).

Notice the extreme insensitivity to temperature and the sharp overload points, making the whole dynamic range useful.<sup>1</sup> We will now discuss the effects of departures from the simple theory due to area mismatch, ohmic resistance, and beta.

#### A. Area Mismatches

Equation (9) assumed all the diodes had equal areas, hence, the same  $I_s$ . In practice, however, this will never be exactly the case. We will define the variable

$$\gamma = \frac{I_{s2}I_{s4}}{I_{s1}I_{s3}}. \quad (13)$$

Reevaluating (9) to include  $\gamma$ , we find that

$$a = \frac{x\gamma}{1 + x(\gamma - 1)} \quad (14)$$

which is *no longer linear* with respect to  $x$ , unless  $\gamma = 1$ , that is, unless the emitter areas of the inner and outer pairs of transistors are *mutually equal*.

Since we are concerned with achieving low distortion in the transfer function, the effects of even small departures from the ideal case must be examined. From (14),

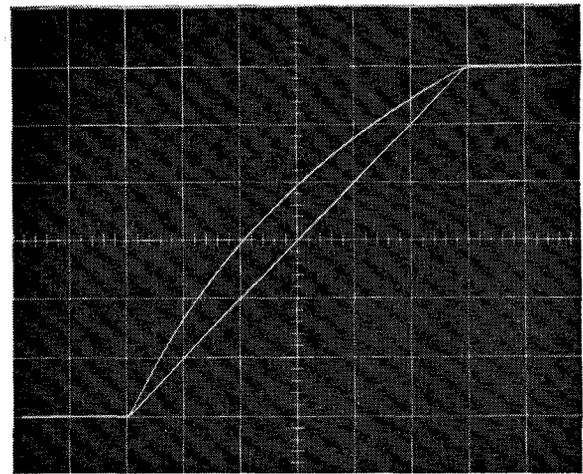
$$\frac{da}{dx} = \frac{\gamma}{[1 + (\gamma - 1)x]^2}. \quad (15)$$

At the extremes of the dynamic range,  $x = 0$  and 1. The incremental slopes at these points are thus  $\gamma$  and  $1/\gamma$ , respectively. To a first approximation, then, small departures from  $\gamma = 1$  cause the slope of the transfer curve to vary linearly over the dynamic range. For example, when  $\gamma = 1.1$ , corresponding to a total offset around the Q1-Q4 loop of 2.7 mV at 300°K, the gain will be 10 percent low at  $x = 0$ , rising to 10 percent high at  $x = 1$ .

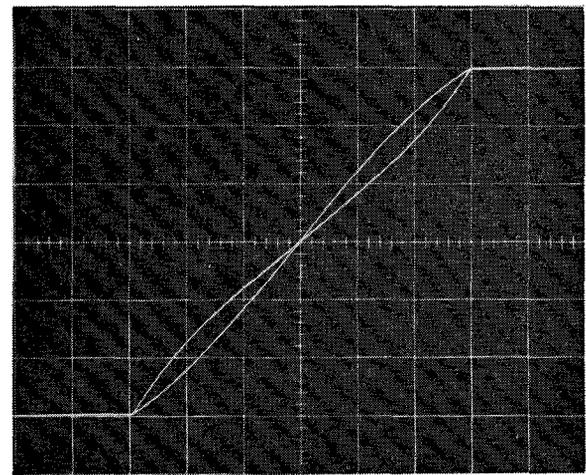
The distortion caused by  $\gamma = 2$  (for example, one of the four emitters has an area twice that of the others) is shown in Fig. 4(a). To confirm that area ratios are synonymous with offset voltages, this photograph also shows that linearity can be restored by introducing an error voltage of 18 mV (26 mV times log 2) into the loop.

It might seem that serious distortion would arise in this kind of amplifier due to the inevitable mismatches,

<sup>1</sup>These results were obtained using a single-sided input configuration (see Section III-G) driven from the oscilloscope sweep output of 100-volt amplitude via a large resistor. The input voltage variations over the dynamic range were thus negligible, and a true current-drive condition obtained.



(a)



(b)

Fig. 4. Demonstration of distortions caused by (a) area mismatch ( $\gamma = 2$ ), linearity is restored by a correction voltage; (b) ohmic resistances.

but measurements are surprisingly reassuring. Slope variations of less than  $\pm 2$  percent are typical, and intermodulation products of  $-60$  dB have been measured for two equal signals at 500 kHz having a peak-combined amplitude of 80 percent of the dynamic range. It should be pointed out that prevalent differential amplifiers of similar gain and bandwidth exhibit far greater distortion for *full-scale* signal swings.

#### B. Ohmic Resistances

In addition to the diode voltages described by (1) there will be components of voltage due to the ohmic (bulk) resistances of the base and emitter diffusions, metalization paths, and contact interfaces. These resistances are also current dependent due to crowding effects and to an extent dependent on the device geometry.

All of these resistive components can be referred to the emitter circuit as an *equivalent ohmic emitter resistance*. Also, we will assume that this resistance scales with emitter area (a safe assumption for many "stripe" geometries). With these resistances inserted into the

circuit of Fig. 3 and (9) suitably modified, we find

$$\frac{mkT}{q} \log \left\{ \frac{x(1-a)}{(1-x)a} \right\} = R_E \{ AI_B(1-2x) - I_E(1-2a) \} \quad (16)$$

where  $R_E$  = equivalent ohmic emitter resistance of each inner transistor

$A$  = area ratio of inner to outer pairs of emitters.

When the right-hand side of this equation vanishes,  $x = a$ . This occurs when  $A = G_0 = I_E/I_B$ , that is, *the areas are scaled in proportion to the drive currents*. If  $A \neq G_0$ , distortion will arise. Equation (16) has no general explicit solution for  $a$ , but a good estimate of the magnitude of the distortion can be readily obtained by a small-signal analysis.<sup>2</sup>

At the quiescent point ( $x = 0.5$ ), the mean normalized slope of the transfer curve is

$$\left. \frac{da}{dx} \right|_{0.5} = \frac{\frac{mkT}{q} + AR_E I_B}{\frac{mkT}{q} + R_E I_E} \quad (17)$$

Also, putting  $x = 0$ ,  $x = 1$  in (16) yields  $a = 0$ ,  $a = 1$ , respectively. Therefore, the *mean* normalized slope is still unity. Thus, when  $A > G_0$  (area ratio too big) the transfer curve will have a *higher* slope at the center than at the extremes, and vice versa. Fig. 4(b) shows transfer curves for  $I_B = 6$  mA,  $I_E = 6$  mA  $\pm$  3 mA using four small-geometry transistors of the same emitter area.

### C. Effects of Beta

For monolithic transistors, the match of the current gain between adjacent transistors is very good and will be assumed to be perfect. A further assumption is that the beta is not a function of  $I_B$  over the current range of interest. In the presence of finite beta, the emitter currents of Q1 and Q4 are modified so that

$$\begin{aligned} I_{B1} &= xI_B - (1 - \bar{\alpha})aI_E \\ I_{B2} &= (1 - x)I_B - (1 - \bar{\alpha})(1 - a)I_E \end{aligned} \quad (18)$$

where  $\bar{\alpha}$  = large-signal common-base dc current gain.

Substituting these values into (9) and reducing the logarithmic terms to products and quotients as before, we have

$$\frac{\{xI_B - (1 - \bar{\alpha})aI_E\}(1 - a)I_E}{\{(1 - x)I_B - (1 - \bar{\alpha})(1 - a)I_E\}aI_E} = 1 \quad (19)$$

which again reduces to  $a = x$ . This is an astonishing result, since it implies that no matter how low the beta is, the gain *to the emitter circuit* is still  $G_0$ . Of course,

<sup>2</sup> Another method of analyzing this kind of distortion is used in [9], where expressions for the form and amplitude of the nonlinearities are derived.

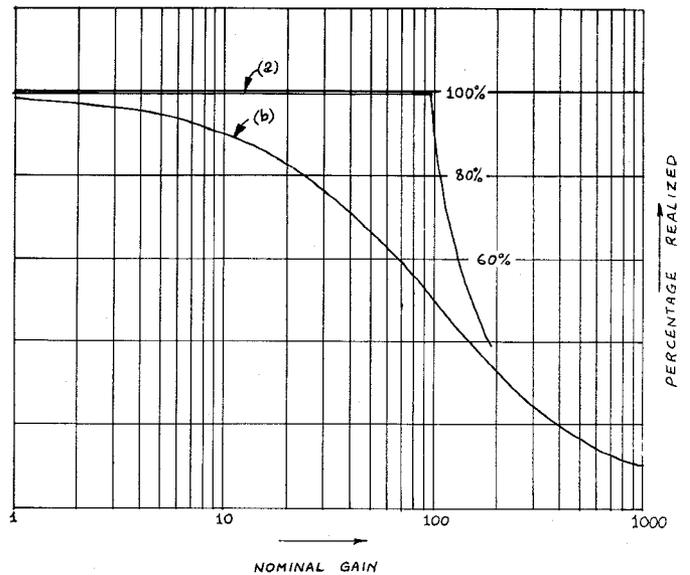


Fig. 5. Gain errors due to beta. Curve (a) shows that accurate gain up to beta is achieved with the new circuit. All prevalent transistor amplifiers have gain errors as shown in curve (b), calculated for  $\beta = 100$ .

$I_B$  must still be sufficient to supply the base currents of Q2 and Q3; any excess flows into Q1 and Q4 to establish the correct drive voltages for linear operation. Since betas of 100 are typical, the gain reduction from emitter to collector is of the order of 1 percent and first order compensation can be readily made.

The significance of the above discussion is that stage gains close to beta in magnitude can be realized with very high accuracy. It will be appreciated that this is not possible with any of the standard circuit forms (including Fig. 1(b)) where the actual gain is given by an expression of the form

$$G'_0 = \frac{\beta}{\beta + (1 + G_0)} \cdot G_0, \quad (20)$$

where  $G_0$  is now the nominal current gain (for example, the ratio of base to emitter impedances in an emitter-degenerated circuit, or the ratio  $\lambda$  in the circuit of Fig. 1(b)).

Measurements on an amplifier like Fig. 3 in which the betas of Q2 and Q3 were 95 showed a gain substantially equal to the ratio  $I_E/I_B$  right up to  $I_E/I_B = 92$ , with no degradation of linearity. The results are shown graphically in Fig. 5.

### D. Collector Saturation Resistance

The input transistors are connected as diodes and thus operate with  $V_{CE} = V_{BE} \approx 0.8$  volts. However, the internal collector bias is reduced to  $V_{BE} - I_C R_{sat}$ , and the transistor will not be a well-behaved diode when this voltage approaches zero. Since  $V_{BE}$  decreases with temperature but  $R_{sat}$  increases, the maximum usable current falls sharply with temperature. Circuits involving high-current operation must use a sufficiently large-geometry device with a buried-layer collector.

### E. Thermal Effects

The analyses so far have assumed that all transistors operate at the same temperature, which need not be the case. For example, the inner pair may be at a higher temperature than the outer pair if they are in a separate package and dissipating considerable power. It can be shown by inserting the appropriate temperatures into (9) that the variable  $a$  becomes

$$a = \frac{x^n}{(1-x)^n + x^n} \quad (21)$$

where

$$n = \frac{\text{temperature of Q1 and Q4 in } ^\circ\text{K}}{\text{temperature of Q2 and Q3 in } ^\circ\text{K}} \quad (22)$$

Another potential source of nonlinearity is thus revealed. A temperature excess of  $60^\circ\text{K}$  will put  $n = 1.2$  and introduce significant distortion, similar in form and magnitude to the effects of bulk resistances shown in Fig. 4(b).

In monolithic structures having close thermal coupling this problem does not arise, but *transient* thermal distortion can be a problem in critical applications. Following a step change of input current, the dissipation of each device in the quad changes, and some time is taken for the circuit to reacquire thermal equilibrium. During this time, the output signal will suffer a transient distortion whose magnitude, waveshape, and duration depend on the circuit-to-sink and transistor-to-transistor thermal impedances, and, of course, the power dissipation.

Measurements show that these effects are typically less than 1 percent of the step amplitude for the circuit being described. The gain cell described later, has less distortion than this.

### F. DC Stability

Equation (14) showed that emitter-area mismatches introduce a shift in the output at the quiescent point ( $x = 0.5$ ), but this shift is not temperature dependent. By adjusting the drive-current balance, the output can be zeroed, and remains this way over the temperature range. This is in contrast with the behavior of prevalent differential amplifiers, where the offset is corrected by a voltage, leaving a drift of  $85 \log \lambda \mu\text{V}/^\circ\text{K}$ .

### G. Unbalanced Drive Currents

The use of *complementary* drive currents is the key to linear large-signal operation of this type of circuit. However, a constant-current *offset* on one (or both) of the inputs will not impair linearity, but there will be a change in gain, and dc unbalance.

An amplitude *ratio* between the two inputs is more serious—for example,  $I_{B1}$  varies from 0 to 1 mA as  $I_{B2}$  varies from 1.5 mA to zero. This is equivalent to an area mismatch, as (14) will reveal, in which case  $\lambda$  represents the input ratio. If the ratio is known, linearity can be restored by an adjustment in the emitter area of one of the transistors.

The need for a pair of complementary currents can be eliminated altogether by connecting the  $(1-x)I_B$  input to ground and supplying a constant current of  $I_B$  to the junction of Q1 and Q4 emitters. Clamping diodes should also be added to the input point to control the input voltage during overload conditions ( $x > 1$  or  $x < 0$ ). This input point is a useful current-summing node close to ground potential since,

$$V_{in} = 26 \log \frac{x}{1-x} \text{ mV at } 300^\circ\text{K} \quad (23)$$

thus

$$|V_{in}| < 36 \text{ mV for } 0.2 < x < 0.8.$$

This circuit no longer possesses the unique beta immunity of the original form and a small offset term ( $a \neq 0.5$  at  $x = 0.5$ ) arises. However, for typical betas, the errors in gain and dc balance are negligible.

## IV. ALTERNATIVE CONFIGURATIONS

Many variants of the basic circuit just discussed have been designed. Space permits only a few of these to be described here, but the generalized statement of the principle, given in the Appendix, will point the way to further forms. These examples will serve to illustrate the versatility of the principle.

### A. Inverted Input Diodes

The input diodes Q1 and Q4 can be inverted and driven from current sinks (instead of current sources), as shown in Fig. 6. Neglecting the effects of beta, area mismatches, and bulk resistances, we can write<sup>3</sup>

$$xI_B a I_E = (1-a)I_E(1-x)I_B \quad (24)$$

or

$$a = 1 - x. \quad (25)$$

The circuit thus has a polarity reversal over the original form. It has the advantage that the input currents are *reusable* at the collectors of Q1 and Q4, a feature that is put to use in the gain cell described later.

This configuration lacks the beta immunity of the original form, because the base current of Q2 and Q3 *add* to the input currents. This, together with the fact that the input that is receiving the *smaller* fraction of the base-current drives the transistor that conducts the *larger* fraction of the emitter current, causes significant beta dependence.

By considering these currents, it is found that the output modulation index  $a$  is reduced to

$$a = \theta(1-x) + \frac{G_0}{\beta} \quad (26)$$

where

$$\theta = \frac{\beta}{\beta + (1 + 2G_0)}. \quad (27)$$

<sup>3</sup> The Appendix shows how these quantities can be equated from an inspection of the circuit.

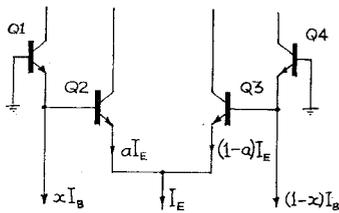


Fig. 6. Inverted form of circuit. Performance is similar, except for phase change in output ( $\alpha = 1 - x$ ).

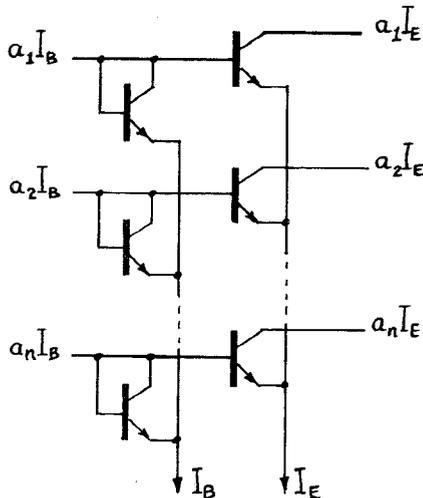


Fig. 7. Multiple-input version of Fig. 3. This circuit is useful to standardize the absolute magnitude of analog signals.

Linearity is not impaired, but the actual stage gain (to the collector circuit) is reduced to

$$G'_0 = \theta G_0, \quad (28)$$

and the output swing is reduced.

For example, with  $G_0 = 2$  and  $\beta = 50$ ,  $G'_0$  is 1.8, 10 percent below the nominal gain, and the limiting values of  $\alpha$  are 0.96 and 0.04.

The "inverted" circuit is identical in behavior to the earlier configuration as regards the effects of area mismatches, ohmic resistances, and temperature. It may also be driven by a single-sided signal, by a similar rearrangement to that suggested for Fig. 3. This time, the collectors and bases of  $Q1$  and  $Q4$  are joined and taken to a positive current source equal to  $I_B$ , and the emitter of  $Q4$  and base of  $Q3$  grounded. The input is then applied to the emitter of  $Q1$ , and must lie in the range 0 to  $I_B$ . Clamping diodes are needed to constrain the input voltage outside of this range.

### B. Multiple-Input Configurations

It is not necessary to limit the input and output to a pair of complementary currents. Fig. 7 shows how  $n$  inputs may be accepted to produce  $n$  outputs. This is useful when the inputs are known to be in a certain ratio but have an absolute value which may vary widely, and it is desired to standardize the signals to a known amplitude. An example might be in connection with lateral  $p$ - $n$ - $p$  transistors as the drivers and level shifters.

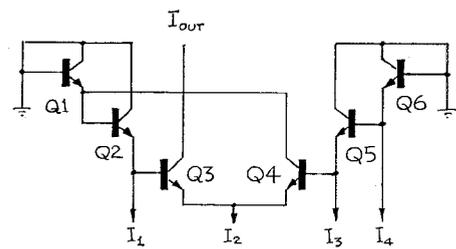


Fig. 8. Product-quotient circuit. By suitable inputs, various functions can be generated.

This circuit also possesses the beta immunity referred to earlier, and can be inverted to the Fig. 6 form.

### C. Product-Quotient Configuration

Fig. 8 shows an interesting variant able to produce an output equal to the products or quotients of several inputs at nanosecond speeds. The significant feature of this circuit is the feedback connection of  $Q4$ 's output to the emitter of  $Q1$ . Neglecting second order effects, we have

$$(I_2 - I_{out})I_1I_{out} = (I_2 - I_{out})I_3I_4 \quad (29)$$

or

$$I_{out} = \frac{I_3I_4}{I_1}.$$

The magnitude of the emitter current,  $I_2$ , although not present in the expression for  $I_{out}$ , determines the maximum value the output may assume.

The circuit may be expanded to accept any odd number of inputs to generate power terms such as  $x^2$ ,  $x^3$ ,  $x^3/yz$ , etc. Of course, there is a practical limit to the circuit complexity determined by the stacking of mismatch errors.

By putting extra diodes in series with the emitters of the center transistors,  $Q3$  and  $Q4$ , and in  $Q1$  and  $Q6$ , we get

$$I_{out} = I_4 \sqrt{\frac{I_3}{I_1}}. \quad (30)$$

The number of circuits that can be devised to perform functions of this kind is legion.

## V. THE "GAIN CELL"

We will now describe what is probably the most useful circuit to have arisen from this work, certainly as far as incorporation into cascaded amplifiers is concerned. It is shown in Fig. 9, and is similar to the "inverted" circuit of Fig. 6, except that the input currents that reappear at the collectors of  $Q1$  and  $Q4$  are added in phase with the outputs of  $Q2$  and  $Q3$ . The gain is thus

$$G''_0 = 1 + G_0 = \frac{(I_B + I_E)}{I_B}. \quad (31)$$

This form is attractive for several reasons. Firstly, the inner stage can operate with a gain less than unity, yet still achieve a net stage gain greater than unity; we

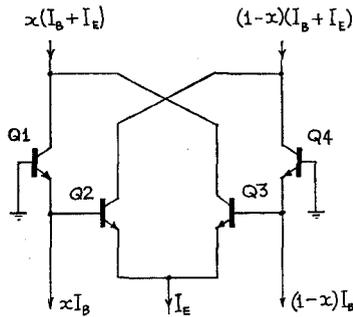


Fig. 9. The "gain cell."

would therefore expect to find this circuit faster for a given gain. Secondly, the cell is well suited to cascading, because the output of one stage can drive the next directly. This leads to a further advantage: all of the current injected at each emitter node contributes to the total output swing. Finally, the bias-voltage circuit for each stage has to supply only the base current for that stage, which is not signal dependent. Hence a low-power diode or resistor string can be used; only 0.5–1 volt per stage is needed.

A. More Exact Gain Expression

Equation (31) omits the effects of beta. It was previously shown that other parameters, such as ohmic resistances and area mismatches, did not affect the mean gain over the full dynamic range, but only introduced distortion. Thus, the accuracy limitations determined by beta are of most concern.

In a cascaded amplifier, the inputs of all stages, except the first, are simply the collector currents from the previous stage. Thus, starting with an input bias current,  $I_B$ , a modulation index,  $x$ , and an emitter supply current  $I_E$ , we can calculate the effective values of  $I'_B$  and  $x'$  for the input to the following stage, the stage gain  $G'''_0$ , and the output swing  $\Delta I_C$ .

The results are

$$\begin{aligned} I'_B &= \alpha I_B + I_E \\ \Delta I_C &= \alpha(I_B + \theta I_E) \\ G'''_0 &= \alpha(1 + \theta G_0) \\ x' &= \frac{1}{2} + \frac{G'''_0}{\alpha + G_0} \left(x - \frac{1}{2}\right). \end{aligned} \tag{32}$$

The practical significance of these expressions is demonstrated by the photographs of Fig. 10, which show the swept gain of a gain cell using (a) transistors with a beta of 40, and (b) transistors with a beta of 200. For these measurements,  $I_B = 200 \mu\text{A}$  and  $I_E$  was swept from 0 to over 1 mA. The input was modulated to a depth of 80 percent at 1 kHz. At more practical operating currents, the effects of bulk resistances must be taken into account, preferably by scaling the emitter areas.

B. Thermal Distortion

In many exacting applications, for example, oscilloscope vertical amplifiers, the transient thermal distortions mentioned earlier can be very disturbing. It can be

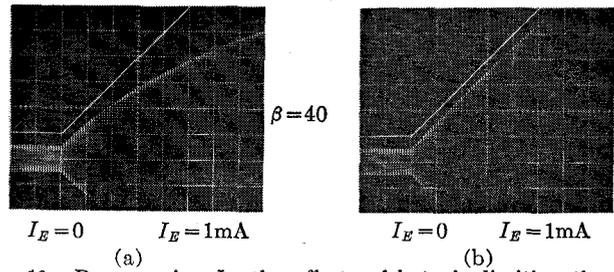


Fig. 10. By sweeping  $I_E$ , the effects of beta in limiting the accuracy of gain are demonstrated.

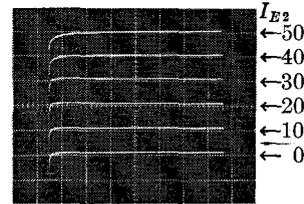


Fig. 11. Thermal distortion of gain cell. Vertical scale expanded to 1 percent/div.  $I_B = 20 \text{ mA}$ .

shown that for the gain cell no thermal distortion arises if the total power dissipated in the outer and inner pairs of transistors is the same. Because the inner pair operates at a higher voltage (by one  $V_{BE}$ ) than the outer pair,  $I_E$  must be less than  $I_B$ , so that to eliminate thermal distortion a gain of less than two must be used.

It can be simply shown that for no distortion

$$I_E = \frac{V_{CB}}{V_{CB} + V_{BE}} \cdot I_B, \tag{33}$$

where  $V_{CB}$  is the bias voltage supplied to each stage and appears across the collector-base junctions of Q1 and Q4. Typically, with  $V_{CB} = 1.6$  volts,  $V_{BE} = 0.8$  volts, we must use a stage gain of 1.67.

In practice, it proves very difficult to induce appreciable distortion even under high-power conditions. Fig. 11 shows the step response of a single integrated gain cell operated at very high currents with the vertical scale expanded to 1 percent per division.

VI. TRANSIENT RESPONSE

An accurate large-signal model for the transistor does not exist, and it is, therefore, not possible to determine the exact step response of this type of circuit. Various small-signal analyses have been made, however, and these indicate, not surprisingly, that the response time is dominated by the  $f_t$  of the devices. Further, if the geometries are scaled to the currents, the effects of  $r_b$  can be eliminated, and for fairly large stage-gains (greater than two or three), an adequate approximation for the stage response of the Fig. 3 circuit is obtained by treating it as a network with a single pole at  $f_t/G_0$ .

The gain-cell response is more complex, and can be approximated by a pole at  $f_t/2$ , flanked by a pole-zero pair. For  $G_0 = 1$ , ( $G''_0 = 2$ ), the pole pair are co-incident and cancel, leaving a single-pole response with a gain-bandwidth product equal to  $f_t$ . For  $G_0 < 1$ , the zero moves toward the origin, while the second pole moves out,

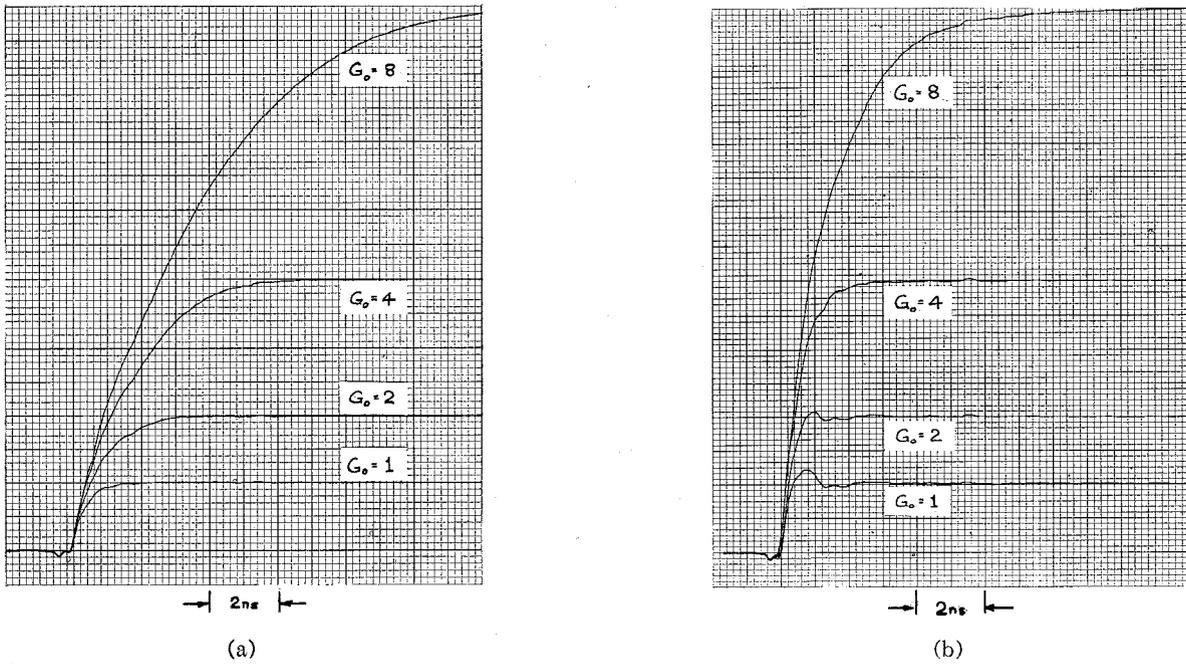


Fig. 12. Measured transient responses of Fig. 3 circuit.

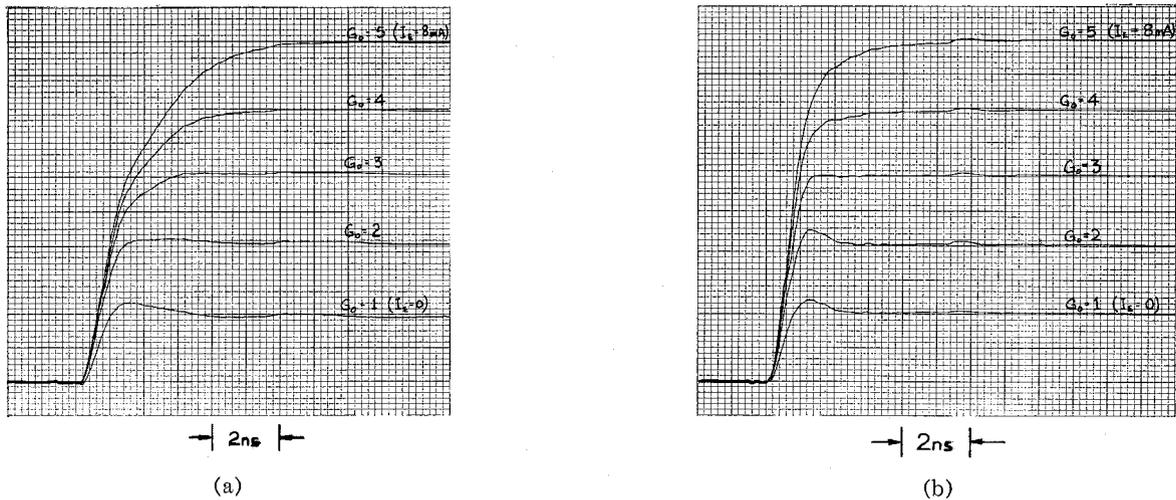


Fig. 13. Measured transient response of the gain cell.

causing an overshoot response. For  $G_0 > 1$ , the reverse situation arises, and the response consists of an initial fast rise followed by a slower time constant. For  $G_0 \gg 1$ , the second pole dominates and is at approximately  $f_t/G_0$ .

Measurements on integrated forms of Fig. 3 and Fig. 9 have been made, using both the standard process (200 ohms-per-square base diffusion) and a shallower 450 ohms-per-square process. The device characteristics are summarized below.

Parameter	200-Ohm Process	450-Ohm Process	Test Conditions
$f_t$	600 MHz	1200 MHz	$V_{CE} = 2$ volts, $I_C = 10$ mA
$C_{CS}$	2 pF	2 pF	$V_{CS} = 10$ volts
$C_{CB}$	0.7 pF	0.7 pF	$V_{CB} = 0$ volts
$\beta$	40	200	$V_{CE} = 2$ volts, $I_C = 5$ mA
$r_b$	15 ohms	27 ohms	$I_{EB} = 10$ mA, $I_C = 0$

Response time was measured in a test system having an overall risetime of less than 75 ps. The waveforms were plotted on an X-Y plotter, which provided better resolution than photography for comparing several collocated responses.

The results confirm the predictions of the small-signal analyses. In Fig. 12, the responses of the Fig. 3 circuits are shown, using (a) the 600-MHz devices, and (b) the 1200 MHz devices. The form of the response closely approximates a single time constant proportional to gain. Fig. 13 shows the more complex "gain cell" response for (a) the 600-MHz devices and (b) the 1200-MHz devices.

It should be mentioned that the test jig incorporated neutralizing capacitances and balun transforms at critical sites to eliminate preshoot and other aberrations in the responses, and those components markedly influenced the

apparent risetimes at the lower gains. The improved performance obtained from the totally integrated amplifier (Section VII-D) supports this conclusion.

## VII. CASCADED AMPLIFIERS

A repetitive problem in dc amplifier design is that of cascading stages without "level shifting," that is, the buildup of supply-voltage requirements as each stage is added. Several techniques are used to overcome this problem. They include the use of

- 1) complementary  $p-n-p-n-p-n$  devices between two supply rails,
- 2) zener-diode or resistive level-shifting circuits,
- 3) very low bias voltages for each stage.

For monolithic designs, the third method is by far the most attractive, but is possible only where voltage swings are small, and hence is eminently suited to the circuits under discussion. Several cascaded forms will be presented that use this approach. In some cases, stages can be cascaded indefinitely without level shifting.

Other factors of interest in a cascaded amplifier are gain stability, drift, overload recovery, linearity, noise level, and transient response. Typical results for these parameters will be given where appropriate.

### A. Bandwidth of Cascaded Stages

It was shown earlier that each stage of an amplifier using the diode transistor circuit can be treated as a single pole at  $f_t/G_0$ , where  $G_0$  is the low-frequency current gain of the stage.

The overall bandwidth,  $F$ , of  $N$  cascaded stages is thus

$$F = \frac{f_t}{G_0} (\sqrt[N]{2} - 1)^{1/2}. \quad (34)$$

Also, if the total current-gain,  $G$ , is shared equally over the  $N$  stages we can write

$$\frac{F}{f_t} = \frac{(\sqrt[N]{2} - 1)^{1/2}}{\sqrt[N]{G}}. \quad (35)$$

This "bandwidth-shrinkage" function is plotted in Fig. 14. It shows three things of interest. Firstly, there is an optimum number of stages required to maximize the bandwidth. Secondly, the maxima are well defined for low gains but vague at high gains. Finally, the bandwidth of a large number of stages is relatively insensitive to overall gain.

In discrete designs, it is frequently necessary to use less than the optimum number of stages for economic reasons. The possibility of integrating the entire amplifier removes this limitation.

### B. A Practical Cascaded Amplifier

The first cascaded form we will discuss is made by taking the Fig. 3 circuit and connecting the collectors of one stage to the bases of the next, as shown in Fig. 15. The amplifying transistors operate with  $V_{CB} = 0$ , just

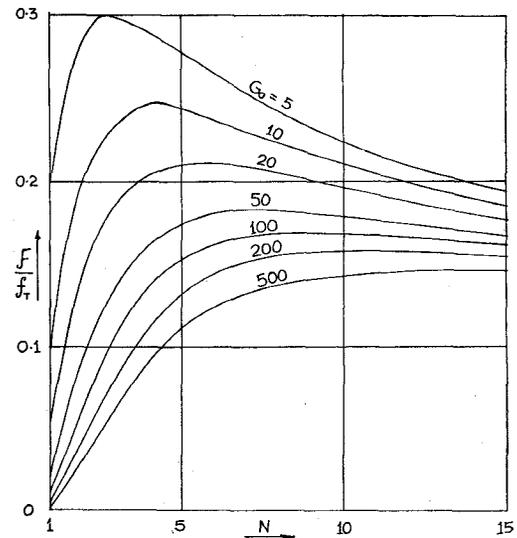


Fig. 14. Curves showing number of stages required to maximize bandwidth.

as the diode-connected transistors do. Currents must be supplied at each interface, and these, together with the emitter currents, determine the gain. The overall gain of  $N$  stages is

$$G = \frac{I_2}{I_1} \cdot \frac{I_4}{I_3 - I_2} \cdot \frac{I_{N+2}}{I_{N+1} - I_N}. \quad (36)$$

Consequently, the gain can be swept over a very wide range (zero to about  $\beta^N$ ), but the gain is a sensitive function of the difference terms in the denominators of (36). Methods of overcoming this problem have been devised, using dependent current sources for the emitters. Diodes (not shown) prevent the transistors saturating at the extremes of the dynamic range.

A limited investigation of this form of amplifier has been made. Some results for a four-stage high-gain (about X1000) design are shown in Fig. 16. They demonstrate the linearity and overload recovery to a  $3\text{-}\mu\text{s}$  ramp (Fig. 16(a)), and the transient response and noise level (Fig. 16(b)). In the second photograph, the responses of 600-MHz and 1200-MHz transistors are compared.

### C. Cascaded Gain Cells

A series of three gain cells is shown in cascade in Fig. 17. The power is supplied as currents (to the emitter nodes) which set both the gain and the output swing capability. A low-power bias string, shown here as pairs of diodes, sets the operating voltage of each stage. The balanced nature of the amplifier results in very small signal currents in the base circuit, and permits a common undecoupled line to supply all the bases in a multistage amplifier. The overall gain, for sensible values of beta, is

$$G' = \frac{I_{E1} + I_{E2} + I_{E3} + I_{E4}}{I_{E1}}. \quad (37)$$

With this method of cascading, the correct static bias conditions to handle the steadily increasing signal ampli-

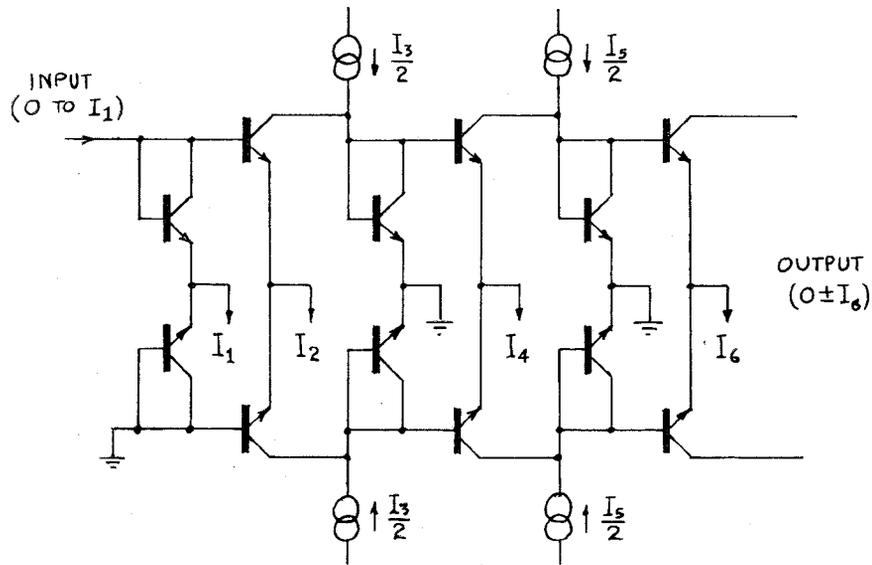


Fig. 15. A cascoded amplifier.

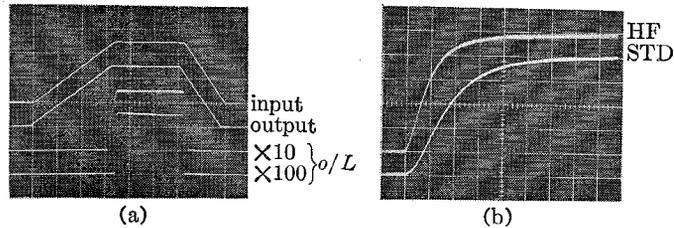


Fig. 16. Performance of amplifier similar to Fig. 15. In (a), a 2.5- $\mu$ A input current (top trace) is accurately reproduced at  $\times 1000$  (second trace). When overloaded ten or a hundred times, (lower traces) recovery is rapid. The top portion of the waveform is within the linear range. Time scale is 1  $\mu$ s/div. Waveforms (b) show transient response at  $\times 1000$ , to a 1- $\mu$ A input step using 600-MHz (top) and 1200-MHz (bottom) transistors. Time scale is 20 ns/div.

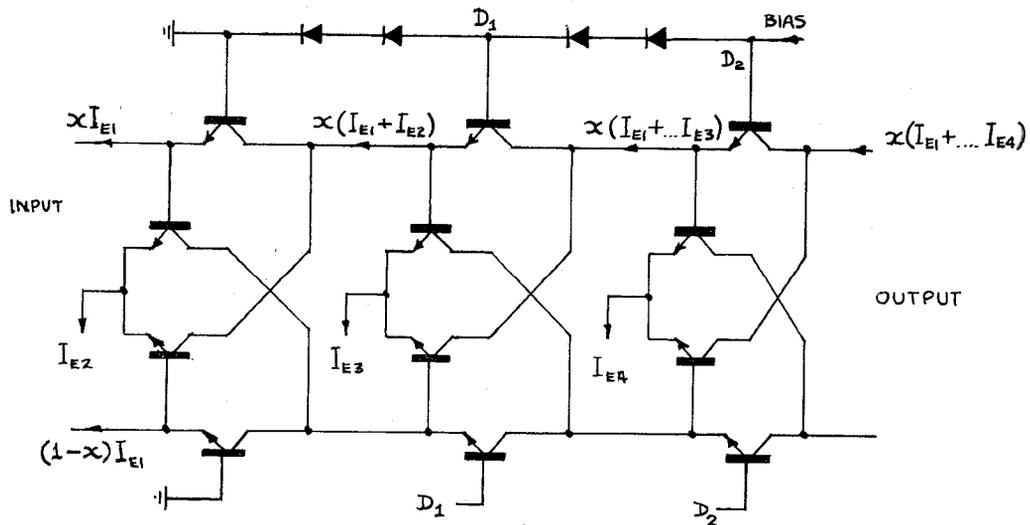


Fig. 17. Cascaded gain cells.

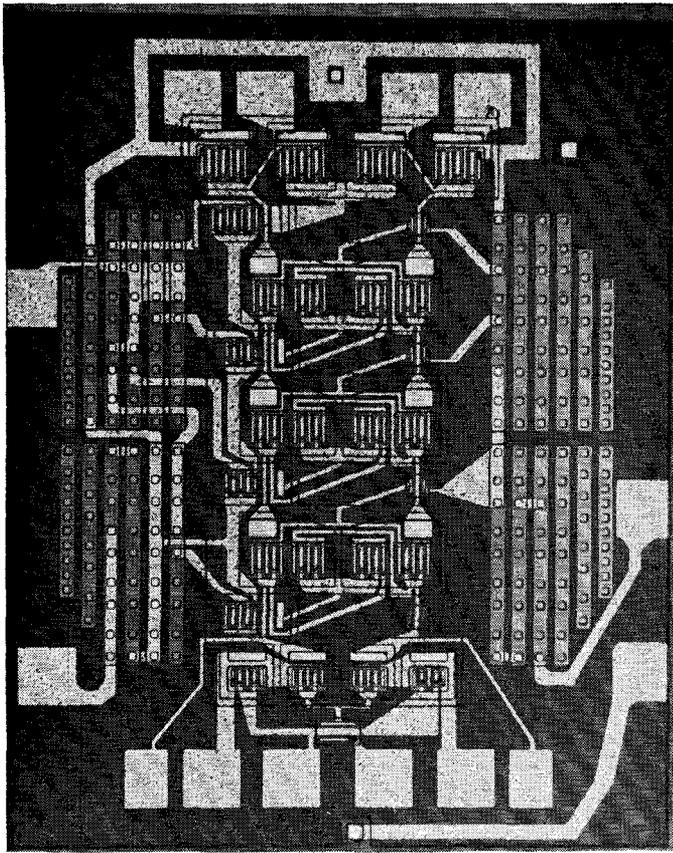


Fig. 18. Photomicrograph of integrated gain-cell amplifier. The emitter currents are supplied by the transistors just left of center, and the signal path is from bottom to top.

tude along the chain are automatically present, because *signal and bias currents increase at the same rate*. This is in contrast with conventional amplifiers where care must be taken to provide adequate range at each stage.

A corollary of this is that all stages overload at the same point on the overall transfer characteristic, when  $x < 0$  or  $x > 1$ . Furthermore, at overload, no transistors saturate, and, because of the modulation reduction discussed above, the input to each successive stage never quite drops to zero. Consequently, the recovery characteristics are good.

#### D. An Integrated Gain-Cell Amplifier

Fig. 18 is a photomicrograph of a five-stage amplifier on a  $50 \times 60$  mil die, using the 1200-MHz transistors. It comprises an input stage of the type shown in Fig. 3 followed by four cascaded gain-cell stages each with a current-source transistor, and a patchwork of resistors which can be used to set the base bias voltages and emitter currents.

The mask layout is of considerable importance in achieving optimum performance. For example, in order to maximize the  $f_t$  of each stage, the transistors must be increased in size from input to output, thus maintaining a constant current density. Also, to reduce collector-substrate capacitance, the gain-cell transistors are paired into two common-collector isolations, which also serve to provide a crossover region for the base circuit. Because of the low impedance level of the circuit, meticulous attention to balancing the metallization resistances is essential to achieve low distortion.

In this circuit, the inner and outer transistors have equal area. For minimum distortion due to bulk resistances a stage gain of two must be used, producing an overall gain of 16. (The input stage operates at a gain of unity.) However, quite large variations from this value can be tolerated without introducing significant distortion.

A summary of the performance of the integrated amplifier is given below, and in the waveforms of Fig. 19. The system risetime used for these measurements was 0.4 ns, and, accordingly, some corrections were necessary to the measured risetimes.

The input to the amplifier was supplied by a push-pull 50-ohm pulser driving Q1 and Q4 via 1-k $\Omega$  resistors, to ensure current-drive conditions. Also, both outputs were connected to the sampling oscilloscope via 50-ohm lines terminated at both ends. Under these conditions, no power gain was realized for current gains below 40. It is, however, entirely practical to match both input and output to 50 ohms.

By suitable choice of current ratios, a wide range of gain from zero to  $\times 1000$  could be obtained, although acceptable high-speed performance was possible only over a limited range. Some idea of the effect of gain on transient response is provided by Fig. 19(b) and (c). Considerable care had to be exercised to eliminate ringing and preshoot, particularly in respect to ground-plane

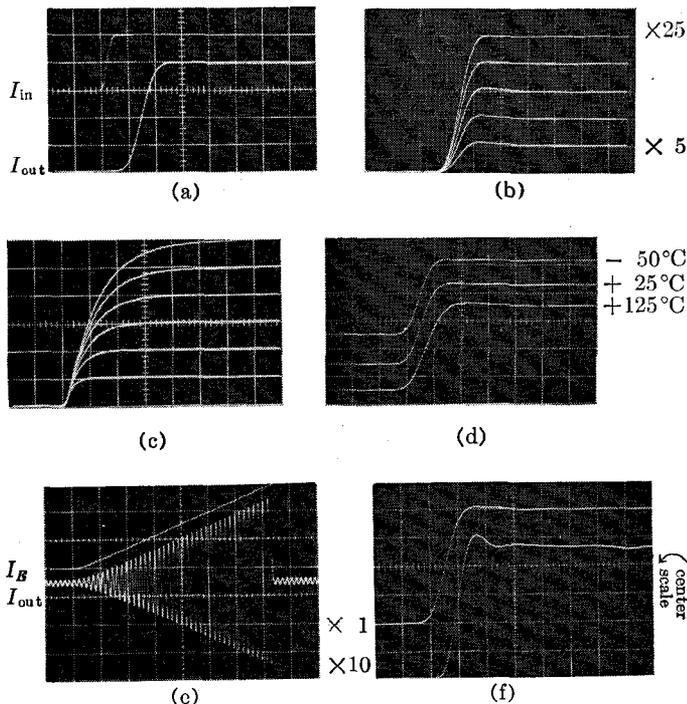


Fig. 19. Performance of the integrated amplifier. (a) Input/output delay at  $\times 20$ , 1 ns/div. (b) Transient-response variations for gains of  $\times 5$  to  $\times 25$ , 1 ns/div. (c) For gains  $\times 50$  to  $\times 300$ , 5 ns/div. (d) Shows transient response for  $\times 25$  at  $-50^\circ\text{C}$ ,  $25^\circ\text{C}$ , and  $125^\circ\text{C}$ . (e) Demonstrates swept-gain linearity,  $\times 1$  to  $\times 30$ . (f) Shows recovery from  $\times 10$  overload.

currents. A bifilar transformer was required to balance the input drive. It is interesting to note the improvement that resulted from putting all the stages on one die. Results published earlier [8] showed a large amount of ringing. These waveforms were for an amplifier in which individually packaged gain cells were breadboarded together. Series damping resistors were required between stages to bring the ringing down to even this level, and these slowed the response.

Here is a summary of the performance of the fully integrated amplifier.

Current Gain	Corrected Risetime (ns)	Overshoot (percent)	Bandwidth (MHz to -3 dB)	Gain-Bandwidth Product (GHz)
5	0.58	12	>500	-
10	0.63	8	>500	-
15	0.69	3	500	7.5
20	0.81	-	420	8.4
25	0.92	-	370	9.2
50	2.0	-	170	8.5
100	4.0	-	92	9.2
200	8.5	-	45	9.0
300	12.0	-	31	9.3

The effect of temperature on transient response was also checked, and, as expected, the risetime decreases at low temperatures, due to increases in  $f_t$ . Also checked were the accuracy of gain in response to a linearly swept current (Fig. 19(e)) and overload recovery (Fig. 19(f)). The latter shows the response for an output equal to 80 percent of the dynamic range, and that for a  $\times 10$  increase in the input.

The CW response is shown in Fig. 20, for current gain from  $\times 5$  to  $\times 50$ , and for a signal equal to 50 percent of the dynamic range. Other performance details, for a gain of  $\times 25$ , are

Output current swing:  $\pm 60$  mA into two 25-ohm loads  
 Output voltage swing: 3 volts peak-peak, differentially  
 Dissipation: 320 mW (64 mA at 5 volts)  
 Gain stability: Within  $\pm 0.35$  dB  
 Zero drift:  $< 0.4$  percent of full scale  $\left\{ \begin{array}{l} \text{over } -35^\circ\text{C} \\ \text{to } +125^\circ\text{C} \end{array} \right.$

These results are very encouraging. Subsequent improvements in device technology will almost certainly permit faster gain-cell amplifiers to be fabricated.

### VIII. SUMMARY

This paper has described what is believed to be a new technique for the design of very linear transistor amplifiers especially suited to monolithic planar fabrication. It demonstrates that useful de-coupled wide-band amplifiers can be made using transistors alone, operating in a strictly current-gain mode. A commercial integrated circuit giving a controllable power gain of 0 to 20 dB with a dc to 500-MHz response, and designed to be cascaded without additional components, is now fully practical. This building-block concept could be expanded

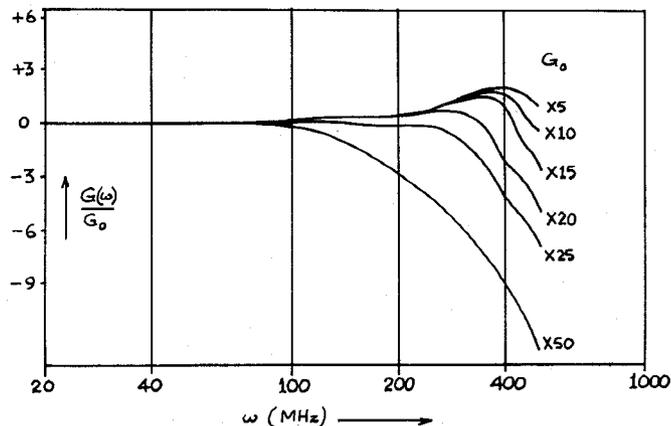


Fig. 20. CW response of integrated amplifier.

to include precision high-impedance input stages, power amplifiers, four-quadrant multipliers, etc., and would fill a long neglected need. Some of these topics will be taken up in subsequent papers.

### APPENDIX

#### The Generalized Principle

The key equation for these circuits is of the form given in (9), an algebraic summation of an even number of logarithmic terms set equal to zero, having a common factor,  $mkT/q$ , which is, thus, of no consequence. The logarithmic arguments may be compounded into a single product-quotient term, and finally the antilogarithm of both sides is taken, leaving the product-quotient term equal to unity, for example

$$\frac{I_{D1}I_{D2}I_{S3}I_{S4}}{I_{S1}I_{S2}I_{D3}I_{D4}} = 1. \quad (38)$$

The saturation currents of the diodes in the loop can be extracted as the ratio

$$\gamma = \frac{\prod I_s(+)}{\prod I_s(-)} \quad (39)$$

where the plus sign indicates all diodes connected in the same direction around the loop, and the minus sign indicates those in the opposite direction. Since  $I_s$  is proportional to area, the  $\gamma$  ratio can also be calculated as the ratio of the areas of the emitter diodes in the loop. Also, every  $I_s$  varies with temperature in the same way, and thus temperature drops out of the analysis at this point, too, explaining the excellent freedom from temperature effects observed in these amplifiers. This reduces the key equation to the form.

$$\prod I(+)=\gamma \prod I(-). \quad (40)$$

Stated at length, in a closed loop containing an even number of perfect exponential diode voltages (not necessarily two-terminal devices) arranged in cancelling pairs, the currents are such that the product of the currents in diodes whose voltage polarities are positive with respect to a node in the loop is exactly proportional to the

product of the currents in diodes whose voltages are negative with respect to that node, the constant of proportionality being the ratio of the product of the saturation currents of the former set of diodes to that of the latter set.

#### ACKNOWLEDGMENT

During the course of this work, the discussions held with L. Larson of the Advanced Instruments Group and G. Wilson of the Integrated Circuits Group proved stimulating and helpful. The experimental work of E. Traa is also gratefully acknowledged.

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## A Precise Four-Quadrant Multiplier with Subnanosecond Response

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**Abstract**—This paper describes a technique for the design of two-signal four-quadrant multipliers, linear on both inputs and useful from dc to an upper frequency very close to the  $f_t$  of the transistors comprising the circuit. The precision of the product is shown to be limited primarily by the matching of the transistors, particularly with reference to emitter-junction areas. Expressions are derived for the nonlinearities due to various causes.

#### I. INTRODUCTION

**A**N IDEAL FOUR-quadrant multiplier would perfectly satisfy the expression

$$Z = \text{constant}, XY \quad (1)$$

for any values of  $X$  and  $Y$ , and produce an output having the correct algebraic sign. Ideally, there would be no limitation on the rate of variation of either input.

All practical multipliers suffer from one or more of the following shortcomings.

- 1) A nonlinear dependence on one or both of the inputs.
- 2) A limited rate of response.

- 3) A residual response to one input when the other is zero (imperfect "null-suppression").
- 4) A scaling constant that varies with temperature and/or supply voltages.
- 5) An equivalent dc offset on one or both of the inputs;
- 6) A dc offset on the output.

In the field of high-accuracy medium-speed multipliers, the "quarter-square" technique has gained favor [1]. This method makes use of the relationship

$$XY = \frac{1}{4}[(X + Y)^2 - (X - Y)^2] \quad (2)$$

and employs elements having bipolar square-law voltage-current characteristics, together with several operational amplifiers.

Much work has been put into harnessing the excellent exponential voltage-current characteristics of the junction diode for multiplier applications, either by using single diodes (or transistors) in conjunction with operational amplifiers [2], or, more recently, pairs of transistors connected as a differential amplifier [3]-[6]. In the majority of cases, the strong temperature dependence of the diode voltage proved a problem, and at least two commercially available multipliers are equipped with an oven to reduce this dependence.